## ROBINSON CRUSOE MEETS WALRAS AND KEYNES ${ }^{1}$

Once upon a time on an idyllic South Seas Isle lived a shipwrecked sailor, Robinson Crusoe, in solitary splendor. The only product of the island, fortunately adequate for Robinson's sustenance, was the wild yam, which Robinson found he could collect by refraining from a life of leisure long enough to dig up his dinner. With a little experimentation, Robinson found that the combinations of yams and hours of leisure he could obtain on a typical day (and every day was a typical day) were given by the schedule shown in Figure 1. ${ }^{2}$

Being a rational man, Robinson quickly concluded that he should on each day choose the combination of yams and hours of leisure on his production possibility schedule which made him the happiest. Had he been quizzed by a patient psychologist, Robinson would have revealed the preferences illustrated in Figure 2, with I1, I2, ... each representing a locus of leisure/yam combinations that make him equally happy; e.g., Robinson would prefer to be on curve I2 rather than I1, but if forced to be on curve I2, he would be indifferent among the points in I2. You can think of Robinson's preferences being represented as a mountain, the higher the happier, and
 Figure 2 is a contour map of this preference mountain, with each contour identifying a fixed elevation. ${ }^{3}$

The result of Robinson's choice is seen by superimposing Figures 1 and 2. The combination of yams and leisure denoted E in Figure 3, the highest preference contour touching the set of production

[^0]possibilities, is the best point he can reach given the technology at his disposal. ${ }^{4}$ Robinson consumes $Y_{E}=6$ pounds of yams at this point and spends $\mathrm{H}_{\mathrm{E}}=18$ hours of leisure out of his available endowment of 24 hours per day. The remaining time, $\mathrm{L}_{\mathrm{E}}=24-\mathrm{H}_{\mathrm{E}}=$ 6 , is spent at labor digging up yams.

Robinson was reconciled to a peaceful, if tedious, existence consuming at the combination E each day. However, he was startled one morning to find wading through the breakers a rather scholarly gentleman in bedraggled formal attire who introduced himself as Leon Walras. It was quickly established that Walras was a student of the organization of economic activity. Walras was at first content to sit on the beach all day, watching Robinson dig yams and making occasional helpful comments on the use of clam shells to sharpen digging sticks. (Walras never partook of yams himself, but disappeared every evening. Robinson suspected Walras of seeking the companionship and table of Gauguin, who was leading a more bohemian existence on a neighboring island.)

One day, Walras remarked "It must be difficult when engaging in a mindless activity like digging yams to remember exactly what yam combination maximizes your preferences." Robinson really had no trouble at all, but to be agreeable he assented. Walras then said "You know, the way you run your life is rather feudal. If you would like, I will help you reorganize using the most modern techniques for market mechanism design,

## 2.Robinson's Preference ContouI




[^1]direct from Paris". ${ }^{5}$ Robinson was rather suspicious of this offer, since he had noted that like most economists, Walras was better at giving advice than really digging. He muttered under his breath, "Those who can, dig; those who can't, teach.". However, Walras' proposal promised a diversion from his daily drudgery, and be was anxious not to appear old-fashioned to his distinguished visitor. So he agreed to go along with the experiment.

Walras then outlined his proposal. "You, Robinson, should form a yam-digging company. Call it Crusoe, Inc. Let me introduce an acquaintance, Mr. Friday, who is standing just behind that palm tree, and whom I recommend you appoint as manager of Crusoe, Inc." Robinson was alarmed at this point at the appearance of a young man dressed, like most professional managers, in a grey flannel Brooks Brothers suit. However, Mr. Friday's ready knowledge of yam-digging and corporate finance quickly reassured Robinson, particularly after Mr. Friday explained that his management fee would be exactly equal to the additional yams dug per hour under his direction. This meant that Robinson would in effect have exactly the same leisure-yams production possibilities shown in Figure 1 as he before, and he would not have to worry about where to dig. So Robinson agreed to the arrangement, and asked Walras to finish describing his plan.
"Each morning," said Walras, "I will call out a wage rate in yams per hour. You should instruct Mr. Friday, as your manager, to offer to hire the amount of labor and produce the quantity of yams which maximize your dividends as owner of Crusoe, Inc. He should report these offers to me, and should inform you of the dividends he expects to pay you. Given this dividend income and the wage rate I have called out, you should inform me of the amount of labor you choose to supply and yams you want to demand. If your supplies and demands don't coincide with Crusoe, Inc., then I will call out another wage rate, and we'll start over. When we finally hit a wage rate where supply equals demand, then I'll stop and you and Mr. Friday can trade the amounts of labor and yams you offered."
"Could you run through that again from the top?" asked Robinson.
"Certainly," said Walras. "Let's start with the instructions to Mr. Friday. Suppose I call out a wage rate of, say, $w=0.3$ pounds per hour. Let's draw a graph (Figure 4) of various combinations of labor in and yams out available to Crusoe, Inc. ${ }^{" 6}$ Robinson noted that this was the same graph as his Figure 1, except that instead of starting from zero leisure and measuring leisure to the right, Walras was placing the origin at zero labor (or 24 hours of leisure) and measuring labor to the left. Walras continued, "Let's see what dividend income you would receive if Mr.
 Friday offered to operate the firm with $\mathrm{L}_{\mathrm{A}}=6$ hours

