

Estimation on stated-preference experiments constructed from revealed-preference choices

Kenneth Train *, Wesley W. Wilson

Department of Economics, University of California, Berkeley, CA 94720-3880, United States

Department of Economics, University of Oregon, United States

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Abstract

Constructing stated-preference (sp) experiments from a choice that the respondent made in a revealed-preference setting can enhance the realism of the sp task and the efficacy of preference revelation. However, the practice creates dependence between the sp attributes and unobserved factors, contrary to the independence assumption that is maintained for standard estimation procedures. We describe a general estimation method that accounts for this non-independence and give specific examples based on standard and mixed logit specifications of utility. We show conditions under which standard estimation methods are consistent despite the non-independence. We illustrate the general methodology through an application to shippers' choice of route and mode along the Columbia/Snake River system.

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1. Introduction

Stated-preference choice experiments are used extensively in economics and public policy (see, e.g., [Louviere et al., 2000](#), for a review of methods and applications). The standard way that stated-preferences are elicited is to construct hypothetical choice situations. Each situation consists of two or more options from which a survey respondent is asked to choose. The attributes of the options are varied over experiments to provide the variation needed for estimation of underlying preference parameters. The stated-preference (sp) data are often pooled with data on choices in actual market situations, called revealed-preference (rp) data, with extra parameters included to account for differences across the two types of data, such as differences in the variance of unobserved factors. Examples of this pooled approach include [Ben-Akiva and Morikawa \(1990\)](#), [Hensher and Bradley \(1993\)](#), and [Hensher et al. \(1999\)](#) within a logit specification and [Brownstone et al. \(2000\)](#) and [Bhat and Castelar \(2002\)](#) using mixed logit.

* Corresponding author. Tel.: +1 415 291 1023; fax: +1 415 291 1020.
E-mail address: train@econ.berkeley.edu (K. Train).

In standard sp experiments, the alternatives are constructed without regard to the respondent's choice in the rp setting (even if rp data are pooled with the sp data in estimation). However, researchers have also developed sp experiments on the basis of the respondent's rp choice.¹ For example, in examining route choice, Rose et al. (2005) ask the respondent to describe a recent trip. Hypothetical routes are constructed with times and costs that are some amount above or below those of the recent trip, and the respondent is then asked to choose among these hypothetical routes. The recent trip with its observed time and cost might be included under the label, e.g., "your recent trip". The procedure is called "pivoting" since the attributes in the sp experiment are created by changing the attributes of the chosen rp alternative. Applications include Hensher and Greene (2003), Hensher (2004, 2006), and Caussade et al. (2005).²

The advantage of pivoting is that it creates more realism in the sp experiments by assuring that the alternatives are similar to that which the respondent has experienced in an rp setting. It also provides greater specificity in the context of the sp choice task, since the respondent can think of the sp alternatives as being the same in unlisted attributes as the rp choice from which the sp alternatives were constructed. For example, the respondent can think about the recent trip when choosing among the alternative routes that are offered in the sp experiment. This specificity, of course, means that unobserved factors in the rp setting can be expected to carry over to the sp choice, creating a form of non-independence in conditional errors that needs to be represented in the estimation process.

The exact nature of the non-independence is worth noting, since its form affects the solution that we provide. In the rp setting, unobserved factors are usually assumed to be independent of the observed attributes of each alternative. However, since the person's choice depends on these unobserved factors as well as the observed variables, the unobserved factors are not independent of the attributes of the *chosen* rp alternative. When sp experiments are based on the chosen rp alternative, this non-independence is inherited: any unobserved factors from the rp setting that carry forward to the sp experiments become, by construction, non-independent of the attributes of the sp alternatives. The situation is analogous to regression models under self-selection where the regression errors, conditional on a discrete choice, are not independent of the explanatory variables (e.g., Heckman, 1979; Dubin and McFadden, 1984). The solution in these models is to enter a new variable that represents the conditional mean of the errors. In our case, the errors entering the sp choice, conditional on the discrete rp choice, are not independent, and we enter a new variable that represents these non-independent errors. However, since the sp choice model is nonlinear, we account for the entire conditional distribution of the errors rather than just their conditional mean.

The respondent's rp choice can be used to facilitate sp revelation in even more ways than have previously been explored by researchers. Under standard utility maximization, a person's rp choice changes only if the attributes of the chosen alternative become worse or the attributes of a non-chosen alternative improve. Sp questions can be designed that change the attributes of the rp alternatives in these directions. For example, in a mode choice situation, consider a respondent who has chosen bus when car, bus, and rail are available for the commute to work. For the sp experiments, the respondent is asked such questions as: "Would you still have chosen bus if the bus fare were \$1.50 instead of \$1.00?" or "Would you have switched to rail if the trains were 10 min faster than they are now?" Whatever the respondent answers, information about preferences is obtained, namely, that the value of the change is greater or less than the original difference in utility (mitigated by any new errors induced by the sp task).

These types of questions can be considered a form of pivoting, where the direction of the pivoting depends on the chosen rp alternative. However, they differ from the pivoted designs cited above in ways that affect the econometrics. To allow for ease of discussion, we call them "sp-off-rp" questions. There are two relevant

¹ Early applications are included in the review by Pearmain et al. (1991).

