MIXED LOGIT WITH REPEATED CHOICES: HOUSEHOLDS' CHOICES OF APPLIANCE EFFICIENCY LEVEL

David Revelt and Kenneth Train*

Abstract—Mixed logit models, also called random-parameters or error-components logit, are a generalization of standard logit that do not exhibit the restrictive "independence from irrelevant alternatives" property and explicitly account for correlations in unobserved utility over repeated choices by each customer. Mixed logits are estimated for households' choices of appliances under utility-sponsored programs that offer rebates or loans on high-efficiency appliances.

I. Introduction

MIXED logit (also called random-parameters logit) generalizes standard logit by allowing the parameter associated with each observed variable (e.g., its coefficient) to vary randomly across customers. The moments of the distribution of customer-specific parameters are estimated. Variance in the unobserved customer-specific parameters induces correlation over alternatives in the stochastic portion of utility. As a result, mixed logit does not exhibit the restrictive forecasting patterns of standard logit (i.e., it does not exhibit independence from irrelevant alternatives). Mixed logit also allows efficient estimation when there are repeated choices by the same customers, as occurs in our application.

Mixed logits have taken different forms in different applications; their commonality arises in the integration of the logit formula over the distribution of unobserved random parameters. The early applications (Boyd and Mellman (1980), Cardell and Dunbar (1980)) were restricted to situations in which explanatory variables do not vary over customers, such that the integration, which is computationally intensive, is required for only one "customer" using aggregate share data rather than for each customer in a sample. Advances in computer speed and in our understanding of simulation methods for approximating integrals have allowed estimation of models with explanatory variables varying over customers. Ben-Akiva et al. (1993), Ben-Akiva and Bolduc (1996), Bhat (1996), and Brownstone and Train (1996) apply a mixed-logit specification like that given below but without repeated choices. Other empirical studies (Berkovec and Stern (1991), Bolduc et al. (1993), and Train et al. (1987)) have specified choice probabilities that integrate a logit function over unobserved terms, but with these terms representing something other than random parameters of observed attributes. In all cases except Ben-Akiva et al. (1993) and Train et al. (1987), the integration is performed through simulation, similar to that described below. These two exceptions used quadrature, which was feasible in their cases because only one- or two-dimensional integration was required in their specifications.

Terminology for these models varies. "Random-coefficients logit" or "random-parameters logit" has been used for obvious reasons (Ben-Akiva and Lerman (1985), Bhat (1996), Train (1996)). The term "error-components logit" is useful since it emphasizes the fact that the unobserved portion of utility consists of several components and that these components can be specified to provide realistic substitution patterns rather than to represent random parameters *per se* (Brownstone and Train, 1996). "Mixed logit" reflects the fact that the choice probability is a mixture of logits with a specified mixing distribution (Brownstone and Train (forthcoming), McFadden and Train (1997), Train (forthcoming)). This term encompasses any interpretation that is consistent with the functional form. We use "mixed logit" in the current paper because of this generality, even though our specification is motivated through a random-parameters concept. Ben-Akiva and Bolduc (1996) use the term "probit with a logit kernel" to describe models where the customer-specific parameters are normally distributed. This term is instructive since it points out that the distinction between pure probits (in which utility is normally distributed) and mixed logits with normally distributed parameters is conceptually minor.

II. Specification

A person faces a choice among the alternatives in set J in each of T time periods or choice situations. The number of choice situations can vary over people, and the choice set can vary over people and choice situations. The utility that person n obtains from alternative j in choice situation t is $U_{njt} = \beta'_n x_{njt} + \epsilon_{njt}$ where x_{njt} is a vector of observed variables, coefficient vector β_n is unobserved for each n and varies in the population with density $f(\beta_n | \theta^*)$ where θ^* are the (true) parameters of this distribution, and ϵ_{njt} is an unobserved random term that is distributed iid extreme value, independent of β_n and x_{njt}. Conditional on β_n, the probability that person n chooses alternative i in period t is standard logit:

$$L_{nit}(\beta_n) = \frac{e^{\beta'_n X_{nit}}}{\sum_j e^{\beta'_n X_{njt}}}. \tag{1}$$

The unconditional probability is the integral of the conditional probability over all possible values of β_n, which depends on the parameters of the distribution of β_n:

$$Q_{nit}(\theta^*) = \int L_{nit}(\beta_n) f(\beta_n | \theta^*) \, d\beta_n.$$

For maximum likelihood estimation we need the probability of each sampled person's sequence of observed choices. Let $i(n,t)$ denote the alternative that person n chose in period t.

Received for publication March 18, 1996. Revision accepted for publication October 9, 1997.

* University of California, Berkeley.

Conditional on β_n, the probability of person n's observed sequence of choices is the product of standard logits:[1]

$$S_n(\beta_n) = \prod_t L_{ni(n,t)t}(\beta_n).$$

The unconditional probability for the sequence of choices is

$$P_n(\theta^*) = \int S_n(\beta_n) f(\beta_n | \theta^*)\, d\beta_n. \tag{2}$$

Note that there are two concepts of parameters in this description. The coefficient vector β_n is the parameters associated with person n, representing that person's tastes. These tastes vary over people; the density of this distribution has parameters θ^* representing, for example, the mean and covariance of β_n. The goal is to estimate θ^*, that is, thepopulation parameters that describe the distribution of individual parameters.

The log-likelihood function is $\text{LL}(\theta) = \Sigma_n \ln P_n(\theta)$. Exact maximum likelihood estimation is notpossible since the integral in equation (2) cannot be calculated analytically. Instead, we approximate the probability through simulation and maximize the simulated log-likelihood function. In particular, $P_n(\theta)$ is approximated by a summation over randomly chosen values of β_n. For a given value of the parameters θ, a value of β_n is drawn from its distribution. Using this draw of β_n, $S_n(\beta_n)$—the product of standard logits—is calculated. This process is repeated for many draws, and the average of the resulting $S_n(\beta_n)$'s is taken as the approximate choice probability:

$$\text{SP}_n(\theta) = (1/\text{R}) \textstyle\sum_{r=1,\ldots,R} \text{S}_n(\beta_n^{r|\theta})$$

where R is the number of repetitions (i.e., draws of β_n), $\beta_n^{r|\theta}$ is the r-th draw from $f(\beta_n | \theta)$, and $SP_n(\theta)$ is the simulated probability of person n's sequence of choices. By construction $SP_n(\theta)$ is an unbiased estimator of $P_n(\theta)$ whose variance decreases as R increases. It is smooth (i.e., twice-differentiable) which helps in the numerical search for the maximum of the simulated log-likelihood function. It is strictly positive for any realization of the finite R draws, such that the log of the simulated probability is always defined.[2]