[^2]per day of labor input, producing $\mathrm{Y}_{\mathrm{A}}=6$ pounds of yams. He would pay you 1.8 pounds per day in wages, the product of the wage $\mathrm{w}=0.3$ and $\mathrm{L}_{\mathrm{A}}=6$. What is left over is Crusoe, Inc.'s profit, which will be paid to you the owner as dividend income. Since we are pricing things in yams, the profit is 6-1.8 $=4.2$, or $\pi_{\mathrm{A}}=\mathrm{Y}_{\mathrm{A}}-\mathrm{w} \cdot \mathrm{L}_{\mathrm{A}}$ in symbols." Walras explained that $\pi$ was the Greek letter "pi," used by economists as a symbol for profits. Since Robinson looked somewhat puzzled, Walras went on to point out that if a line with slope $-w$ were drawn on the graph through the point $\left(-\mathrm{L}_{A}, \mathrm{Y}_{\mathrm{A}}\right)$, then the points $\left(-L_{A}, Y_{A}\right),\left(0, Y_{A}\right)$, and $\left(0, Y_{A}-\mathrm{wL}_{A}\right)$ form a triangle whose horizontal side is $\mathrm{L}_{\mathrm{A}}$, and whose vertical side is wL.A , the wage bill or cost of labor. "Clearly," said Walras, "given your instructions to maximize profit, Mr. Friday will choose to operate at the point $B$ in Figure 5 where $L_{B}=16.67$ and $Y_{B}=10$." With a little scribbling, Robinson convinced himself that every line with slope $-\mathrm{w}=$ -0.3 parallel to the line through A was a locus of input-output combinations that yielded the same profit, the profits of Crusoe, Inc. would be maximized by getting to the most northeasterly line possible while staying on the transformation function, and this was indeed achieved at point B at which a line with slope $-\mathrm{w}=-0.3$ just touches the transformation function. ${ }^{7}$

Walras said, "Please note that the amount of labor demanded, yams supplied, and maximum profits depend on the wage rate w. As I call out various wage rates, Mr. Friday will respond (in accord with your instructions to maximize profits) with schedules something like those graphed in Figure 6 (labor demand), Figure 7 (yam supply), or Figure 8 (profits or dividend income)."


Walras told Robinson that as a consumer he need not worry about the details of these curves, as he would need only to respond only to non-wage income and the wage rate that were called out.

[^3]However, Walras suggested that if Robinson were curious, he could look at the offers that Mr. Friday would make at different wage rates by finding the profit-maximizing points on Crusoe, Inc.'s transformation function at various wages, and translating this onto the graphs of labor demand, output supply, and dividend income at the various wage levels, as illustrated in Figure 9. Robinson made a brief show of interest, but soon Walras was off again. "Now you, Robinson, will face the wage rate w I call out and the non-wage or dividend income $\pi(\mathrm{w})$ reported to you by Mr. Friday. Your opportunities will be represented by the budget line in Figure 10. If you supply no labor, then you receive $\mathrm{H}=24$ hours of leisure and an amount of yams $\mathrm{Y}=\pi(\mathrm{w})$ equal to your dividend income. For each hour of leisure you give up to dig yams, you receive w additional yams worth of labor income. As a rational man, you will clearly want to choose the yam/leisure combination E which maximizes your preferences. Then you will offer to supply $\mathrm{L}=24-\mathrm{H}$ hours of labor, the difference between the total amount of leisure you are endowed with ( 24 hours per day) and the amount you choose to consume, $H$. You will demand $Y=\pi(w)+w(24-H)$ pounds of yams."
"Nov wait a minute," said Robinson, "I may not be from Paris, but I wasn't born yesterday. I know that there are points on that budget line which couldn't possibly work. If I don't supply any labor, then Crusoe, Inc. can't produce any yams, and its profits will be zero, not $\pi(\mathrm{w})$."
"No problem," replied Walras, "the whole point of organizing your economy this way is that you don't need to worry about whether Crusoe, Inc. can actually provide bundles on your budget line. I just need information from you on what you would like if you had this budget line, your supplies and demands. It's $m y$ job as the operator of this market mechanism to see that when we reach a final wage

10. Robinson's Budget
 rate, your desires will be consistent with what Crusoe, Inc. provides." Walras continued "Don't you see what I have done for you? I have freed you from the nagging anxiety that your choices might not be consistent with the production possibilities of your economy. All you have to do is act like a modern rational consumer, calling out your supplies and demands each time you are given a budget line. Neither you nor Friday have to reveal to me your hidden opportunities and desires, and I don't have to keep track of such information. All I need are notes from the two of you telling me your supply and demand offers you make in response to each wage rate I call out." Robinson suspected that market mechanism design might not be as simple as Walras suggested, and some special conditions might be needed to ensure that each respondent was truthful and did not have an incentive to game the system. However, Walras looked like he was prepared to go into a lengthy economic discourse on the topic, so Robinson said "Oh yes, I agree. Will I have demand and supply schedules like Mr. Friday's?
"Yes indeed," said Walras, "but of course of somewhat different shape. When the wage rate is very high, your dividend income will be low but your potential wage income is very high. If you are like many people, you will then choose to work only a small amount. Because you would be so well paid, you would end up consuming a great deal of both yams and leisure. An economist would say that your income effect, which makes you want to consume both yams and leisure in greater quantities when your income rises, has outweighed your substitution effect, which makes you want to consume more yams and less leisure (i.e., supply more labor) when the wage rate, which is also the relative price of leisure, rises. On the other hand, if the wage rate is very low, your dividend income is very high and the relative price of leisure is very low, leading both your income and substitution effects to push you in the direction of consuming a lot of leisure, and supplying very little labor. Your yam demand will again be high because of the income effect. At intermediate wage levels, you are likely to offer somewhat more labor, so that your consumption of both yams and leisure will be lower than at the extreme wages.
"You can try varying the wage rate w and dividend income $\pi$ in Figure 10," continued Walras, "and see if your own preferences give yam demand and labor supply that fit my description. Of course, their exact shape, and how they behave at extreme wage rates and dividend income levels are very sensitive to the degree to which you are willing to substitute leisure for yams, and whether your tastes for both yams and leisure are normal in the sense that you want more of both when income rises. Robinson did these calculations for a few cases, moving the budget line in Figure 10 around by varying the wage rate and the level of dividend income. Time passed and Walras became impatient. He said "This is a little awkward. The idea behind my market mechanism is that I can collect the offers from all economic actors simultaneously, and the only messages we need to exchange are prices and net demands for the two goods, leisure and yams. However, I am finding that I have to either call out trial values of dividend income in addition to prices, which is a lot more information to exchange, or else ask you to send me your complete schedule of offers at all possible dividend levels at each wage I call out, which is also a lot of information as well as a burden on numerically challenged people like yourself. I tell you what. I am going to give you Crusoe, Inc.'s schedule of the dividends they expect to pay at each wage rate, which you saw briefly in Figure 8. When I call out a wage rate, just read off from this schedule the corresponding dividend income, and then tell me your offers from the budget set these determine. Just don't mention this to any other economists you meet. We have this little game we play, completely harmless, about the information content of the market mechanisms we devise. If they knew I was slipping you this on the side, it could damage my reputation." Robinson made himself a mental note to blow the whistle on Walras if he should ever be unfortunate enough to be dropped into a convention of mechanism designers, but for the moment he was thankful to be able to shortcut his calculations.