² Other studies, e.g., Hensher and Rose (2007), and Greene et al. (2006), have created sp alternatives by pivoting off the rp alternatives, but do so in a way that is independent of which rp alternative is chosen. The endogeneity that we address in this paper is not an issue in this type of pivoting, since the sp attributes do not depend on the rp choice. However, the concepts in Section 2.2 can nevertheless be used with these experimental designs to calculate probabilities for the sp choices conditional on the rp choice, with the unobserved attributes of each rp alternative entering the utility of each corresponding sp alternative. In fact, Bradley and Daly (2000) argue that unobserved attributes of the rp alternatives can carry over to the sp alternatives even in standard experiments where the sp attributes are constructed without reference to the rp alternatives other than through the use of labels, such as bus, rail, and car.

differences. First, with the pivoted experiments, the respondent faces whatever number of alternatives the researcher constructs and presents to the respondent in the sp task, whereas in sp-off-rp questions the respondent faces the same number of alternatives in the sp task as in the rp task. Second, and related to the first, in sp-off-rp questions, there is a one-to-one correspondence of the sp to the rp alternatives, whereas in the pivoted experiments cited above each of the sp alternatives corresponds to either one rp alternative (the chosen one) or no specific rp alternative. This difference affects the treatment of unobserved attributes, as we discuss below.

The advantages of sp-off-rp questions are that they are easy for the respondent to understand and contain a realism that might not be attained by either standard and pivoted sp experiments, since respondents face the same choice situation with the same alternatives in the sp-off-rp questions as in the rp setting. Also, as stated above, by changing attributes in the directions that are needed to induce a change in the respondent's rp choice, the respondent's answer to each sp-off-rp question necessarily reveals some information about preferences. The disadvantages are that these questions might be more subject than standard and pivoted experiments to prominence issues (by which asking about an attribute gives it more prominence in a respondent's decision than would occur in a real-world choice) and order bias (by which the order of the questions affect the response).³

In this paper, we describe econometrics that are applicable when sp experiments are based on an rp choice. To our knowledge, no previous study has explicitly delineated the implications of the non-independent errors inherent in these types of experiments or utilized estimation methods that account for them.⁴ We give general notation first, followed by specific examples for sp-off-rp questions and pivoted experiments, using both fixed and random coefficient utility specifications. We show that under certain conditions standard estimation methods are consistent for pivoted experiments, despite the endogeneity. More generally, however, alternative procedures are required. We illustrate the methods on data from sp-off-rp questions regarding shippers' choice of route and mode on the Columbia/Snake River system.

2. Econometrics

2.1. General specification

An agent faces J alternatives in an rp setting. The utility of each alternative depends on observed variables, denoted x_j for alternative j (with the subscript for the agent omitted for simplicity), and unobserved random factors denoted collectively as ε with density $f(\varepsilon)$. Utility of alternative j is denoted $U_j(x_j, \varepsilon)$, where the function U_j determines which elements of ε enter the utility for alternative j . Denote the chosen alternative as i and let A_i denote the set of ε 's that result in alternative i being chosen: $A_i = \{\varepsilon | U_i(x_i, \varepsilon) > U_j(x_j, \varepsilon) \forall j \neq i\}$. The probability that the agent chooses alternative i is $P_i = \text{Prob}(\varepsilon \in A_i) = \int I(\varepsilon \in A_i) f(\varepsilon) d\varepsilon$, where $I(\cdot)$ is an indicator of the event in parentheses occurring. The density of ε conditional on alternative i being chosen is $f(\varepsilon | \varepsilon \in A_i) = I(\varepsilon \in A_i) f(\varepsilon) / P_i$.

The researcher presents the agent with a series of sp experiments in which the attributes of the alternatives are constructed on the basis of the agent's rp choice. The researcher constructs T experiments, with attributes

³ Fowkes and Shinghal (2002) have implemented an sp procedure for assessing freight demand, called the Leeds adaptive stated-preference (LASP) method, that combines pivot and sp-off-rp concepts. The chosen rp alternative is used as a base, like pivoted experiments. A sequence of sp experiments is administered where an alternative that was favored in a previous experiment is made worse and/or an alternative that was disfavored in a previous experiment is improved. This adaptation uses the concept of sp-off-rp questions but applies it to each sp experiment based on previous experiments. Bradley and Daly (2000) discuss the endogeneity that arises from the procedure when respondents' data are combined for estimation, illustrating the inconsistency through Monte Carlo simulations. Fowkes and Shinghal argue that when the method is applied separately to each respondent to obtain each respondent's own utility coefficients, then endogeneity bias does not arise. Applications include Bergantino and Bolis (2006), Fowkes et al. (2004), and Shinghal and Fowkes (2002).

⁴ Other sources and forms of non-independent errors in choice models have been examined by, e.g., Rivers and Vuong (1988), Villas-Boas and Winer (1999), Berry et al. (2004), Blundell and Powell (2004), and Manchanda et al. (2004).

\tilde{x}_{jt}^i for alternative j in experiment t based on alternative i having been chosen in the rp setting. The agent is asked to choose among the alternatives in each sp experiment. The agent's choice can be affected by unobserved factors that did not arise in the rp situation, reflecting, e.g., inattention by the agent to the task, pure randomness in the agent's responses, or other quixotic aspects of the sp choices. These factors are labeled collectively as η with density $g(\eta)$. The relative importance of these factors will be estimated, as described below. The agent obtains utility $W_{jt}(\tilde{x}_{jt}^i, \varepsilon, \eta)$ from alternative j in sp experiment t , where the function W_{jt} determines which elements of ε and η enter this utility. Without loss of generality, we treat η as independent of ε , since any dependence can be captured by the function W_{jt} . In each experiment, the agent chooses the alternative with the greatest utility. The chosen alternative in experiment t is denoted k_t and vector $k = \langle k_1, \dots, k_T \rangle$ collects the sequence of sp choices.