The simulated log-likelihood function is constructed as $\text{SLL}(\theta) = \Sigma_n \ln(\text{SP}_n(\theta))$, and the estimated parameters are those that maximize SLL.[3] Lee (1992) and Hajivassiliou and Ruud (1994) derive the asymptotic distribution of the maximum simulated likelihood estimator based on smooth probability simulators with the number of repetitions increasing with sample size. Under regularity conditions, the estimator is consistent and asymptotically normal. When the number of repetitions rises faster than the square root of the number of observations, the estimator is asymptotically equivalent to the maximum likelihood estimator. Note that, even though the simulated probability is an unbiased estimate of the true probability, the log of the simulated probability with fixed number of repetitions is not an unbiased estimate of the log of the true probability. The bias in SLL decreases as the number of repetitions increases. We use 500 repetitions in our estimation.[4]

The simulated score for each person is

$$\begin{aligned}\text{SS}_n(\theta) \equiv \frac{\partial \ln \text{SP}_n(\theta)}{\partial\theta} &= \left[\frac{1}{\text{SP}_n(\theta)}\right]\left[\frac{1}{\text{R}}\right] \\ &\times \sum_r \text{S}_n(\beta_n^{r|\theta}) \\ &\times \left[\sum_t \sum_j (\text{d}_{njt} - \text{L}_{njt}^{r|\theta}) \frac{\partial \beta_n^{r|\theta\prime} \text{X}_{njt}}{\partial\theta}\right]\end{aligned} \tag{3}$$

[1] Our specification assumes that the person's tastes, as represented by β_n, are the same for all choice situations. The model can be generalized to allow the coefficient vector to vary over t as well as n. Our data consist of repeated choices within a survey, such that the assumption of β_n constant over choices seems reasonable.

[2] The simulated probabilities for a sequence of choices sum to one over all possible sequences. Similarly, simulated choice probabilities in each time period (that is, simulated versions of $Q_{nit}(\tau)$) sum to one over alternatives, which is useful in forecasting.

[3] Software to estimate mixed logits is available on K. Train's home page at http://elsa.berkeley.edu/~train.

[4] Other estimation procedures could be applied. The method of simulated moments (MSM) (McFadden, 1989) has the advantage of being consistent with a fixed number of repetitions when the weights in the moment condition are independent of the residuals; however, it is inefficient unless the ideal weights are used. When the ideal weights are simulated with the same draws as the probabilities, then MSM is equivalent to our procedure with maximum simulated likelihood (MSL): the weights and residuals are not independent, and the procedure is not consistent for a fixed number of repetitions. Simulating the weights separately from the probabilities (i.e., using separate draws for each) provides a consistent and asymptotically efficient estimator. However, anecdotal evidence indicates that the finite sample properties of this estimator are poor (Hajivassiliou, personal communication). Furthermore, MSM requires simulation of the probability of each possible sequence of responses, which in our situation would involve calculation of over 300,000 probabilities for each customer. Method of simulated scores (Hajivassiliou and McFadden (forthcoming)) is consistent if an unbiased simulator for the score is used; however, an unbiased score simulator is difficult to develop. The score takes the form $(1/P)dP/d\theta$. An unbiased simulator for $dP/d\theta$ is readily available; however, obtaining an unbiased simulator of $(1/P)$ is difficult. In particular, the reciprocal of an unbiased simulator of P is not unbiased for $1/P$. Usually, MSS estimators are called asymptotically unbiased, meaning that their bias disappears when the number of repetitions increases without bound, which is the same as MSL. Our procedure using MSL is a MSS estimator with $(1/P)$ simulated as the reciprocal of the simulated probability.

A reviewer suggested an alternative procedure that has desirable characteristics. First, obtain a consistent estimate using, e.g., MSM with exogenous weights or MSS with an unbiased score simulator. Then, apply one bhhh step to this consistent estimator. This estimator is efficient when the number of repetitions increases without bound along with sample size, the same as MSL. However, the asymptotic properties can perhaps be attained more readily with this approach than with MSL. In particular, since only one bhhh step is used (i.e., one iteration in MSL), the number of repetitions can be increased enormously for this one step while still utilizing the same computer time as with MSL. In our application, we used 500 repetitions, and approximately twenty iterations were needed to reach convergence. The alternative procedure could use 10,000 repetitions for its one iteration. The difficulty, of course, would be obtaining the initial consistent estimate. MSM is infeasible in our setting, since, as stated, it would involve simulation of hundreds of thousands of probabilities for each customer. In other settings, however, MSM could be utilized; importantly, the inefficiency that arises from nonideal weights would not be a concern since the MSM estimator is followed by a bhhh step using a very large number of repetitions.

where $d_{njt} = 1$ if person n chose alternative j in period t and zero otherwise, and $L_{njt}^{r|\theta}$ is the logit formula (1) evaluated with $\beta_n^{r|\theta}$. The score is easy to compute, which speeds the iteration process. We found that calculating the Hessian from formulas for the second derivatives resulted in computationally slower estimation than using the bhhh or other approximate-Hessian procedures.

In general, the coefficient vector can be expressed as $\beta_n = b + \eta_n$, where b is the population mean, and η_n is the stochastic deviation that represents the person's tastes relative to the average tastes in the population. Then $U_{njt} = b' x_{njt} + \eta'_n x_{njt} + \epsilon_{njt}$. In contrast to standard logit, the stochastic portion of utility ($\eta'_n x_{njt} + \epsilon_{njt}$) is in general correlated over alternatives and time due to the common influence of η_n. Mixed logit does not exhibit the independence from irrelevant alternatives property of standard logit, and very general patterns of correlation over alternatives and time (and hence very general substitution patterns) can be obtained through appropriate specification of variables and parameters. In fact, McFadden and Train (1997) show that any random-utility model can be approximated to any desired degree of accuracy with a mixed logit through appropriate choice of explanatory variables and distributions for the random parameters.[5] In the application below, we estimate models with normal and log-normal distributions for elements of β_n; other distributions are of course possible.

III. Application

Demand-side management (DSM) programs by electric utilities have relied heavily on rebates as a mechanism for promoting energy efficiency. As the electricity industry moves toward greater competition, the feasibility of rebates is questionable. Low-interest loan programs are being considered as alternatives. Potentially, loans can provide an incentive for efficiency (and so serve the goals of DSM) and yet generate profits as long as the interest rates on the loans are above the firm's cost of capital.

Using data from Southern California Edison (SCE), we estimate the impact of rebates and loans on residential customers' choice of efficiency level for refrigerators. Since loans have not been offered by SCE in the past, and since there has been little variation in rebate levels, data on actual purchases by SCE customers do not provide the information that is needed to estimate choice models with loan terms and rebate levels as explanatory variables. Stated-preference data were collected to estimate such models. In particular, a sample of SCE's residential customers was presented in a survey with a series of choice experiments. In each experiment, two or three refrigerators with different efficiency levels were described, with a rebate, loan, or no incentive offered on the high-efficiency units. The customer was asked which appliance he/she would choose. For customers who had bought a refrigerator within the last three years, these stated-preference data were supplemented, insofar as possible, with information on the efficiency level of the refrigerator that the customer actually purchased. Mixed logits are estimated on the stated-preference data; the models are then adjusted, or "calibrated," to reflect the limited revealed-preference data. The calibrated models are then used to forecast the impact of various loan programs.