Robinson's budget constraint was

$$
\left[\begin{array}{c}
\text { Yams } \\
\text { Bought }
\end{array}\right]+\left[\begin{array}{c}
\text { Wage } \\
\text { Rate }
\end{array}\right] *\left[\begin{array}{c}
\text { Leisure } \\
\text { Bought }
\end{array}\right]=\left[\begin{array}{c}
\text { Dividend } \\
\text { Income }
\end{array}\right]+\left[\begin{array}{c}
\text { Wage } \\
\text { Rate }
\end{array}\right] *\left[\begin{array}{c}
\text { Leisure } \\
\text { Endowment } \\
\text { Sold }
\end{array}\right]
$$

or in algebraic shorthand, $\mathrm{Y}+\mathrm{wH}=\pi+\mathrm{w} \cdot 24$. Since $\mathrm{L}=24-\mathrm{H}$ is labor sold, the budget constraint can also be written $Y=\pi+w \cdot L$. Using Figure 8 to obtain $\pi=\pi(w)$ for each wage, Robinson worked out curves for his labor supply and yam demand, shown in Figures 11 and 12, respectively.
11. Supply of Labor

12. Demand for Yams


He showed them to Walras, who said "The fact that your supply of labor is always increasing when the wage rate rises suggests that your substitution effect dominates your income effect." "Is that serious?," asked Robinson. "Oh no," said Walras, "things can go either way, nothing abnormal. However, the fact that you are utilizing the schedule in Figure 8 for dividend income makes your case a little different than the one we usually treat in textbooks. That is why your demand for yams has a U-shape. Some textbook authors might accuse you of being Giffonish, a rare economic affliction, but personally I think you are bending over backwards to be normal. You are really a pretty decent fellow, for a non-economist.
"Perhaps," Walras continued enthusiastically, "you would like me to run through my advanced microeconomics course that I give in France. Then I can show you precisely how we define income and substitution effects, and how consumers in your situation with various tastes might behave."
"I wouldn't want to put you to any trouble," Robinson said hastily. "Let's postpone that and try out your new mechanism." The hour was late, so Robinson, Walras, and Mr.. Friday agreed to start the following morning. Early the next day, Walras called Robinson and Mr. Friday together, and began calling out wages. "One yam per hour," said Walras. Mr. Friday consulted his schedules in Figures 6-8, and said "I want to buy 1.5 hours of labor and sell 3 pounds of yams, and I will deliver 1.5 pounds of dividend income to my owner." After a quick calculation, Robinson replied "I want to supply 10.5 hours of labor and buy 12 pounds of yams. "Aha!" said Walras, "supply of labor exceeds the demand for labor. Lower wages are the ticket.", and he called out a wage of $0.23 . \mathrm{Mr}$. Friday said "I want to buy 24 hours of labor and sell 12 pounds of yams. Dividends will be 6 pounds. Robinson said, "I don't want to sell any labor, and I will buy 6 pounds of yams with my dividend income." "Whoops," said Walras, "too far. Now demand for labor exceeds supply of labor." After several iterations, Walras found that at a wage rate of 0.5 yams per hour, the labor demand of Crusoe, Inc. and the labor supply of Robinson were both 6 hours, and demand equaled supply at 6 pounds of yams. He then told Robinson and Mr. Friday to trade these amounts. Figures

13 and 14 show the demand and supply curves Walras uncovered by calling our various wage rates; these can also be obtained by overlaying Figures 6 and 11, and overlaying Figures 7 and 12...
"Both the labor market and the yam market have supply and demand equal at the same price," said Robinson. "What would you have done if they had balanced at different prices?" "Oh, that can't happen," said Walras, "because of a law I discovered. Look at the formula for the profit of Crusoe, Inc., $\pi_{C}=Y_{C}-w \cdot L_{C}$, where the subscripts denote Crusoe's choices. Now look at the formula for your budget constraint, $\mathrm{Y}_{\mathrm{R}}=\pi_{\mathrm{C}}$ $+\mathrm{w} \cdot \mathrm{L}_{\mathrm{R}}$, where the subscripts denote your choices. Plug the profit expression from Crusoe into your budget constraint, and you get $Y_{R}-Y_{C}=w\left(L_{R}-L_{C}\right)$. This formula holds for every value of w . It says that if for some w , supply equals demand in one of the markets, making one side of the equation zero, then the other side of the equation must also be zero, and supply must equal demand in the second market."
"Very interesting," said Robinson, "so the fact that supply equals demand at $w=0.5$ in both Figure 13 and Figure 11 was no accident."
"Right! It always works out," said. Walras. "By the way, are you happy with the yam/leisure combination you finally obtained under my scheme?"
"Why, it's just the same as the combination I was choosing before you arrived," said Robinson. "I guess I am exactly as happy as I was then, although it is of course interesting to have Mr. Friday telling me where to dig yams."
"That's no accident either," said Walras, "and that is the beauty of my scheme. Let me show you why it works. In Figure 15, I have redrawn Figure 3 which showed the optimal yam/leisure combination E you chose when you were completely on your own. Suppose I draw a straight line through E that is just tangent to both the production possibility curve and to the indifference curve I3 that just touches it. This is called a separating hyperplane, although in this case of two dimensions it is just a line, because it separates the production possibilities, which are entirely on one side of the line, and the set of points that are better for you
13. Labor Supply and Demand