We describe classical estimation by maximum likelihood or maximum simulated likelihood. Bayesian estimation procedures can be developed by adapting the methods in [Albert and Chib \(1993\)](#) and [Allenby and Rossi \(1999\)](#) with the concepts we develop here for classical methods. The joint probability of the agent's rp choice and sequence of sp choices can be expressed as the product of the probability of the agent's rp choice and the probability of the sp choices conditional on the rp choice. The probability of the rp choice is the standard choice model, $P_i = \text{Prob}(\varepsilon \in A_i)$, whose functional form is determined by the density of ε . The probability of the sp choices conditional on alternative i being chosen in the rp setting is

$$P_{k|i} = \text{Prob}(k|\varepsilon \in A_i) = \int \int I(\eta \in B_{k|i}(\varepsilon))g(\eta)f(\varepsilon|\varepsilon \in A_i)d\eta d\varepsilon \quad (1)$$

where $B_{k|i}(\varepsilon)$ is the set of η 's that, given ε , give rise to the agent's sequence of sp choices. That is,

$$B_{k|i}(\varepsilon) = \{\eta | W_{k_t t}(\tilde{x}_{k_t t}^i, \varepsilon, \eta) > W_{jt}(\tilde{x}_{jt}^i, \varepsilon, \eta) \forall j \neq k_t, \forall t\}.$$

The joint probability of the agent's rp and sp choices, which enters the log-likelihood function for maximum likelihood estimation, is

$$P_{ki} = \int I(\varepsilon \in A_i)f(\varepsilon) d\varepsilon \cdot \int \int I(\eta \in B_{k|i}(\varepsilon))g(\eta)f(\varepsilon|\varepsilon \in A_i) d\eta d\varepsilon. \quad (2)$$

Specific models are obtained by specifying distributions for ε and η and functional forms for U and W .

2.2. Sp-off-rp

We describe sp-off-rp experiments first because the one-to-one correspondence between sp and rp alternatives facilitates notation and specification.

2.2.1. Fixed parameters logit

Let $U_j(x_j, \varepsilon) = \beta x_j + \varepsilon_j$, where ε_j is iid extreme value with unit scale. The rp choice is, therefore, a standard logit. Let $W_{jt}(\tilde{x}_{jt}^i, \varepsilon, \eta) = \beta \tilde{x}_{jt}^i + \varepsilon_j + \eta_{jt}$, where the subscript j on ε_j refers to the corresponding alternative in the rp setting. Under this specification, the agent evaluates each sp alternative using the same utility coefficients and same ε_j as in the rp situation,⁵ but with the addition of a new error to account for quixotic aspects of the sp task. Let η_{jt} be iid extreme value with scale $(1/\alpha)$. A large value of parameter α indicates that there are few quixotic aspects to the sp choices and that the agent chooses essentially the same as in an rp situation under the new attributes. Utility can be equivalently expressed as $W_{jt}(\tilde{x}_{jt}^i, \varepsilon, \eta) = \alpha(\beta \tilde{x}_{jt}^i + \varepsilon_j) + \eta_{jt}$ where now η_{jt} is iid extreme value with unit scale. The sp choices are, therefore, standard logits with ε_j as an extra explanatory variable. Since the ε_j 's are not observed, these logits must be integrated over their conditional distribution, as follows.

⁵ The hypothesis that the same utility coefficients apply can be tested as well as whether ε_j enters utility in the sp choice.

The probability of alternative k_t in sp experiment t , conditional on i being chosen in the rp choice is

$$\begin{aligned}
 P_{k_t|i} &= \text{Prob} \left[\alpha(\beta \tilde{x}_{k_t}^i + \varepsilon_{k_t}) + \eta_{k_t} > \alpha(\beta \tilde{x}_{j_t}^i + \varepsilon_j) + \eta_{j_t} \forall j \neq k_t, \quad \forall t | \beta x_i + \varepsilon_i > \beta x_j + \varepsilon_j \forall j \neq i \right] \\
 &= \int \frac{e^{\alpha \beta \tilde{x}_{k_t}^i + \alpha \varepsilon_{k_t}}}{\sum_j e^{\alpha \beta \tilde{x}_{j_t}^i + \alpha \varepsilon_j}} f(\varepsilon | \beta x_i + \varepsilon_i > \beta x_j + \varepsilon_j \forall j \neq i) d\varepsilon.
 \end{aligned}
 \tag{3}$$

This probability is a mixed logit, mixed over the conditional density of ε .⁶ It can be simulated by taking draws of ε from its conditional density, calculating the logit probability for each draw, and averaging the results.

Draws of ε from its conditional density are easy to obtain, given the convenient form of the conditional density of extreme value deviates. In particular, the density of ε_i conditional on alternative i being chosen in the rp setting is extreme value with mean shifted up by $-\ln(P_i)$ (Anas and Feng, 1988.) A draw is obtained as $-\ln(P_i) - \ln(-\ln(\mu))$ where μ is a draw from a uniform between zero and one. Conditional on ε_i and on i being chosen, the density of each $\varepsilon_j, j \neq i$, is extreme value truncated above at $\beta x_i - \beta x_j + \varepsilon_i$. A draw is obtained as $-\ln(-\ln(m(\varepsilon_i)\mu))$, where μ is a draw from a uniform between zero and one, and $m(\varepsilon_i) = \exp(-\exp(-(\beta x_i - \beta x_j + \varepsilon_i)))$. Details are given in the Appendix. Since draws of ε are constructed analytically from draws from a uniform (as opposed to by accept–reject methods), variance reduction procedures can readily be applied, such as Halton draws (Bhat, 2001; Train, 2003), (t,m,s)-nets (Sándor and Train, 2004), and modified Latin hypercube sampling (Hess et al., 2006).