In the stated-preference choice experiments, each sampled customer was offered a series of binary choices, followed by a series of trinary choices. For the binary choices, the purchase price and operating cost of a standard-efficiency and a high-efficiency refrigerator were described, and the customer was asked to choose between them. The high-efficiency unit was offered without any incentive, with a rebate, or with a financing package detailing the interest rate, amount borrowed, repayment period, and monthly payment. Trinary choices were then offered to the customer. In these experiments, the customer was offered three high-efficiency units: one with no incentive, one with a rebate, and one with financing. The purchase price and operating cost of the units differed, such that the unit with no incentive was not dominated. In total, responses to 6,081 choice experiments were obtained from 401 surveyed customers, with each customer providing responses to twelve binary choice experiments and up to four trinary experiments. The 6,081 experiments consist of the following types: 1,604 pair a standard unit with a high-efficiency unit that has no incentive; 1,626 pair a standard unit with a high-efficiency unit on which a rebate is available; 1,602 pair a standard unit with a high-efficiency unit on which a loan is offered; and 1,249 include three high-efficiency units with no incentive, a rebate, and a loan.[6]

The choice experiments were designed to provide plausible attributes, orthogonal over experiments, and with no

[5] This result differs critically and is stronger than the "mother logit" theorem, which states that any choice model can be approximated by a model that takes the form of a standard logit (McFadden (1975)). In the mother logit theorem, any choice model can be expressed as a standard logit if attributes of one alternative are allowed to enter the "representative utility" of other alternatives. However, when cross-alternative attributes are entered, the logit model is no longer a random utility model (i.e., it is not consistent with utility maximizing behavior), since the utility of one alternative depends on the attributes of other alternatives. In the theorem regarding mixed logit, any random utility model can be approximated by a mixed logit without entering cross-alternative variables, or, more precisely, while still maintaining consistency with utility-maximizing behavior.

[6] A note concerning identification is warranted. In the binary choice experiments, the variance in β_n induces heteroskedasticity in the difference in utility between the two alternatives. The parameters of the distribution of β_n, i.e., θ, are identified by the heteroskedasticity over experiments (that is, by the variation in the variance in the utility-difference). In the trinary experiments, the variance in β_n induces heteroskedasticity and covariance in the two utility-differences (that is, in the utility for two of the alternatives minus the utility for the remaining alternative.) Essentially, each person has a variance in utility differences in a binary situation, and two variances and a covariance between two utility-differences in a trinary situation. If these terms were fixed over people and normalized to account for the arbitrary scale of utility, then estimation of at most two parameters would be possible with trinary experiments and none with binary experiments. However, with mixed logit, these terms vary over people in a way that depends on θ and the variables; θ is thereby identified even in binary experiments and even when its dimension is greater than two in trinary experiments.

experiment containing a dominated alternative. The variables that enter the models below are:

(a) *Price of the refrigerator, net of any rebate, in hundreds of dollars.* (For a standard-efficiency unit and high-efficiency units without a rebate, this variable is the price of the unit. For high-efficiency units with a rebate, it is the price of the unit minus the rebate.[7])

(b) *Savings, in hundreds of dollars.* This variable is zero for the standard unit and, for the high-efficiency units, is the annual dollar reduction in operating cost that the unit provides relative to the standard unit. (That is, savings in any experiment is the operating cost of the standard unit minus the operating cost of the high-efficiency unit.)

(c) *Amount borrowed, in hundreds of dollars.* This variable is zero for standard units and for high-efficiency units for which no loan is offered. For high-efficiency units on which a loan is offered, this variable is the maximum dollar amount that the customer is allowed to borrow. The percent of the purchase price that the customer is able to borrow varies over experiments.

(d) *Interest rate, in digits* (i.e., 4% interest is entered as 0.04). This variable is zero for standard units and for high-efficiency units for which no loan is offered. For high-efficiency units with a loan available, the variable is the interest rate that is offered for the loan. The interest rate varies over experiments.

(e) *Efficiency dummy.* This variable takes the value of zero for standard units and one for high-efficiency units.

(f) *Rebate dummy,* taking the value of one for high-efficiency units on which a rebate is provided, and zero otherwise.

(g) *Finance dummy,* taking the value of one for high-efficiency units for which a loan is provided, and zero otherwise.

The means of these variables over the choice experiments are given in table 1. Details of the survey design and variables are provided in SCE (1994).

A. Model Estimation

We specify the price coefficient to be fixed while allowing the other coefficients to vary. The willingness to pay for each attribute (which is the ratio of the attribute's coefficient to the price coefficient) is thereby distributed in the same way as the attribute's coefficient, which is convenient for interpretation of the model.[8]

TABLE 1.—MEANS OF EXPLANATORY VARIABLES

Price of standard-efficiency refrigerator	875.94
Price of high-efficiency refrigerator	1127.89
Annual savings in operating cost for high-efficiency units relative to standard	116.89
Rebate (when rebate is offered)	125.75
Amount borrowed (when loan is offered)	698.50
Interest rate (when loan is offered)	0.0505

We first specify all the nonprice coefficients to be independently normally distributed. The coefficient vector is expressed as $\beta_n = b + W\mu_n$ in which W is a diagonal matrix whose elements are standard deviations (with the top-left element being zero, for the price coefficient), and μ_n is a vector of independent standard normal deviates. For simulation, draws of μ_n are obtained from a pseudorandom number generator, and the corresponding draws of β_n are calculated for any given values of the means b and standard deviations W. With this specification, the derivatives that enter the score (3) are $\partial\beta_n^{r|\theta\prime} x_{njt}/\partial b_k = x_{k,njt}$ and $\partial\beta_n^{r|\theta\prime} x_{njt}/\partial W_k = \mu_k^r x_{k,njt}$, where the subscript k denotes the k-th element. Subsequent models allow correlation among the coefficients and specify log-normal distributions for some of the coefficients.

Table 2 provides the estimation results for this model, along with the results for a standard logit model. The mean coefficients in the mixed logit are consistently larger than the fixed coefficients in the standard logit model. This result reflects the fact that the mixed logit decomposes the unobserved portion of utility and normalizes parameters on the basis of part of the unobserved portion. Suppose true utility is given by the mixed logit: $U_{njt} = b'x_{njt} + \mu'_n Wx_{njt} + \epsilon_{njt}$. The parameters b are normalized such that ϵ_{njt} has the appropriate variance for an extreme value error. The standard logit model treats utility as $U_{njt} = b'x_{njt} + \xi_{njt}$ with b normalized such that ξ_{njt} has the variance of an extreme value deviate. The extreme value term in the standard logit model incorporates any variance in the parameters. In the mixed logit, the variance in parameters is treated explicitly as a separate component of the error ($\mu'_n Wx_{njt}$) such that the remaining error (ϵ_{njt}) is "net" of this variance. Since the variance in the error term in the standard logit is greater than the variance in the extreme value component of the error term in the mixed logit, the normalization makes the parameters in the standard logit model smaller in magnitude than those in the mixed logit. The fact that the parameters rise by a factor of three or more implies that the random

[7] When the rebate enters as a separate variable, rather than subtracted from price, its mean coefficient is similar in magnitude and opposite in sign to the mean coefficient of price. The hypothesis cannot be rejected at reasonable significance levels that the rebate is considered a reduction in price.