14. Yam Supply and Demand


than E, which are entirely on the other side. It is always possible to draw such a hyperplane through the point where two sets just touch when they have the shapes shown in Figure 16, with the production possibilities shaped like an overturned bowl and the points preferred to E shaped like an upright bowl. These are called convex sets. It is not hard to tell from Figure 10 that the slope of this line is -0.5 . If you look at that line from Mr. Friday's point of view, the point $E$ with $L_{E}=6$ and $Y_{E}$ $=6$ corresponds to a profit maximum at the wage rate given by the slope of the line, and determines dividend income of 3 . From your own point of view, E is the best point on the budget determined by this line. Hence both you and Mr. Friday will choose at this wage rate to stay at E, exchanging 6 hours of labor for 6 pounds of yams.
"Suppose, on the other hand, that I call out some other price, as in Figure 16. Then, Mr. Friday will want to go to B , and with the budget line this establishes, you will want to go to A . In this case, labor supply exceeds labor demand, and we cannot have an equilibrium."
"Well, it is a relief to me that your scheme works," said Robinson. "Frankly, I was skeptical at first. However, I know that before this started, I was as well off as I could possibly be on this island. I am happy your markets get me back to the same place. Tell me, will your scheme always work?"
"It will in all nice economies like yours," replied Walras. "However, if you look back at Figure 15, you will see that a key to finding a price to clear the market is that the budget line fitted through the mutual tangency between your preference contour at E and Crusoe, Inc.'s transformation function. The
 transformation function is not cut anywhere else by the line, nor is your preference contour. The way these curves are shaped in your economy this could never happen. However, in an economy with what economists call 'increasing returns' in production or 'non-convex' preferences, my market mechanism may break down. In some of the advanced economies there may also be a problem with things like pollution, noise, and congestion which lead to what economists call 'externalities.' In this case, my market system may lead to an equilibrium, but in failing to take account of externalities may leave consumers less well off than they might possibly be under some form of intervention in the process of allocating goods."
"Fascinating," said Robinson, munching on a yam, "some other time you must tell me more about it."

Under Walras's supervision, the markets for labor and yams were operated each morning. As the months went by, Robinson found himself satisfied in every way. Variations from day to day in his tastes for yams or the digging prospects of Crusoe, Inc., were quickly accommodated by the markets, and Robinson always found himself in the comfortable situation of not having to search outside the market for ways to improve his lot. Then, one morning, Robinson and Mr. Friday arrived at the beach to find no Walras. A quick search turned up a bottle floating in the surf with a note: "Tied up organizing a market for artist's models. Back tomorrow. Leon."
"What shall we do?" Mr. Friday asked Robinson.
"This is a sticky situation," said Robinson. "I suppose we could try trading at the wage rate we traded at yesterday.". Mr. Friday agreed, and each quickly calculated his demands and supplies. Because of shifts in tastes and production possibilities, yesterday's equilibrium wage rate which was $\mathrm{w}=1$ led to a situation like that shown in Figure 16. Mr. Friday offered to hire labor and sell yams at point B , and given the budget line this established, Robinson offered to sell labor and buy yams at point A . The result was an excess supply of labor and excess demand for yams at this price. Figures 12 and 14 show this clearly, since they correspond to today's production possibilities and preferences.
"I'm afraid I can use only the amount of labor $L_{B}=1.5$ hours," said Mr. Friday. "I would lose money if I offered you more work, and that would violate my fiducial responsibility as manager of Crusoe, Inc. I am going to have to lay you off for the rest of the time $L_{A}-L_{B}=10.5-1.5=9$ hours that you want to work. I really empathize with your situation, and in recognition of your many months of meritorious service, Crusoe, Inc. would like to present you with this digging stick with a carved mahogany handle."
"That's quite a shaft," said Robinson.
"Thank you," replied Mr. Friday. "Crusoe, Inc. also regrets that due to the high cost of labor, we are unable to fill your order for $\mathrm{Y}_{\mathrm{A}}=12$ yams.".
"That's just as well," said Robinson. "Since I can't work as much as I would like, I don't have the income to buy yams that I thought I would. In fact, I can only afford $\mathrm{Y}_{\mathrm{B}}=3$ pounds of yams."
"How fortunate, that's exactly what we can provide," said Mr. Friday, "it's been a pleasure to serve you." This seemed to settle the matter, but Robinson was unhappy. He would really have preferred to work a little more and have more yams to eat. He began to long for the good old days, when he was his own boss. "Perhaps you would like to barter a little labor for a few more yams," he said to Mr. Friday. "I'll make you a good price, lower than the wage rate w = 1 that Walras left us with."
"That's very appealing," said Mr. Friday, "and certainly in the spirit of free enterprise. Crusoe, Inc. is always interested in seeing lower wages. Our motto is 'What's good for Crusoe is good for Robinson'."
"Perhaps we could operate the markets ourselves, then," said Robinson, "using the bidding and arbitrage activities of competitive forces to lead to market clearing prices. If supply of labor exceeds demand, as now, the wage will be bid down. On the other hand, if demand exceeds supply, the wage will be bid up."
"Bid up?" said Mr. Friday, "I want to make it perfectly clear that Crusoe, Inc. stands four-square behind free enterprise and the preservation of capitalism. Consequently, we are against cutthroat competition which would drive up the cost of labor, starting us down the road to Socialism. I am a compassionate conservative. I say 'Leave no capitalist behind'."
"What about the preservation of workers?" asked Robinson. "I don't think they should be left behind either."
"Dangerous talk," said Mr. Friday. "You workers are lazy, no-good bums, always looking for a way to shirk work and rip off the capitalists' hard-earned money. You need the specter of starvation to give you sufficient incentives to do your job."

Since Robinson owned Crusoe, Inc., he considered himself a capitalist (every worker's secret dream), so he agreed with Mr. Friday on the perfidy of workers, and once again broached the subject of barter. Robinson was pretty sure that the bidding process he had suggested would work, and that Mr. Friday's rhetoric was only an attempt to get Robinson to agree to laws and regulations which would give Crusoe, Inc. an advantage in bargaining. However, to avoid further argument, he said "Perhaps we should stop trading for today, and let Walras resume tomorrow." Mr. Friday agreed.