Combining these results, and using the independence of η_{jt} over t , the probability of the agent’s rp choice and the sequence of sp choices is

$$P_{ki} = \int [L_{1|i}(\varepsilon) \dots L_{T|i}(\varepsilon)] f(\varepsilon | \beta x_i + \varepsilon_i > \beta x_j + \varepsilon_j \forall j \neq i) d\varepsilon \cdot \frac{e^{\beta x_i}}{\sum_j e^{\beta x_j}}
 \tag{4}$$

where

$$L_{t|i}(\varepsilon) = \frac{e^{\alpha \beta \tilde{x}_{k_t}^i + \alpha \varepsilon_{k_t}}}{\sum_j e^{\alpha \beta \tilde{x}_{j_t}^i + \alpha \varepsilon_j}}.$$

This probability is simulated by taking draws of ε from its conditional density as described above, calculating the product of logits within brackets for each draw, averaging the results, and then multiplying by the logit probability of the rp choice.

Note that as $\alpha \rightarrow \infty$ the simulator for the probability of the sp choices approaches an accept–reject simulator based on the respondent’s utility function in the rp setting with no additional errors (McFadden, 1989; Train, 2003, Sections 5.6.2 and 6.5). Seen in this light, for large α , the logit formula for the sp-off-rp choices can be seen as a smoothed accept–reject simulator based on the true utility $\beta \tilde{x}_{j_t}^i + \varepsilon_j$, whose purpose is to improve numerical optimization rather than having a behavioral interpretation.

2.2.2. Random coefficients logit

Utility is as above except that β is now random with density $h(\beta)$ that depends on parameters (not given in the notation) that represent, e.g., the mean and variance of β . Define $A_i(\beta) = \{\varepsilon | \beta x_i + \varepsilon_i > \beta x_j + \varepsilon_j \forall j \neq i\}$. Then the probability for the rp choice is

$$P_i = \int \text{Prob}(\varepsilon \in A_i(\beta)) h(\beta) d\beta = \int L_i(\beta) h(\beta) d\beta,
 \tag{5}$$

where $L_i(\beta) = \frac{e^{\beta x_i}}{\sum_j e^{\beta x_j}}$. This is a standard mixed logit. The density of β conditional on i being chosen is $h(\beta | \beta x_i + \varepsilon_i > \beta x_j + \varepsilon_j \forall j \neq i) = L_i(\beta) h(\beta) / P_i$.

For the sp choices, let $L_{t|i}(\varepsilon, \beta)$ be the same as $L_{t|i}(\varepsilon)$ defined above but with β treated as an argument. The probability of the sequence of sp choices conditional on the rp choice is

⁶ See Train (2003, Chapter 6), for a description of mixed logit with historical references.

$$\begin{aligned}
P_{k|i} &= \int \int L_{1|i}(\varepsilon, \beta) \dots L_{T|i}(\varepsilon, \beta) f(\varepsilon | \varepsilon \in A_i(\beta)) h(\beta | \beta x_i + \varepsilon_i > \beta x_j + \varepsilon_j \forall j \neq i) d\beta d\varepsilon \\
&= \int \int L_{1|i}(\varepsilon, \beta) \dots L_{T|i}(\varepsilon, \beta) f(\varepsilon | \varepsilon \in A_i(\beta)) L_i(\beta) h(\beta) d\beta d\varepsilon / P_i.
\end{aligned} \tag{6}$$

The probability of the rp choice and the sequence of sp choices is P_i times expression (6), which gives:

$$P_{ki} = \int \left[\int L_{1|i}(\varepsilon, \beta) \dots L_{T|i}(\varepsilon, \beta) f(\varepsilon | \varepsilon \in A_i(\beta)) d\varepsilon \right] L_i(\beta) h(\beta) d\beta. \tag{7}$$

This probability is simulated by

1. Draw a value of β from its unconditional density.
2. Calculate the logit probability for the rp choice using this β .
3. Draw numerous values of ε from its conditional density given β using the method described above. Calculate the product of logit formulas for the sp choices for each draw of ε and average the results.
4. Multiply the result from step 3 by the result from step 2.
5. Repeat steps 1–4 numerous times and average the results.

In theory, only one draw in step 3 is required for each draw in step 1; however, taking more than one draw in step 3 improves accuracy for each draw of β and is relatively inexpensive from a computational perspective. Variance reduction procedures, as given above, can be applied in steps 1 and 3.

2.3. Pivoted experiments

In pivoted experiments, each sp alternative corresponds either to the chosen rp alternative or to no specific rp alternative, depending on how the respondent views the sp alternatives. We consider each possibility.

2.3.1. Fixed coefficients logit

Utility in the rp setting is the same as above: $U_j(x_j, \varepsilon) = \beta x_j + \varepsilon_j$, where ε_j is iid extreme value with unit scale. The attributes of each alternative j in experiment t are constructed from the attributes of chosen rp alternative i (rather than from rp alternative j as for the sp-off-rp experiments). Let us first assume that the respondent evaluates each sp alternative utilizing the unobserved attributes of its chosen rp alternative, ε_i . That is, let $W_{jt}(\tilde{x}_{jt}^i, \varepsilon, \eta) = \alpha \beta \tilde{x}_{jt}^i + \alpha \varepsilon_i + \eta_{jt}$. This is the same as for rp-off-sp experiments except that now the same ε_i enters W_{jt} for all j whereas with sp-off-rp experiments a different ε_j enters each W_{jt} . Since only differences in utility matter, utility can be equivalently expressed without ε_i as $W_{jt}(\tilde{x}_{jt}^i, \varepsilon, \eta) = \alpha \beta \tilde{x}_{jt}^i + \eta_{jt}$. The probability of the sp choice is a standard logit, which can be estimated with standard estimation methods. The endogeneity of the sp attributes takes a form that cancels out of the behavioral model. Using just the sp data, the product $\alpha \beta$ is identified, such that utility parameters β that determine rp choices are estimated up to a scale factor. If the sp data are pooled with the rp data, then α and β are separately identified.