[8] When all coefficients are allowed to vary in the population, identification is empirically difficult, for the reasons given by Ruud (1996). In particular, if the stochastic portion of utility is dominated by the random parameters such that the iid extreme-value term has little influence, then the scaling of utility by the variance of the extreme-value term becomes unstable and an additional scaling is needed. At an extreme, where the extreme-value term has no influence (i.e., zero variance), the simulated probability becomes an accept/reject simulator, and a scaling of the remaining utility (that is, utility without the extreme-value term) is required. We chose the price coefficient to be fixed, since, as stated, this restriction allows easy derivation of the distribution of the willingness to pay. Models with all coefficients varying did not converge in any reasonable number of iterations, as expected by Ruud's observation.

TABLE 2.—STANDARD AND MIXED LOGIT WITH ALL NORMALLY DISTRIBUTED COEFFICIENTS

		Standard Logit Estimates	*Mixed Logit Estimates*
Price net of rebate	Coefficient	−0.379 (0.0360)	−1.23 (0.108)
Savings	Mean coefficient	0.807 (0.0609)	3.03 (0.345)
	Standard deviation of coefficient	—	2.24 (0.281)
Amount borrowed	Mean coefficient	0.0701 (0.0176)	0.392 (0.066)
	Standard deviation of coefficient	—	0.489 (0.057)
Interest rate	Mean coefficient	−6.87 (4.03)	−48.5 (10.09)
	Standard deviation of coefficient	—	44.4 (7.53)
Efficiency dummy	Mean coefficient	1.33 (0.101)	3.70 (0.421)
	Standard deviation of coefficient	—	3.20 (0.398)
Rebate dummy	Mean coefficient	0.229 (0.109)	0.022 (0.212)
	Standard deviation of coefficient	—	1.30 (0.204)
Finance dummy	Mean coefficient	−0.0175 (0.264)	0.156 (0.621)
	Standard deviation of coefficient	—	0.284 (0.475)
Likelihood ratio index		0.275	0.461
Willingness to pay in higher purchase price, calculated at estimated mean coefficients, for:			
	$1 extra savings	2.13	2.46
	$1 extra of amount borrowed	0.19	0.32
	1% reduction in interest rate	18.13	39.43

Notes: Standard errors in parentheses.
Price, savings, and amount borrowed are in hundreds of dollars. Interest rates are in digits (e.g., 4% is entered as 0.04).

parameters constitute a very large share of the variance in unobserved utility.

In the mixed logit, the estimated standard deviations of coefficients are highly significant, indicating that parameters do indeed vary in the population. Also, the likelihood ratio index[9] rises substantially from allowing the parameters to vary, indicating that the explanatory power of the mixed logit is considerably greater than with standard logit.[10] The magnitudes of the estimated standard deviations are reasonable relative to the estimated means. For example, the distribution of the savings coefficient has an estimated mean of 3.03 and an estimated standard deviation of 2.24. Given the estimated price coefficient, the model implies that the willingness to pay for $1.00 of annual savings, on the margin, is normally distributed in the population with mean of $2.46 and standard deviation of $1.81—which is a fairly substantial variation in willingness to pay. The standard logit model implies a willingness to pay of $2.12. If customers consider refrigerators to have a 10-year lifespan, and expect no real growth in energy prices, a willingness to pay of $2.12 implies a discount rate of 46%, and $2.46 implies a discount rate of 39%.[11] These implicit discount rates, while high relative to interest rates, are consistent with previous findings on residential customers' choice of refrigerator efficiency levels (e.g., Cole and Fuller (1980), McRae (1980), Meier and Whittier (1983)).

The mixed logit implies that approximately 9% of the population place a negative coefficient on savings. This implication could reflect reality or could be an artifact of the assumption of normally distributed coefficients. It is possible that some customers are highly skeptical of energy-conservation claims and become more mistrustful the greater the claim of savings is. In this case, negative coefficients for savings reflect the mistrust of these customers and are an accurate representation of reality. On the other hand, the assumption of a normal distribution implies that some share of the population has negative coefficients for savings, whether or not this is true. This issue is addressed below with a model that specifies a log-normal distribution for the coefficients of savings and other variables.

The parameters associated with the amount borrowed imply that the mean willingness to pay for being able to borrow an extra dollar is $0.32 and the standard deviation is $0.40. Interest rates are denoted in digits (e.g., an interest rate of 9% is denoted as 0.09). The mean willingness to pay for a 1% reduction in interest rate is therefore $39 with a standard deviation of $36. For both the interest rate and amount borrowed, the variation in coefficients is fairly substantial, implying that different people respond quite differently to loan terms.

An efficiency dummy enters the utility of high-efficiency refrigerators, whether or not an incentive is offered on the

[9] The likelihood ratio index is a measure of goodness-of-fit, defined as 1-[SLL(θ_e)/SLL(0)], where SLL(θ_e) is the value of the simulated log-likelihood function at the estimated parameters, and SLL(0) is the value with all parameters equal to zero. The index ranges from zero (for a model that is no better than chance, such that SLL(θ_e) = SLL(0)) to 1 (for a "perfect" model that provides a simulated probability of one to the chosen sequence of choices of each sampled decision-maker, such that SLL(θ_e) = 0).

[10] Atherton and Train (1995) estimated a nested logit model on these data, with the high-efficiency options nested together. As such, their model is analogous to a mixed logit with fixed coefficients for all variables except the efficiency dummy, whose coefficient varies randomly over customers and, importantly, over time for each customer. (The random coefficient for the efficiency dummy induces correlation in the stochastic portion of utility over the high-efficiency options (namely, refrigerators with rebates, loans, or no incentive), without inducing correlation with the utility of a standard refrigerator—as in the nested logit. The nested-logit model treats the stochastic portion of utility as independent over repeated choices, which requires assuming that each customer's coefficient for the efficiency dummy is independent over choice situations.) The mixed logit in table 2 obtains a considerably higher log-likelihood than the nested logit of Atherton and Train. This is expected of course, since, in the mixed logit, the standard deviations in the coefficients of variables other than the efficiency dummy are highly significant. One could construct nested-logit models that have richer correlation patterns than in Atherton and Train. Brownstone and Train (1996) compare probit with mixed logit using data on households' choice of cars.