The next day, Walras returned. "I hope you gentlemen were able to trade in my absence," he said.
"I was unable to sell all the labor I wanted to," said Robinson.
"The wage rate was too high," said Walras. "Such things can happen when conditions in the economy are changing so fast that the market manager can't catch up, or when the economy adopts rules and regulations such as wage and price controls which keep prices from adjusting to equate supply and demand. Or, of course, when the market adjustment mechanism is very sluggish, due to difficulties in passing along information, as happened yesterday when I was away. These are the classical reasons that prices might fail to adjust rapidly, leading to trade outside Walrasian equilibrium."
"What's Walrasian equilibrium?" asked Robinson.
"Oh, that's what I call the combination of prices and quantities where supplies and demands are equal," said Walras, "and these supplies and demands come from the schedules of profit-maximizing firms and preference-maximizing consumers. As your experience has shown, in Walrasian equilibrium the consumer is as well off as he could possibly be. However, in trading situations outside of equilibrium, this will usually not be true. Further, you are likely to find that your demand and supply schedules don't tell the whole story about your trades when you are out of equilibrium."
"That's true," said Robinson. "When I found out I couldn't work as much as I wanted, I realized that I couldn't afford to buy the amount of yams I had originally offered."
"You might say that your effective or ex post demand was unequal to your notional or ex ante demand," said Walras.
"You took the words right out of my mouth," said Robinson.

The following week, Walras disappeared again for a day, leaving instructions to trade at the previous day's wage rate. This time, the changes in tastes and technology led to too low a wage rate, as shown in Figure 17. Crusoe, Inc. offered point B, where $L_{B}=$ 16.67, $\mathrm{Y}_{\mathrm{B}}=10$, and $\pi_{\mathrm{B}}=5$, while Robinson offered point $A$ where $L_{A}=0$ and $Y_{A}=5$. Labor demand exceeded labor supply, and yarn supply exceeded yam

demand. This time Robinson refused to work more than $\mathrm{L}_{\mathrm{A}}$. Soon, he received a note from Crusoe, Inc. "Due to circumstances beyond our control, labor shortages have caused earnings this period to fall below the previously anticipated levels. We will not be paying a dividend" Faced with zero non-wage income rather than the 5 pounds than he had expected to receive, Robinson made some further calculations and revised his offer of labor supply upward and yam demand downward to maximize preferences on his new budget line. This led to a further exchange of communications with Crusoe, Inc., and further revision of dividends until it was agreed that Robinson would choose a point C at which he would supply an amount of labor $\mathrm{L}_{\mathrm{C}}=2.16$ hours, receive a dividend of 2.952, and buy $\mathrm{Y}_{\mathrm{C}}=3.6$ pounds of yams, which was exactly what Crusoe could deliver. In Figure 18, C is the point on the production possibility curve where Robinson's preference contour through this point is tangent to a line with slope -w $=-0.3$, corresponding to the specified wage rate.

This would have been the end of the matter had not Mr. Friday noted with some alarm the gap between the amount of yams $\mathrm{Y}_{\mathrm{B}}=10$ he had initially expected to sell at the quoted wage w , and the amount $\mathrm{Y}_{\mathrm{C}}=3.6$ he was actually able to sell. From his vantage point, it seemed entirely possible that with trade at prices which did not clear all markets, he could contract to hire a great deal of labor, say
 $\mathrm{L}_{\mathrm{B}}=16.67$ to produce a large amount of yams that would be left unsold to spoil. He was concerned that he would end up in the position D in Figure 18 where $L_{D}=16.67$ but $Y_{D}=3.6$. Then, instead of achieving $B$ which would yield maximum profits $\pi=5$ for the firm, he would actually achieve a profit of $3.6-0.3 \cdot 16.67=-1.4$. Mr. Friday feared that such a performance could endanger his standing with Robinson, and possibly ruin his promising career as a business executive in the yam industry and his dream of founding a yam trading company, which he hoped to call "Enron". Therefore, he formulated a plan and presented it to Robinson at the next stockholder's meeting.
"The management wishes to report that it has secured an attractive rate of dividends without taking undue risks during the past year," said Mr. Friday. "However, because of market fluctuations in the recent past, the management proposes to institute policies to avoid commitments to produce yams in quantities which are in excess of prospective sales, thereby protecting the interests of the owners against danger of excessive inventories of unsold goods. The management emphasizes that under this policy the company will continue to pay high dividends with minimum risk."
"Would you repeat that in plain English?" asked Robinson.
"The management can respond to stockholders questions only if they are submitted in advance in writing," replied Mr. Friday.
"Come off it, Friday," said Robinson.
"Very well," Friday said reluctantly, "I don't intend to offer to produce more than I think I can sell."
"That seems reasonable enough," said Robinson. "You have my blessing."
The following day, Mr. Friday put his plan into action. On the basis of past experience and the first wage rate $w$ called out by Walras, he would pick an amount of yams $\mathrm{Y}^{*}$ that was the most he could reasonably expect to sell. The offers he made to buy labor and sell yams would then be consistent with $\mathrm{Y}^{*}$. In mathematical terms, he made offers that maximized his profit at the quoted wage rate, subject to the constraints imposed by the transformation function and the expected upper bound $\mathrm{Y}^{*}$ on yam sales. However, Mr. Friday wanted to be flexible, so he decided that he would adjust $Y^{*}$ in light of the information on demand he learned from responses to wage rates that were being called out by Walras. If a demand offer $\mathrm{Y}_{\mathrm{D}}$ exceeded $\mathrm{Y}^{*}$, then he would gradually increase $\mathrm{Y}^{*}$, and if a demand offer $\mathrm{Y}_{\mathrm{D}}$ were less than $\mathrm{Y}^{*}$, then he would slowly decrease $\mathrm{Y}^{*}$. A mathematician would describe his adjustment process in the following way. Let $t$ denote the "clock time" during which Walras is conducting the auction to determine the wage, and suppose the seconds are measured $t=1,2,3, \ldots$. Then, the change in $Y^{*}(t)$, defined as $\Delta Y^{*}(t)=Y^{*}(t+1)-Y^{*}(t)$, is a signpreserving function of $\mathrm{Y}_{\mathrm{D}}(\mathrm{t})-\mathrm{Y}^{*}(\mathrm{t})$. In fact, Mr. Friday used a very simple rule of this type, $\Delta \mathrm{Y}^{*}(\mathrm{t})$ $=\beta\left(Y_{D}(t)-Y^{*}(t)\right)$, where $\beta$ is a positive constant. This is termed a difference equation Walras's method of adjusting the real wage can also be described by a simple rule, the change in the wage $\Delta \mathrm{w}(\mathrm{t})=\alpha \mathrm{w}(\mathrm{t})\left(\mathrm{L}_{\mathrm{D}}(\mathrm{t})-\mathrm{L}_{\mathrm{S}}(\mathrm{t})\right)$, where $\alpha$ is a positive constant, $\mathrm{L}_{\mathrm{D}}$ is labor demand, and $\mathrm{L}_{\mathrm{S}}$ is labor supply. Then, Walras increased wages when demand exceeded supply, and lowered wages when demand was less than supply. If you look back at Figure 13, you will see that at least before Mr. Friday started trying to take account of expected sales, this adjustment process for wages would always lead to equilibrium, so long as $\alpha$ was small enough (i.e., the adjustment process was slow enough) so that any oscillations would damp out.