Often pivoted experiments include the chosen rp alternative with its unchanged attributes as one of the sp alternatives, with this alternative labeled, e.g., “your recent trip”. In this situation, the respondent might evaluate this alternative using the unobserved attributes of the chosen alternative in the rp setting and, yet, evaluate the other sp alternatives without using these unobserved attributes. That is, the respondent might use ε_i in evaluating the sp alternative that is the same as the chosen rp alternative but not for the other sp alternatives. In this case, utility of the sp alternatives is $W_{jt}(\tilde{x}_{jt}^i, \varepsilon, \eta) = \alpha \beta \tilde{x}_{jt}^i + \alpha \delta_{ji} \varepsilon_i + \eta_{jt}$, where $\delta_{ji} = 1$ if sp alternative j is the same as rp alternative i , and $=0$ otherwise. Since ε_i enters only one of the sp alternatives, it does not drop out of utility differences. The probability of the respondent choosing sp alternative k_t given that the respondent chose rp alternative i is:

$$P_{k_t|i} = \int \frac{e^{\alpha \beta \tilde{x}_{k_t}^i + \alpha \delta_{k_t i} \varepsilon_i}}{\sum_{\ell} e^{\alpha \beta \tilde{x}_{\ell}^i + \alpha \delta_{\ell i} \varepsilon_i}} f(\varepsilon_i | \beta x_i + \varepsilon_i > \beta x_j + \varepsilon_j \forall j \neq i) d\varepsilon_i \tag{8}$$

where the summation over ℓ is over the sp alternatives. This is the same as the choice probability for sp-off-rp experiments except that ε_j is replaced with $\delta_{ji}\varepsilon_i$. Estimation is performed the same as described above for sp-off-rp experiments, by drawing from the conditional distribution of ε_i . The hypothesis that respondents utilize ε_i in all sp alternatives (or none) can be tested by determining whether $\delta_{ji}\varepsilon_i$ enters the sp choice probability significantly.

Given that this estimation is more difficult than standard logit estimation, it is perhaps advisable for the researcher to put special effort into assuring that the respondent uses the same unobserved attributes when evaluating the sp alternatives, whether or not the alternative is labeled as, e.g., “your last trip.” For example, the researcher might instruct the respondent to suppose that all the sp alternatives are the same as the respondent’s last trip except in regard to the attributes that are listed. However, results by Huber and McCann (1982), Feldman and Lynch (1988), Broniarczyk and Alba (1994) and Bradlow et al. (2004), e.g., suggest that respondents might not be able or willing to do so even when instructed.

2.3.2. Random coefficients logit

Assume that the respondent does indeed use ε_i in evaluating all of the sp experiments, such that it does not enter utility differences. Under this assumption, random coefficient specifications for pivoted experiments can be estimated with standard mixed logit estimation routines, as long as the sp data are pooled with the rp data. Estimation on sp data alone are inconsistent when coefficients are random since the conditional distribution of coefficients (i.e., conditional on the rp choice) differs over respondents and cannot be calculated without the rp data. Using the notation above, β has unconditional density $h(\beta)$, and its density conditional on rp alternative i being chosen is $L_i(\beta)h(\beta)/P_i$. The probability of sp alternative k_i conditional on rp alternative i being chosen is

$$P_{k_i|i} = \int \frac{e^{\alpha\beta x_{k_i t}}}{\sum_{\ell} e^{\alpha\beta x_{\ell t}}} h(\beta | \beta x_i + \varepsilon_i > \beta x_j + \varepsilon_j \forall j \neq i) d\beta = \int \frac{e^{\alpha\beta x_{k_i t}}}{\sum_{\ell} e^{\alpha\beta x_{\ell t}}} L_i(\beta) h(\beta) d\beta / P_i. \quad (9)$$

The rp data are required for calculation of this sp choice probability, since the conditional density of β is a function of $L_i(\beta)$. The joint probability of the sp and rp choices is a standard mixed logit (the above formula multiplied by P_i , thereby cancelling the division by P_i), which can be estimated with standard software.

If the respondent utilizes ε_i in evaluating one sp alternative but not the others, and utility coefficients are random, then the joint probability of sp and rp choices is the same as for rp-off-sp experiments except that ε_j is replaced by $\delta_{ji}\varepsilon_i$.

3. Application: route and mode choice on the Columbia/Snake river

As an illustration of the method, we examine agricultural shippers in the Pacific Northwest using sp-off-rp questions. Eastern Washington is one of the primary wheat producing regions in the United States and has the largest wheat-producing county, Whitman Country, in the US (Jessup and Casavant, 2004a). The region has an interconnected transportation system that consists of a series of rail lines and the Columbia-Snake River basin. Nearly all of the wheat travels to ocean terminals located in or near Portland, Oregon. Our analysis examines the mode and route choice of shippers in this area, given that the destination is Portland.

A survey of warehouses was conducted in October of 2004 by the Social and Economic Sciences Research Center at Washington State University. Details of the survey instrument and sampling methodology are given by Jessup and Casavant (2004b). The survey was pre-tested and reviewed by academics and target survey recipients. The survey included both grain and non-grain shippers. Grain shippers represent the bulk of the population (over 80%) and the bulk of the respondents (over 85%). Only two of the survey recipients refused to participate in the survey. Responses were obtained for 181 warehouses, which constitutes 46% of the 391 eligible warehouses in the area.