[11] The discount rate is calculated by solving for d in WTP = $[1/(1 + d)] + [1/(1 + d)^2] + \ldots + [1/(1 + d)^{10}]$ where WTP is the willingness to pay for $1 of extra savings annually, and the lifespan of the refrigerator is assumed to be ten years.

unit. Its mean coefficient indicates that, on average, customers choose the high-efficiency unit in the choice experiments more readily than can be explained by the price, savings, and other financial matters. The standard deviation indicates that 88% of the population have a "high-efficiency preference." This "preference" is largely an artifact of the experiments, with customers perhaps feeling that the interviewer wants them to say they would purchase the high-efficiency unit, or would think well of them if they did. When the model is calibrated against revealed-choice data below, the mean drops considerably. However, it is still significantly different from zero, indicating that there is some preference for high-efficiency units, independent of price and savings, even in customers' actual choices. This preference might indicate that customers think that high-efficiency is correlated with higher quality, greater durability, less noise, or other desirable attributes.

Rebates can be viewed by customers in a variety of ways independent of the reduction in price that they provide. Customers seem to be skeptical of information from their energy utility, including information about the supposed savings that high-efficiency appliances provide (Constantzo et al. (1986); Bruner and Vivian (1979); Craig and McCann (1978)). For some customers, the offer of a rebate lends credibility to the savings claim: these customers interpret the rebate as evidence that the utility is willing to "put its money where its mouth is" (Train, 1988). For these customers, the rebate dummy has a positive coefficient. Other customers might see the rebate as the opposite kind of signal, namely, as a sign that the appliances are too poor to sell on their own merit. These customers have a negative coefficient for the rebate dummy. Table 2 indicates that the mean coefficient for the rebate dummy is slightly positive but not significantly different from zero, while the standard deviation is fairly large and highly significant. These results indicate that customers hold a wide variety of views about rebates, with about as many seeing the rebates as a negative signal as those who see it as a positive signal. Note that the standard logit model masks this reality: its slightly positive coefficient for the rebate dummy would be interpreted as indicating that customers, in general, view rebates as a slightly positive signal, while in reality, many customers view rebates as a negative signal (and many view it as a strong positive signal). It is simply that the customers who take the rebate as a negative signal nearly balance the customers who take it as a positive signal, such that the mean effect is only slightly positive.

The coefficient of the financing dummy obtains an insignificant mean and standard deviation: the hypothesis that customers examine loans only on the basis of their financial terms cannot be rejected. The difference in how customers respond to loans versus rebates is plausible. Rebates are a "giveaway"; customers naturally wonder about the motivation for the giveaway and tend to read a signal into it even if there is none. Loans are not a giveaway; the customer realizes that the lender makes money from the loans. The customer need not read a signal into the offer of loans, since the motivation for the offer is clear. Several variations on this basic model were estimated to explore particular issues. These models are described below.

TABLE 3.—MIXED LOGIT WITH DEMOGRAPHIC VARIABLES

		Parameter Estimates
Price net of rebate for respondents with	Some college, income <$25,000	−1.17 (0.184)
	Some college, income $25,000–$50,000	−1.49 (0.196)
	Some college, income >$50,000	−1.54 (0.100)
	No college, income <$25,000	−0.399 (0.181)
	No college, income $25,000–$50,000	−0.530 (0.159)
	No college, income >$50,000	−2.40 (0.326)
Savings	Mean coefficient	3.35 (0.376)
	Standard deviation of coefficient	2.79 (0.321)
Amount borrowed	Mean coefficient	0.348 (0.108)
	Standard deviation of coefficient	0.504 (0.074)
Interest rate	Mean coefficient	−47.8 (13.4)
	Standard deviation of coefficient	52.6 (8.65)
Efficiency dummy	Mean coefficient	3.99 (0.446)
	Standard deviation of coefficient	3.41 (0.449)
Rebate dummy	Mean coefficient	−0.146 (0.188)
	Standard deviation of coefficient	0.775 (0.217)
Finance dummy	Mean coefficient	0.275 (0.771)
	Standard deviation of coefficient	0.222 (0.607)
Number of respondents		375
Likelihood ratio index		0.471

Notes: See table 2 for definitions of variables. Standard errors in parentheses.

The estimates in table 2 indicate that parameters vary greatly in the population. However, the specification does not include observed characteristics of the customer. Variations in parameters that are related to observed characteristics can be captured in standard logit models through interaction of customer characteristics with attributes of the alternatives. The question arises, therefore: to what extent can the variation in parameters that is evidenced in table 2 be captured through the inclusion of customer characteristics? Table 3 presents a model that includes the income and the education level of the customer interacted with the price of the refrigerator. This specification follows Atherton and Train (1995) and was obtained after extensive testing with the demographic variables that were available from the survey. In this model, willingness to pay for each attribute varies with income and education, since the price variable is interacted with these factors. The standard deviations are still large and significant, which indicates that the willingness to pay varies more than is captured by the income and education of customers. There are probably other potentially observable characteristics that relate to willingness to pay; the fact that only education and income enter this model reflects the limited nature of the sociodemographic information that was available from the survey.

The model in table 2 specifies the coefficients to be independently distributed while, in reality, one would generally expect correlation. For example, customers who are especially concerned about savings in their monthly energy bill might also be concerned about interest rates, particularly since the loan payments will appear on their monthly energy bill. To investigate these possibilities, we specify $\beta_n \sim N(b,\Omega)$ for general Ω. The coefficient vector is expressed $\beta_n = b + L\mu_n$ where L is a lower-triangular Choleski factor of Ω, such that $LL' = \Omega$. We estimate b and L, and calculate standard errors for elements of Ω with the derivative rule.[12] The ratios of estimated means are very similar to those in table 2, with similar levels of significance; their magnitudes are somewhat higher, reflecting the fact that allowing for covariances captures more variance in the unobserved portion of utility, such that ϵ has less variance and the normalization raises the parameters. The estimates of b and L are not reported, since the estimates of b have the same interpretation as for table 2, and the estimates of L have no meaning in themselves. Table 4 gives the estimated covariance matrix, t-statistics for the estimated covariance matrix, and point estimates for the correlation matrix. Five covariances have t-statistics over 1.6.

TABLE 4.—COVARIANCES AMONG COEFFICIENTS IN MIXED LOGIT
COEFFICIENTS OF: 1. SAVINGS
2. AMOUNT BORROWED
3. INTEREST RATE
4. EFFICIENCY DUMMY
5. REBATE DUMMY
6. FINANCE DUMMY

Estimated Covariance Matrix:					
12.12	−0.890	−115.4	−5.300	−1.205	5.259
		−10.46	1.371	−0.004	−1.259
		6,032.00	113.2	19.16	−88.12
			17.92	0.3740	−12.20
				3.074	4.375
					18.11
T-Statistics for Estimated Covariances:					
4.09	1.36	1.59	2.57	1.96	1.39
	3.22	0.50	2.96	0.02	1.45
		2.88	1.33	0.75	0.77
			3.97	0.26	1.94
				3.41	1.90
					1.52
Correlation Matrix:					
1.0	−0.26	−0.43	−0.35	−0.20	0.35
	1.0	−0.14	0.32	−0.002	−0.30
		1.0	0.34	0.14	−0.27
			1.0	0.05	−0.68
				1.0	0.59
					1.0

(i) The savings coefficient is negatively correlated with the coefficient of the efficiency dummy. This estimate implies that customers who value savings highly tend not to be motivated by the label of high-efficiency independent of savings.