Each morning, Robinson, Mr. Friday, and Walras would start from the previous day's wage and expected maximum sales, and let this adjustment process run on until $\mathrm{w}(\mathrm{t})$ and $\mathrm{Y}^{*}(\mathrm{t})$ approached limiting values. Then, trade would occur at these values. The results were alarming! Instead of always achieving the Walrasian equilbrium where Robinson was as well off as he could possibly be, the adjustment process often led to situations in which Robinson was offered a very low wage, and as a result chose to work very little and consume only a few yams. He complained bitterly to Walras that something was going wrong, and he was clearly not as happy as in the good old days.
"Very strange," said Walras, "supply equals demand in both markets. You must be mistaken, and just think you are less than happy. My economic theory shows you must be just as happy as possible."
"I'm hungry, and I can't eat economic theories," retorted Robinson. "You must do something!"
"I never ran into anything like this in my classroom in France," said Walras. "Come to think of it, I'm rather homesick for Paris."
"Robinson is just lucky that things aren't worse," interjected Mr. Friday. "Due to my astute management, I have avoided saddling him with losses occasioned by the production of goods which can't be sold."
"You mean you base your production decisions on what you think is the maximum amount you can sell, rather than what would maximize profits on the assumption that anything produced could be sold?" asked Walras.
"Exactly," answered Mr. Friday proudly.
"Robinson, I believe we have located your problem," said Walras, "Expectations."
"I'm hungry," whimpered Robinson.
"I believe you deserve a full explanation of expectations and how they might influence equilibrium," said Walras. "I could of course provide the complete story, even though the topic is out of my area. However, I really must be getting back to Paris, and fortunately I see just approaching in a fast cutter a young economic colleague who has thought deeply about these matters. He will be happy ,I am sure, to give you the details." With this, Walras jumped up, dashed into the surf, and swam briskly in the direction of Paris, soon disappearing over the horizon. Robinson's attention was now diverted to the cutter, which rapidly approached the shore and rode gracefully up on the sand, assisted by two lanky seamen. A sprightly gentleman bounced out. "I am Lord Keynes," he said, "and these seaman, A.L. and H.V., are my interpreters."
"Do you have any food?" asked Robinson. "I am in a very depressed state."
"Depression!" exclaimed Keynes, "How interesting! You must tell me more."
Robinson recounted his history on the island, first as a centralized economy, then operated under the guidance of Walras, and then finally the coming of darker days with low wages and little production. He concluded by stressing how he could think better on a full stomach, and by mentioning Walras's cryptic comment about expectations.
"A most unusual case," said Keynes, "one that I have only hinted at in my theories."
"Oh, you said it, sir, after your fashion," chorused A.L. and H.V.
"You are getting ahead of me gentlemen," replied Lord Keynes. Turning to Robinson, he asked, "Would you like all this explained to you?"
"I'd rather eat," said Robinson.
"By all means," said Lord Keynes, "first food for the mind, then for the body. In my customary style, I am going to explain the situation in words, many words. I find that circumlocution and ambiguity are useful in economic discourse. The listener is entertained by guessing my meaning, and I leave room for my interpreters to find employment and fulfillment."
"Does this mean it's going to be a long time before we eat?' asked Robinson.
"Absolutely!" said Lord Keynes, "Food for the mind takes time to digest."
"Isn't there a faster way?" Robinson asked hungrily.
"Well, yes," Keynes said reluctantly, "it is possible to describe your situation mathematically in just a few pages, but I have to recommend against it."
"Why" asked Robinson.
"It makes things too simple, " replied Keynes, "deceptively simple. There is a danger you will understand the mathematical skeleton, but only words can put meat on the bones and give you the whole picture. Besides, Walras told me you are mathematically challenged."
"That may be, but I am even more econo-speak-challenged," said Robinson, "I would say that a short, really short mathematical presentation, followed by dinner, is my comparative advantage."
"Spoken like an economist," said Keynes. "Very well, with the help of my interpreters, I will give you the mathematical synopsis."
"First," said Keynes, " let's introduce some shorthand notation:

| Y | Yams in pounds |
| :--- | :--- |
| L | Labor in hours |
| H | Leisure in hours |
| w | Wage rate in yams per hour |
| $\pi$ | profit in pounds of yams |