Shippers were asked to provide information on the last shipment that they had made. Six alternatives constitute the universe of alternatives available to shippers in the area, with each individual shipper facing a subset of these six:

1. truck to Pasco and barge to Portland;
2. truck to another barge port and barge to Portland;

Table 1
Revealed choice data

Option	<i>N</i>	Available (percent)	Choice (percent)	Rate (per ton-mile)	Time (days)	Reliability
Truck to Pasco-Barge to Portland	120	61.3	7.3	5.0	11.2	77.3
Truck to Port-Barge to Portland	107	54.7	32.7	4.2	4.1	90.5
Rail to Portland	65	33.4	16.1	3.7	10.4	63.2
Truck to Rail-Rail to Portland	95	50.9	13.7	4.2	11.3	73.0
Barge to Portland	22	12.3	8.3	2.6	1.1	88.1
Other	12	11.8	21.9	13.1	4.4	90.1

3. rail to Portland;
4. truck to a rail terminal and rail to Portland;
5. barge to Portland;
6. other.

Shippers were asked which of these options were available to them and which one they used for their last shipment.⁷ For each available option, respondents were also asked to provide rates, transit-times and reliability measures.⁸ Transit-times were specified to include the scheduling, waiting time for equipment, and travel time. Reliability was measured by asking the shippers to estimate the percentage of time that shipments like this arrive “on-time” at the final destination. These data constitute the rp data for the analysis.

Table 1 provides summary statistics on the responses by option. As expected, the rate per ton-mile by barge to Portland is the lowest of all options. It is somewhat unexpected that the transit-times are also lowest for barge. However, transit-times include scheduling and waiting for equipment, and multi-modal shipments require added scheduling, waiting for equipment, etc. Finally, movements that involve barge-only or a truck-barge combination yield the most reliable service, while railroad-alone and truck-rail entail the lowest reliability.

The respondents were asked a series of sp-off-rp questions. Each shipper was first asked if they would stay with their original choice or switch to another alternative if the rate on their chosen option were X percent higher, where X was random selected from 10, 20, 30, 40, 50 and 60.⁹ For shippers who stated they would switch, the alternative that they said they would switch to was determined. Similar questions were asked for an increase in transit-time and a decrease in reliability. About two-thirds (68%) of the responses to these questions were that the respondent would switch, with the switching rate being higher for the rate changes than the time and reliability changes.

Table 2 gives the estimated parameters of a standard logit model that was estimated on the rp data alone. The estimated coefficients of rate, time, and reliability all take the expected signs, and the rate and reliability coefficients are significant at the 95% confidence level. The ratios of coefficients imply that a day of extra transit-time is considered equivalent to about 27 cents per ton in higher rates and that decreasing reliability by 1% point is considered equivalent to 26 cents per ton in higher rates. These two estimated values being nearly the same seems unreasonable. First, note that, absent risk aversion, the expected value of a 1% increase in the chance of a one-day delay is 1/100 the expected value of one day of extra transit-time. While unexpected delays can be more burdensome than an anticipated increase in transit-time, and the delay may be for more than a day, it seems doubtful that these factors are sufficient to counteract the 100-fold difference in these expected

⁷ Over a quarter of the respondents (51) reported that they have only one alternative available – the one that they used for their last shipment. This fairly larger share of reportedly captive shippers was also found in an analysis of Upper Mississippi river shippers (Train and Wilson, 2004). Since these shippers state that they have no choice, they were not included in the econometric analysis.

⁸ These type of self-report data can be subject to a variety of biases, including respondents reporting better attributes than are real for their chosen alternative as a way of justifying the choice. Identifying any such biases is beyond the scope of the current paper; however, it is worth noting that some of the empirical results presented below, such as the high value of reliability implied by the rp model, could perhaps be an artifact of this data collection method.

⁹ An important area for future research is the optimal design of sp-off-rp questions. Burgess and Street (2005) and Rose (2005), e.g., describe efficient designs for standard sp experiments, and Rose et al. (2005) describe efficient design of pivoted sp experiments. Similar concepts applied to sp-off-rp questions can be expected to increase efficiency relative to randomly selected changes in attributes.

Table 2
Fixed coefficients logit model of route/mode choice estimated on rp data only

Explanatory variable	Estimated parameter	Standard error	<i>t</i> -statistics
Rate, in dollars per ton	−0.1252	0.0633	1.977
Time, in days	−0.0342	0.0320	1.070
Reliability	0.0322	0.0114	2.839
Constant for alt. 1	−1.7421	0.5579	3.123
Constant for alt. 3	1.0753	0.5103	2.107
Constant for alt. 4	−0.6748	0.3963	1.703
Constant for alt. 5	−0.4564	0.7818	0.584
Constant for alt. 6	−0.5962	1.0561	0.565
Mean log likelihood at convergence	−0.838280		

Table 3
Fixed coefficients logit model of route/mode choice estimated on rp and sp-off-rp data

Explanatory variable	Estimated parameter	Standard error	<i>t</i> -statistics
Rate, in dollars per ton	−0.2086	0.0371	5.625
Time, in days	−0.1483	0.0233	6.356
Reliability	0.0282	0.0046	6.127
Constant for alt. 1	−0.1037	0.3378	0.307
Constant for alt. 3	0.9921	0.3965	2.502
Constant for alt. 4	−0.1021	0.3073	0.332
Constant for alt. 5	−0.9890	0.0775	1.276
Constant for alt. 6	−0.9287	1.0711	0.867
Scale of additional sp errors (α)	5.5874	1.6223	3.444
Mean log likelihood at convergence	−2.34026		

values. Second, previous studies on shippers' values (Shinghal and Fowkes, 2002; Bergantino and Bolis, 2006) have found that that a day of time savings is worth more than a 1% reduction in the chance of delay.