(ii) The savings coefficient is negatively correlated with the rebate dummy coefficient, implying that customers who value savings highly tend not to be motivated by rebates beyond the reduction in price that the rebates provide.

(iii and iv) The efficiency dummy coefficient is positively correlated with the coefficient of amount borrowed and negatively with the finance dummy coefficient. Customers who like high-efficiency per se (independent of savings) like being able to borrow and are not motivated by the offer of a loan independent of its terms.

(v) The coefficients of the rebate and finance dummies are positively correlated: customers who are motivated by rebates beyond the reduction in price that the rebates provide are also motivated by the offer of a loan beyond the terms of the loan.

The normal distribution allows coefficients of both signs. For some variables, such as savings, it is reasonable to expect that all customers have the same sign for their coefficients. We estimate a model with log-normal distributions for the coefficients of savings, amount borrowed, and interest rates. The coefficients for the efficiency, rebate, and finance dummies are kept as normals, since these coefficients can logically take either sign for a given individual. Let k denote an element of β_n that has a log-normal distribution. This coefficient is expressed $\beta_{nk} = \exp(b_k + s_k\mu_{nk})$ where μ_{nk} is an independent standard normal deviate. The parameters b_k and s_k, which represent the mean and standard deviation of $\log(\beta_{nk})$, are estimated. The median, mean, and standard deviation of β_{nk} are $\exp(b_k)$, $\exp(b_k + (s_k^2/2))$, and mean * $\sqrt{(\exp(s_k^2) - 1)}$, respectively. Savings and amount borrowed enter directly, such that all customers' coefficients are positive, and the negative of interest rates is entered such that all customers' coefficients of interest rate are negative. Table 5 gives the estimation results. The results are similar qualitatively to those obtained with all normal distributions. Each of the three log-normal distributions has median and mean that bracket the mean that is obtained with a normal distribution. For example, from table 5, the estimated median willingness to pay for savings is \$1.81 with an estimated mean of \$3.23, while the

[12] The general result is: for $\psi = f(\rho)$, $\mathrm{Var}(\psi) = (\partial\psi/\partial\rho)'\ \mathrm{Var}(\rho)\ (\partial\psi/\partial\rho)$ where ψ and ρ are vectors. In our case, ρ is the vector composed of the elements of L, ψ is the vector composed of the elements of Ω, and f is LL′ expressed in vector form.

TABLE 5.—MIXED LOGIT WITH LOG-NORMAL DISTRIBUTION FOR COEFFICIENTS OF SAVINGS, AMOUNT BORROWED, AND INTEREST RATE

		Parameter Estimates	Median and Mean for Log-Normally Distributed Coefficients, Calculated at Estimated b and s: Median	Mean
Price net of rebate	Coefficient	−1.22 (0.0964)		
Savings	b	0.79 (0.152)	2.20	3.95
	s	1.08 (0.165)		
Amount borrowed	b	−1.17 (0.244)	0.310	0.686
	s	1.26 (0.24)		
Interest rate (neg.)	b	3.78 (0.301)	43.82	73.7
	s	1.02 (0.227)		
Efficiency dummy	Mean coefficient	3.43 (0.337)		
	Standard deviation of coefficient	2.47 (0.285)		
Rebate dummy	Mean coefficient	0.199 (0.188)		
	Standard deviation of coefficient	1.13 (0.214)		
Finance dummy	Mean coefficient	−0.191 (0.533)		
	Standard deviation of coefficient	0.479 (0.586)		
Likelihood ratio index		0.448		
Willingness to pay in extra purchase price, calculated at mean and median coefficients, for	$1 extra savings		1.80	3.24
	$1 extra amount borrowed		0.25	0.56
	1% reduction in interest rate		35.92	60.41

Notes: See table 2 for definitions of variables. Standard errors in parentheses.

mean/median with a normal distribution is $2.46.[13] It is interesting to note that the log-likelihood value is lower for the model with log-normal distributions than the comparable model (see table 2) with all normally distributed coefficients. A possible reason is discussed in footnote fourteen. For calibration and simulation, we utilize both models.

B. *Calibration to Revealed-Preference Data*

Once estimated, the models are calibrated to the limited revealed-preference data that were available. Each surveyed customer was asked whether he/she had purchased a refrigerator during the last three years. Those who responded in the positive were asked to locate the serial number or other identifying information for the unit that they purchased. With this information, we determined, using product specification sheets, the efficiency level of the refrigerator. Program files were then used to determine which of the customers who had purchased a high-efficiency refrigerator had received a rebate. In combination, this information identified whether the customer had chosen standard-efficiency, high-efficiency without a rebate, or high efficiency with a rebate. The information was obtained for 163 of the 401 surveyed customers. Of course, since financing had not been offered by SCE's programs, a high-efficiency unit with utility financing was not available.

Actual choices are expected to differ from stated choices for two primary reasons. First, customers might have a tendency to say that they would purchase a high-efficiency refrigerator more readily than they actually do. This would evidence itself in the coefficient for the high-efficiency dummy being higher with the stated-preference data than is true for actual choices. Second, any time or effort that the customer must expend to receive a rebate, or any lack of awareness about the program, is not reflected in the stated-preference data. In the hypothetical situation, the customer is informed about the rebate and does not have to do anything to receive it. As a result, the estimated coefficient for the rebate dummy is expected to be higher in the stated-preference models than in reality. To account for these issues, the parameters associated with the efficiency and rebate dummies were reestimated on the revealed-preference data, holding the other parameters at the values obtained with the stated-preference data. The results are given in table 6. As expected, the mean and standard deviation of the efficiency dummy coefficient drop considerably—the mean from 3.70 to 0.785, and the standard deviation from 3.20 to 0.213 for the model with all normally distributed coefficients, and comparable amounts for the model with log-normal distributions for some coefficients. The mean of the rebate dummy coefficient decreases, but the standard deviation increases. This result is consistent with rebates being more burdensome to obtain in the real world

TABLE 6.—CALIBRATION TO REVEALED-PREFERENCE DATA

		Parameter Estimates: Model with all normals	Model with log-normals and normals
Efficiency dummy	Mean coefficient	0.785 (0.182)	0.713 (3.64)
	Standard deviation of coefficient	0.213 (2.86)	0.397 (0.322)
Rebate dummy	Mean coefficient	−3.70 (7.31)	−3.01 (2.17)
	Standard deviation of coefficient	2.84 (8.30)	2.03 (1.70)
Number of respondents		163	163
Likelihood ratio index		0.122	0.121

Note: Standard errors in parentheses.