Demand for yams will be denoted $Y_{D}$, and supply will be denoted $Y_{S}$. Demand for labor will be denoted $L_{D}$, and supply will be denoted $L_{S}$. Mr. Friday's beliefs about the amount he can sell will not be introduced right away, but when they are, they will be denoted $\mathrm{Y}^{*}$.
"Mr. Friday's transformation function, giving the amount of yams that can be produced from each amount of labor input, efficiently used, is $Y=(6 \mathrm{~L})^{1 / 2}$. Points below this curve are also possible, but less efficient, ways to produce. Therefore, the transformation function is the boundary of a set which we will call the production possibility set, the set of all input-output combinations that it is possible to reach. Just remember, this particular curve is only an illustration, although it represents your actual possibilities today. You can look back at Figure 4 to remind yourself how this curve is shaped.
"Mr. Friday’s instructions are to maximize the profits of Crusoe, Inc., given the wage rate w called out by the market coordinator; i.e., to maximize $\pi=\mathrm{Y}-\mathrm{wL}$ over the (-L,Y) points in the production possibility set. Since $Y=(6 L)^{1 / 2}$, the expression to be maximized is $\pi=(6 L)^{1 / 2}-w L$ Then $d \pi / d L=(3 / 2 L)^{1 / 2}-w$, which is a decreasing function of $L$ that is zero at $L_{D}=3 / 2 w^{2}$. At this labor input, the output is $\mathrm{Y}_{\mathrm{S}}=3 / \mathrm{w}$, and the profit achieved is $\pi=3 / 2 \mathrm{w}$.
"Now consider what happens when Mr. Friday expects he can sell no more than $\mathrm{Y}^{*}$. If $\mathrm{Y}^{*} \geq$ $3 / \mathrm{w}$, the profit-maximizing output calculated above, then this expectation is not binding, and he will offer to hire the $3 / 2 \mathrm{w}^{2}$ hours of labor and sell the $3 / \mathrm{w}$ pounds of yams that maximize profit at wage w. However, when Mr. Friday expects he can sell no more than $Y^{*}$ and $Y^{*}<3 / w$, then he will offer to sell only $\mathrm{Y}^{*}$, and buy the amount of labor needed to produce it, $\left(\mathrm{Y}^{*}\right)^{2} / 6$. We can put these cases together and say that labor demand is $\mathrm{L}_{\mathrm{D}}=\min \left(3 / 2 \mathrm{w}^{2},\left(\mathrm{Y}^{*}\right)^{2} / 6\right)$ and yam supply is $\mathrm{Y}_{\mathrm{S}}=\min \left(3 / \mathrm{w}, \mathrm{Y}^{*}\right)$. The resulting dividend income is $\pi=3 / 2 \mathrm{w}$ when $\mathrm{Y}^{*} \geq 3 / \mathrm{w}$, and $\pi=\mathrm{Y}^{*}-\mathrm{w}\left(\mathrm{Y}^{*}\right)^{2 / 6}$ otherwise. If you
look back at Figure 18, point B corresponds to Friday's offers at profit maximization when the expectation $\mathrm{Y}^{*}$ is not binding, and point C corresponds to Friday's offers when the expectation $\mathrm{Y}^{*}$ is binding at the level corresponding to C . Note that dividend income at C is lower than at B ; this follows from the fact that B gave maximum profit or dividend income.
"Now, Robinson, let's consider your behavior. You have preferences that can be summarized by the function $\mathrm{u}=\ln (\mathrm{Y})+\mathrm{H} / 12$, where ' $\ln$ ' stands for natural logarithm, Y is your yam consumption, H is your leisure consumption, and u is an index of how desirable a yam-leisure combination is, the higher the better. You want to maximize your preferences subject to your budget constraint, $\mathrm{Y}=\mathrm{wL}+\pi$ and the accounting condition that $\mathrm{H}+\mathrm{L}=24$, your total endowment of leisure each day. Substituting the accounting condition and the budget constraint into your utility function gives the preference level you attain at each $L$, given $w$ and $\pi$ :