Table 3 gives the estimated parameters of a fixed-coefficients logit estimated on the combined rp and sp choices. Simulation was performed with 1000 pseudo-random draws of the conditional extreme value terms, with different draws for each observation. As expected, the level of significance for the coefficients of rate, time, and reliability rise considerably. The scale parameter α is estimated to be about 5.6, which implies that the standard deviation of the additional unobserved portion of utility that affects the sp choices is less than a fifth as large as the standard deviation of unobserved utility in the rp choices. As discussed above, if there were no quixotic aspects to the sp choices such that the respondent answered the same as in the rp setting with the changed attributes, then the standard deviation would be zero (α unbounded high.) The relatively small estimated standard deviation implies that respondents were apparently paying careful attention to the sp tasks and answering similarly to how they would behave in the rp setting.¹⁰

The relative values of time and reliability seem more reasonable when the sp data are utilized. In particular, the value of time rises from 27 to 71 cents per ton, and the value of reliability drops from 26 to 14 cents per ton. The magnitudes of these changes, though large from a policy perspective, are not unreasonable given the standard errors in Table 2. In fact, the changes confirm the purpose of utilizing sp data, which is to augment rp data when the rp data contain insufficient variation to estimate parameters precisely.

We next examine a random coefficients specification. The time and reliability coefficients are specified to be distributed normally with censoring at zero.¹¹ That is, the coefficient of time is specified as $\min(0, \beta_2)$, where β_2 is normally distributed with mean and standard deviation that are estimated; and the coefficient of reliability is $\max(0, \beta_3)$ with normal β_3 . This specification assures that the time and reliability coefficients have the expected

¹⁰ This strict interpretation should not be taken too far, of course, since small estimated standard deviations could arise for other reasons.

¹¹ See Train and Sonnier, 2005, for a discussion and application of censored normals and other distributions with bounded support within mixed logit models.

Table 4
Random coefficients logit model of route/mode choice estimated on rp and sp-off-rp data

Parameter	Estimate	Standard error	<i>t</i> -statistics
Rate	−0.2325	0.0306	7.610
Time:mean	−0.3031	0.0603	5.027
Time:standard deviation	0.2235	0.0648	3.448
Reliability:mean	0.03674	0.0054	6.756
Reliability:standard deviation	0.0170	0.0045	3.777
Constant for alt. 1	−0.2006	0.3734	0.537
Constant for alt. 3	1.1227	0.4326	2.595
Constant for alt. 4	−0.3469	0.3759	0.923
Constant for alt. 5	−1.2563	0.7883	1.594
Constant for alt. 5	−0.9684	1.1192	3.448
Mean log likelihood at convergence	−2.22959		

Scale of sp choices held at $\alpha = 10$.

sign throughout their support. Also, by having a mass at zero, the specification allows for the possibility that some shippers do not care about time or reliability (at least within the ranges that are relevant). The rate coefficient is held fixed, following Goett et al. (2000) and Hensher et al. (2005, 2006), which implies that the distribution of the value of time and reliability is simply the distribution of these variables' coefficients scaled by the fixed price coefficient.¹²

When we attempted to estimate the random coefficients model with all parameters free, the value of α rose without bound in the iterative maximization process. This result, taken at face value, implies that no additional errors enter the sp choices, beyond the unobserved portion of utility in the rp choices. Since a bounded α was obtained with the fixed coefficients model, the unbounded value in the random coefficients model implies that differences in coefficients account for the sp responses that seem quixotic in a fixed coefficients model. That is, sp responses that appear quixotic when all shippers are assumed to have the same coefficients for rate, time and reliability are found not actually to be quixotic when shippers are allowed to have different coefficients.

Table 4 gives the estimated parameters for a random coefficients model with α set at 10. Simulation was performed with 1000 draws of the random coefficients and 10 draws of the extreme value terms for each draw of the random coefficients (for 10,000 draws of the extreme value terms in total for each observation). As described above, the large value of α can be interpreted as providing a logit-smoothed accept–reject simulator of the probability of the sp choices, which aids numerical maximization without reflecting the existence of any additional errors. The estimated mean value of time is \$1.34 per ton with a standard deviation of 0.89, and the estimated mean value of reliability is 16 cents with a standard deviation of 7.2 cents.¹³ The mean value of time is higher than that obtained with fixed coefficients (\$1.34 versus \$0.71), while the mean value of reliability is about the same (16 cents versus 14 cents). Fewer than 9% of shippers are estimated not to care about transit-time (i.e., the mass at zero is less than 0.09), and fewer than 2% are estimated not to care about reliability.

4. Conclusion

We have described estimation methods based on fixed and random coefficients logit specifications for sp experiments that are constructed from the respondent's rp choice. The method is illustrated on the choice of mode and route by shippers in the Pacific Northwest. It is found to perform well, giving expected results

¹² Ruud (1996) points out that a random coefficients model with all random coefficients is nearly unidentified empirically, especially with only one or a few observed choices per agent, since only ratios of coefficients are behaviorally meaningful. Holding the price coefficient fixed assists with empirical identification. Train and Weeks (2005) discuss reasons for and against holding the price coefficient fixed and compare estimation methods when the price coefficient is random.

¹³ These statistics are unconditional moments, i.e., for the population as a whole. Moments conditional on each shipper's choices can be calculated using the procedures in Train (2003, Chapter 11).

and providing an improvement over the use of rp data alone. An instructive next step will be to implement standard, pivoted, and sp-off-rp questions in the same setting and compare results across the three methods. Of course, the use of one method does not preclude the others, and researchers can, if they wish, use two or three of the methods in a given setting, adapting the estimation methods in this paper accordingly to account for pooling of the various sp data.