[13] Convergence was very slow with the log-normal distributions, taking nearly a hundred iterations. We reparameterized the likelihood function and gradient to operate in the means and standard deviations of the log-normal distributions themselves, but this reparameterization did not materially reduce the number of iterations.

than in the hypothetical experiments, and the value that people place on the time and hassle required to obtain the rebate varying considerably across customers. In simulation, the mean and standard deviation of the financing dummy coefficient are adjusted by the same amount by which the calibration adjusted the rebate dummy's mean and standard deviation. This adjustment reflects the presumption that the hassle associated with obtaining rebates will also occur for obtaining a loan.

Our calibration procedure that adjusts only the distribution of constants is analogous to the procedure used by Atherton and Train (1995), which adjusts the constants and nesting parameter in a nested logit (the nesting parameter in their model is equivalent to the variance of the efficiency dummy in our mixed logit). This correspondence allows us to compare our forecasts with those of Atherton and Train. Other procedures that could be pursued are estimation of the model on the combined stated- and revealed-preference data with mixed or Bayesian procedures that weight the two sources of data, or estimation on the revealed-preference data of a scale parameter that adjusts all the parameters obtained on the stated-preference data (e.g., Swait and Louviere (1993), Hensher and Bradley (1993)).

C. Predictions

We use the calibrated models to predict the effect of DSM programs. Consider first the impact of the rebate program. From the mixed logit with all normal coefficients, 15.8% of refrigerator purchasers obtained a rebate, 46.1% purchased a standard-efficiency unit, and 38.1% purchased a high-efficiency unit but did not obtain a rebate. The average rebate is \$64. With no DSM program (i.e., without the option of purchasing a high-efficiency unit with a rebate), 54.6% of customers are predicted to purchase a standard unit with the other 45.4% buying a high-efficiency unit without a rebate. These predictions imply that the rebates reduced the standard efficiency share from 54.6% to 46.1%, such that the rebate program is predicted to have induced 8.5% of buyers to switch from a standard- to a high-efficiency refrigerator. The cost per induced switch is therefore \$119 (\$64 × 0.158/0.085). Predictions from the model with log-normal distributions are essentially identical.

Consider now the impact of loan programs. Table 7 presents predictions under various interest rates for loans offered on the full price of high-efficiency units. Zero interest loans are predicted to attract about 40% of refrigerator purchasers, which is far greater participation than the rebate program. Compared to no program, such loans would induce 22.6% of buyers to switch from standard to high efficiency, which is nearly three times greater than the rebate program's impact. The average loan in this scenario is \$1,031, such that cost to the utility is \$64 at a 6% cost of funds and a two-year repayment period—the same as the average rebate. The cost per induced switch is \$112, which is

TABLE 7.—PREDICTED CHOICES OF REFRIGERATOR BUYERS WHEN LOANS ARE OFFERED ON HIGH EFFICIENCY UNITS

	Mixed Logit with All Normal Distributions			Mixed Logit with Log-Normal and Normal Distributions		
Interest Rate	Standard Efficiency	High Efficiency Without Loan	High Efficiency With Loan	Standard Efficiency	High Efficiency Without Loan	High Efficiency With Loan
0%	0.320	0.283	0.397	0.317	0.280	0.403
2%	0.354	0.314	0.332	0.361	0.319	0.320
4%	0.381	0.336	0.283	0.387	0.342	0.272
6%	0.402	0.351	0.246	0.405	0.357	0.238
8%	0.418	0.362	0.220	0.419	0.370	0.212
10%	0.430	0.370	0.201	0.430	0.379	0.191
12%	0.438	0.375	0.186	0.439	0.387	0.174

slightly lower than the rebate program. The total outlay by the utility is higher with the loans than with the rebates, since participation is greater.

The utility earns a profit on loans when the interest rate is above its cost of funds. At 8% interest, 19% to 22% of refrigerator purchasers are predicted to obtain the loans, depending on which model is used in prediction. At 12% interest, the predicted share is 14% to 17%. In all scenarios, more than half of the customers who obtain loans would have purchased a standard unit without the loans. So, a loan program that finances the entire price of the high-efficiency unit at a rate that allows the utility to make a profit is predicted to induce 8.4% to 13% of customers to switch from a standard- to a high-efficiency unit. The loans have a larger impact than the rebates and also generate profit for the firm: a "win-win" situation.[14]

[14] The loans could also induce customers to buy larger refrigerators than they otherwise would. This effect would reduce the energy savings but increase consumer surplus.

An interesting phenomenon arises when predicting the effect of loans that cover the incremental price of the high-efficiency unit (i.e., the price of the high-efficiency unit minus the standard unit's price) rather than the full price. For interest rates above 4%, the mixed logit with normally distributed coefficients predicts the share of customers who obtain a loan to rise as the interest rate rises. Recall that this model implies that 14% of the population have positive coefficients for interest rates. When the amount borrowed is only the incremental price and the interest rate is above 4%, the share of customers obtaining loans is so small that the tail of the distribution dominates. The customers who supposedly like to pay interest are primarily the ones predicted to obtain loans, and for these customers a rise in the interest rates makes the loans more attractive. This phenomenon does not occur when the amount borrowed is sufficiently high to make the loans attractive to a large share of the population. In short, when predicted shares are small, the tails of the distribution drive the results, and so the plausibility of the tails is important; when predicted shares are large, the tails are less determinative, and the distribution can be treated as a reasonable approximation.

With the log-normal distribution, the share obtaining loans necessarily decreases as the interest rate rises. It is important to note that the basic issue does not disappear with a log-normal distribution, it is just made less obvious. The basic issue is that an unrestricted distribution necessarily gives implausible results for some share of the population. With the normal distribution, the implausibility of, for example, positive interest rate coefficients, is obvious. However, the normal distribution also provides implausibly large coefficients of the correct sign, which might not be so obvious (or, more precisely, the cutoff for what is plausible cannot be so easily discerned.) The log-normal avoids the obvious problem with the

Atherton and Train (1995) performed the same kind of predictions with their nested logit model. They obtain practically the same shares for the base situation of the rebate program. This is expected, since both models were calibrated to this base situation on the same revealed-preference data. In predicting beyond the base situation, Atherton and Train (A-T) predict essentially the same shares as we do for the situation without a DSM program; however, their model predicts about half as many participants as our model for the loan programs. The reasons for these results are directly traceable to the specification of the models. The change in shares from the base situation to the no-DSM situation is determined primarily by the correlation between the stochastic portion of utility for a rebated high-efficiency unit and that of a nonrebated high-efficiency unit. (If the correlation is zero, then the shares for standard and nonrebated high-efficiency units increase nearly proportionately when the rebated high-efficiency unit is eliminated as an option, as required in a logit model with the independence from irrelevant alternatives property.) Both the nested logit model of A-T and our mixed logit include a correlation between the utilities of these alternatives; the two models obtain similar forecasts as a result. The predicted share for a loan program depends largely on the coefficients of the loan-related variables (amount borrowed, interest rate, and finance dummy), since these coefficients determine how attractive the loans are to people. A-T have fixed coefficients for these variables, which can be considered to reflect the tastes of the average person. The mixed logit reflects the distribution of tastes and obtains large standard deviations for the loan-related coefficients, indicating a wide divergence of tastes. Stated loosely, the results from the two models indicate that while the loans do not appeal greatly to the average tastes, a sizable share of the population have tastes such that the loans are attractive.

These predictions should not be overinterpreted. An important limitation is the implicit assumption that only the utility offers loans on appliance purchases, whereas in reality retailers offer credit and customers can use their credit cards. These loans are available for standard-efficiency units as well as high-efficiency units. To induce buyers to switch from standard- to high-efficiency units when loans are available on both, better loan terms must be offered on the high-efficiency units. The interest rates on credit cards and retailers' loans are fairly high, certainly above the utilities' cost of funds. However, whether the difference represents a premium for nonpayment and management, which the utility must also bear, is a critical issue. In this context, the analysis can perhaps best be taken simply as a indication that loans might be an avenue to generate profits and greater energy efficiency, and that attention to this potential by utilities and regulators is warranted.

normal but retains and actually exacerbates the less obvious problem (since its upper tail is thicker.) This difference at the high end might be the reason the model with all normal distributions obtains a higher log-likelihood value than the model with log-normals. A distribution with an bounded support, such as the beta distribution, might be worth exploring.

REFERENCES

Atherton, T., and K. Train, "Rebates, Loans, and Customers' Choice of Appliance Efficiency Level: Combining Stated- and Revealed-Preference Data," *Energy Journal,* Vol. 16, No. 1 (1995), 55–69.

Ben-Akiva, M., and D. Bolduc, "Multinomial Probit with a Logit Kernel and a General Parametric Specification of the Covariance Structure," working paper, Department of Civil Engineering, MIT (1996).

Ben-Akiva, D. Bolduc, and M. Bradley, "Estimation of Travel Choice Models with Randomly Distributed Values of Time," *Transportation Research Record* 1413 (1993), 88–97.

Ben-Akiva, M., and S. Lerman, *Discrete Choice Analysis,* Cambridge, MA: MIT Press (1985).

Berkovec, J., and S. Stern, "Job Exit Behavior of Older Men," *Econometrica,* Vol. 59, No. 1 (1991), p 189–210.

Bhat, C., "Accommodating Variations in Responsiveness to Level-of-Service Measures in Travel Model Choice Modeling," working paper, Department of Civil Engineering, University of Massachusetts at Amherst (1996).

Bolduc, D., B. Fortin, and M.-A. Fournier, "The Impact of Incentive Policies on the Practical Location of Doctors: A Multinomial Probit Analysis," Cahier de recherche numero 93-05 du Groupe de Recherche en Politique Economique (1993), Department d'economique, University Laval, Quebec, Canada, G1K 7P4.

Boyd, J., and R. Mellman, "The Effect of Fuel Economy Standards on the U.S. Automotive Market: An Hedonic Demand Analysis," *Transportation Research,* Vol. 14A, No. 5–6 (1980), 367–378.

Brownstone, D., and K. Train, "Forecasting New Product Penetration with Flexible Substitution Patterns," *Journal of Econometrics* (forthcoming).

Bruner, R., and W. Vivian, *Citizen Viewpoints on Energy Policy,* Ann Arbor: University of Michigan, Institute of Public Studies (1979).

Cardell, N., and F. Dunbar, "Measuring the Societal Impacts of Automobile Downsizing," *Transportation Research,* Vol. 14A, No. 5–6 (1980), 423–434.

Cole, H., and R. Fuller, "Residential Energy Decision Making: An Overview with Emphasis on Individual Discount Rates and Responsiveness to Household Income and Prices," Hittman Associates report, Columbia, MD (1980).

Constantzo, M., D. Archer, E. Aronson, and T. Pettigrew, "Energy Conservation Behavior: The Difficult Path from Information to Action," *American Psychologist* 41 (1986), 521–528.

Craig, C., and J. McCann, "Assessing Communication Effects on Energy Conservation," *Journal of Consumer Research* 5 (1978), 82–88.

Hajivassiliou, V., and D. McFadden, "The Method of Simulated Scores for the Estimation of LDV Models," *Econometrica* (forthcoming).

Hajivassiliou, V., and P. Ruud, "Classical Estimation Methods for LDV Models Using Simulation," *Handbook of Econometrics,* Vol. IV, R. Engle and D. McFadden (Eds.), Elsevier Science B.V., New York (1994).

Hensher, D., and M. Bradley, "Using Stated Response Data to Enrich Revealed Preference Discrete Choice Model," *Marketing Letters,* Vol. 4, No. 2 (1993), 39–152.

Lee, L., "On Efficiency of Methods of Simulated Moments and Maximum Simulated Likelihood Estimation of Discrete Response Models," *Econometric Theory* 8 (1992), 518–552.

McFadden, D., "On Independence, Structure, and Simultaneity in Transportation Demand Analysis," working paper no. 7511, Urban Travel Demand Forecasting Project, Institute of Transportation and Traffic Engineering, University of California, Berkeley (1975).

——— "A Method of Simulated Moments for Estimation of Discrete Choice Models without Numerical Integration," *Econometrica* 57 (1989) 995–1026.

McFadden, D., and K. Train, "Mixed Multinomial Logit Models for Discrete Response," working paper, Department of Economics, University of California, Berkeley (1997).

McRae, D., "Rational Models for Consumer Energy Conservation," in Burby and Marsden (Eds.), *Energy and Housing,* Oelgeschleger: Gunn and Hain Publishers (1980).

Meier, A., and J. Whittier, "Consumer Discount Rates Implied by Purchases of Energy-Efficient Refrigerators," *Energy,* Vol. 8, No. 12 (1983), 957–962.

Ruud, P., "Approximation and Simulation of the Multinomial Probit Model: An Analysis of Covariance Matrix Estimation," working paper, Department of Economics, University of California, Berkeley (1996).

Southern California Edison, *Customer Decision Study: Analysis of Residential Customer Equipment Purchase Decisions,* report prepared by Cambridge Systematics (1994).

Swait, K., and J. Louviere, "The Role of the Scale Parameter in the Estimation and Use of Multinomial Logit Models," *Journal of Marketing Research* 30 (1993), 305–314.

Train, K., "Incentives for Energy Conservation in the Commercial and Industrial Sectors," *Energy Journal,* Vol. 9, No. 3 (1988), 113–128.

——— "Recreation Demand Models with Taste Differences Over People," *Land Economics,* Vol. 74, No. 2 (1998), 230–239.

——— forthcoming, "Mixed Logit Models for Recreation Demand," in C. Kling and J. Herriges (Eds.), *Valuing the Environment Using Recreation Demand Models,* Elgar Press.

Train, K., D., McFadden, and A. Goett, "Consumer Attitudes and Voluntary Rate Schedules for Public Utilities," *Review of Economics and Statistics,* Vol. LXIX, No. 3 (1987) 383–391.