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Appendix. The density of extreme value errors conditional on the chosen alternative

Domencich and McFadden (1975), Ben-Akiva (1976) and Williams (1977) give the distribution of $\max(U_j, j = 1, \dots, J)$ when each U_j is distributed extreme value. Anas and Feng (1988) build upon these results to derive the distribution of U_i given that $U_i > U_j \forall j \neq i$, which, when the observed portion of utility is subtracted, gives the conditional density of ε_i conditional on i being chosen. However, Anas and Feng’s result is not widely known, and, though the extension is straightforward such that their work can be considered to provide the entire conditional density, they do not explicitly describe the conditional density of U_j for non-chosen alternatives $j \neq i$. We therefore thought that it might be useful to provide a derivation here.

For notational convenience, let the chosen alternative be $i = 1$. Utility is $U_j = V_j + \varepsilon_j$ for $j = 1, \dots, J$, where ε_j is iid extreme value with density $f(\varepsilon_j) = \exp(-\exp(-\varepsilon_j))\exp(-\varepsilon_j)$ and distribution $F(\varepsilon_j) = \exp(-\exp(-\varepsilon_j))$. We want to draw from the distribution of $\varepsilon = \langle \varepsilon_1, \dots, \varepsilon_J \rangle$ conditional on alternative 1 being chosen. Define $V_{1j} = V_1 - V_j$ and $D_1 = \sum_{j=1}^J \exp(-V_{1j})$. Denote by A_1 the set of ε ’s for which $\varepsilon_j < V_{1j} + \varepsilon_1$ such that alternative 1 is chosen. The logit choice probability is $P_1 = Pr(\varepsilon \in A_1) = 1/D_1$.

The joint density of ε conditional alternative 1 being chosen is

$$f(\varepsilon | \varepsilon \in A_1) = \frac{f(\varepsilon_1) \dots f(\varepsilon_J) I(\varepsilon \in A_1)}{P_1} = D_1 f(\varepsilon_1) \dots f(\varepsilon_J) I(\varepsilon \in A_1),$$

where $I(\cdot)$ is an indicator that the statement in parentheses is true. The marginal density of ε_1 conditional on alternative 1 being chosen is then

$$\begin{aligned} f(\varepsilon_1 | \varepsilon \in A_1) &= \int_{\varepsilon_2, \dots, \varepsilon_J} f(\varepsilon | \varepsilon \in A_1) d\varepsilon_2 \dots d\varepsilon_J = D_1 f(\varepsilon_1) F(V_{12} + \varepsilon_1) \dots F(V_{1J} + \varepsilon_1) \\ &= D_1 \exp(-D_1 \exp(-\varepsilon_1)) \exp(-\varepsilon_1) \end{aligned}$$

such that its conditional distribution is $F(\varepsilon_1 | \varepsilon \in A_1) = \exp(-D_1 \exp(-\varepsilon_1))$. A draw is obtained from this distribution by taking a draw, μ , from a uniform distribution between 0 and 1 and calculating the inverse of this distribution function, that is, $\varepsilon_1 = \ln(D_1) - \ln(-\ln(\mu))$. The draws are the same as from an unconditional extreme value distribution, but with the mean raised by $\ln(D_1) = -\ln(P_1)$.

Draws of the other errors are then conditioned on the draw of ε_1 . The density of $\langle \varepsilon_2, \dots, \varepsilon_J \rangle$ conditional on ε_1 and on alternative 1 being chosen is

$$f(\varepsilon_2, \dots, \varepsilon_J | \varepsilon \in A_1, \varepsilon_1) = \frac{f(\varepsilon_1) f(\varepsilon_2) \dots f(\varepsilon_J) \cdot I(\varepsilon \in A_1)}{P_1 \cdot f(\varepsilon_1 | \varepsilon \in A_1)} = \frac{f(\varepsilon_2) \dots f(\varepsilon_J) \cdot I(\varepsilon \in A_1)}{F(V_{12} + \varepsilon_1) \dots F(V_{1J} + \varepsilon_1)}.$$

The marginal density of $\varepsilon_j, j > 1$ (i.e., marginal over $\varepsilon_k \forall k \neq 1, j$), conditional on alternative 1 being chosen and on ε_1 , is then

$$f(\varepsilon_j | \varepsilon \in A_1, \varepsilon_1) = \frac{F(V_{12} + \varepsilon_1) \dots f(\varepsilon_j) \dots F(V_{1J} + \varepsilon_1) I(\varepsilon_j < V_{1j} + \varepsilon_1)}{F(V_{12} + \varepsilon_1) \dots F(V_{1j} + \varepsilon_1) \dots F(V_{1J} + \varepsilon_1)} = \frac{f(\varepsilon_j) I(\varepsilon_j < V_{1j} + \varepsilon_1)}{F(V_{1j} + \varepsilon_1)}$$

with distribution $F(\varepsilon_j | \varepsilon \in A_1, \varepsilon_1) = \frac{F(\varepsilon_j)}{F(V_{1j} + \varepsilon_1)} = \frac{\exp(-\exp(-\varepsilon_j))}{\exp(-\exp(-(V_{1j} + \varepsilon_1)))}$ for $\varepsilon_j < V_{1j} + \varepsilon_1$. A draw is obtained by taking a draw, μ , from a uniform between 0 and 1 and calculating $\varepsilon_j = -\ln(-\ln(m(\varepsilon_1) \cdot \mu))$, where $m(\varepsilon_1) = \exp(-\exp(-(V_{1j} + \varepsilon_1)))$. This is the same as taking a draw from an unconditional extreme value distribution truncated from above at $V_{1j} + \varepsilon_1$.

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