$$
\mathrm{u}=\ln (\mathrm{wL}+\pi)+2-\mathrm{L} / 12 .
$$

The derivatives of this function are $d u / d L=w /(w L+\pi)-1 / 12$ and $d^{2} u / d L^{2}=-w^{2} /(w L+\pi)^{2}$. The function will be maximized either at a corner $\mathrm{L}=0$ with $\mathrm{du} / \mathrm{dL}=\mathrm{w} / \pi-1 / 12 \leq 0$, or at an interior solution with $\mathrm{L}>0$ and $\mathrm{du} / \mathrm{dL}=\mathrm{w} /(\mathrm{wL}+\pi)-1 / 12=0$, or $\mathrm{L}=12-\pi / \mathrm{w}$. The second derivative is negative at any interior solution, a sufficient condition for the solution to be a maximum. Putting the cases together, your labor supply is $L_{S}=\max (12-\pi / \mathrm{w}, 0)$, and your yam demand is $\mathrm{Y}_{\mathrm{D}}=$ $\max (12 \mathrm{w}, \pi)$. Robinson, you should work through the cases and verify that this last formula describes your demand for yams."
"Oh, I'll do my homework right after dinner," said Robinson.
"All right," said Keynes. "Let's now put your yam demand and Crusoe's yam supply together, and find the equilibria where demand equals supply. First, consider the case where $\mathrm{Y}^{*}$ is very large, so that we can be sure it is not going to be binding. In this case, Friday offered supply $\mathrm{Y}_{\mathrm{S}}=3 / \mathrm{w}$ and dividend income $3 / 2 \mathrm{w}$. Your yam demand is $\mathrm{Y}_{\mathrm{D}}=\max (12 \mathrm{w}, \pi)=\max (12 \mathrm{w}, 3 / 2 \mathrm{w})$. In equilibrium, $Y_{S}=Y_{D}$, or $3 / w=\max (12 w, 3 / 2 w)$. Look back at Figure 14, Robinson, and convince yourself that this equation has exactly one solution, at $w=1 / 2$. Thus, the mathematics confirms the result you were obtaining under Walras's market management before Friday started in with expectations, with yam demand and supply of 6 pounds, and labor demand and supply of 6 hours."
"Yes," said Robinson, that was the wage that put me at the point E in Figure 15 where I was as well off as I could be, and convinced me for a while that there was something to this Walrasian mechanism."
"No resource-allocation mechanism is idiot-proof," said Keynes, "and don't forget that your industrialist friend Mr. Friday has been involved in some creative accounting. In the case I just described, Mr. Friday would see demand of 6 , and would adjust $Y^{*}$ downward until it reached this level. That would not disturb the equilibrium as long as nothing changes, but it is going to effect Mr . Friday's offers in the face of future changes in tastes or production possibilities.
"Let's consider the case where $\mathrm{Y}^{*}$ is less than 6 . We found in this case that Mr. Friday would offer to supply $\mathrm{Y}^{*}$ yams, demand $\left(\mathrm{Y}^{*}\right)^{2} / 6$ hours of labor, and offer $\mathrm{Y}^{*}-\mathrm{w}\left(\mathrm{Y}^{*}\right)^{2} / 6$ in dividends. In response, Robinson offers to buy $\mathrm{Y}_{\mathrm{D}}=\max (12 \mathrm{w}, \pi)=\max \left(12 \mathrm{w}, \mathrm{Y}^{*}-\mathrm{w}\left(\mathrm{Y}^{*}\right)^{2} / 6\right)$ pounds of yams. If we were to find that in this case Robinson's demand for yams exceeded $\mathrm{Y}^{*}$ at every wage, then Mr. Friday would adjust $Y^{*}$ upward until $Y^{*}=6$, where we know that $w=1 / 2$ clears the market. However, what actually happens can be quite different. To equate yam supply and demand, a wage w must satisfy $\mathrm{Y}^{*}=\max \left(12 \mathrm{w}, \mathrm{Y}^{*}-\mathrm{w}\left(\mathrm{Y}^{*}\right)^{2} / 6\right)$. But $\mathrm{w}=\mathrm{Y}^{*} / 12$ achieves this, since at this wage, 12 w $\left.=\mathrm{Y}^{*}>\mathrm{Y}^{*}-\mathrm{w}\left(\mathrm{Y}^{*}\right)^{2} / 6\right)$. Thus, at any $\mathrm{Y}^{*}<6$, the wage $\mathrm{w}=\mathrm{Y}^{*} / 12<1 / 2$ equates supply and demand for yams. Labor supply and demand will also be equal, because Walras' law still holds even when Mr. Friday uses expectations in forming his offers. You can also verify this directly, since labor demand is $\left(\mathrm{Y}^{*}\right)^{2} / 6$ and labor supply is $\max (12-\pi / \mathrm{w}, 0)=\max \left(12-\left(\mathrm{Y}^{*}-\mathrm{w}\left(\mathrm{Y}^{*}\right)^{2} / 6\right) / \mathrm{w}, 0\right)=\max (12-$ $\left.\mathrm{Y}^{*} / \mathrm{w}+\left(\mathrm{Y}^{*}\right)^{2} / 6,0\right)=\max \left(\left(\mathrm{Y}^{*}\right)^{2} / 6,0\right)=\left(\mathrm{Y}^{*}\right)^{2} / 6$. Then, every value of $\mathrm{Y}^{*}$ less than 6 and the associated wage $\mathrm{w}=\mathrm{Y}^{*} / 12$ are an equilibrium in this economy, in the sense that the markets clear, and neither Mr. Friday in his rules for adjusting $\mathrm{Y}^{*}$ or the market manager with rules for the wage rate being called out would want to change anything. As you have noticed, your preference level in these situations is lower than at the original Walrasian equilibrium. In that case, you had $Y_{E}=6$ and $L_{E}$ $=6$, giving you utility $\mathrm{u}=\ln (6)+18 / 12=3.29$, whereas at $\mathrm{Y}^{*}<6$, you achieve $\mathrm{u}=\ln \left(\mathrm{Y}^{*}\right)+2$ $\left(\mathrm{Y}^{*}\right)^{2} / 72$. Note that du/dY* $=1 / \mathrm{Y}^{*}-\mathrm{Y}^{*} / 36=\left[1-\left(\mathrm{Y}^{*} / 6\right)^{2}\right] / \mathrm{Y}^{*}>0$, so that the lower $\mathrm{Y}^{*}$, the lower your utility. Thus, the distress you have suffered recently is explained by the existence of equilibria in this economy in both prices and expectations that are less desirable than the optimal resource allocation.
"So far, I have described how your economy with Mr. Friday's expected sales can have multiple equilibria, some of them clearly less desirable for you than others. It might help if I went further and spelled out how this works over the course of the dynamic process of getting to market-clearing prices and expectations. Let t denote the clock time over which the market adjustments within a day take place; think of the adjustment process as being continuous. Suppose the market manager's wage adjustment obeys an equation of motion $d \ln (\mathrm{w}(\mathrm{t})) / \mathrm{dt}=\alpha\left(\mathrm{L}_{\mathrm{D}}(\mathrm{t})-\mathrm{L}_{\mathrm{s}}(\mathrm{t})\right.$ ), where $\mathrm{L}_{\mathrm{D}}(\mathrm{t})$ and $\mathrm{L}_{\mathrm{S}}(\mathrm{t})$ are labor demand and labor supply at time $t$, and $\alpha$ is a small positive constant rate of adjustment. This can also be written $\mathrm{dw}(\mathrm{t}) / \mathrm{dt}=\alpha \mathrm{w}\left(\mathrm{L}_{\mathrm{D}}(\mathrm{t})-\mathrm{L}_{\mathrm{S}}(\mathrm{t})\right)$, and says that wages adjust at a rate proportional to the difference in the wage bill demanded and the wage bill supplied, with wages increasing when demand exceeds supply, and vice versa. Recall that your budget constraint was $\mathrm{Y}_{\mathrm{D}}+\mathrm{w}\left(24-\mathrm{L}_{\mathrm{S}}\right)=$ $\mathrm{w}(24)+\mathrm{Y}_{\mathrm{S}}-\mathrm{wL}_{\mathrm{D}}$, which says that your expenditure on yams and leisure equals your income from selling your endowment of 24 hours of leisure plus your dividend income. Rearranging terms, $\mathrm{Y}_{\mathrm{S}}$ -$Y_{D}=w\left(L_{D}-L_{S}\right)$. Then, the equation of motion for wage can also be written $d w(t) / d t=\alpha\left(Y_{S}(t)-\right.$ $\mathrm{Y}_{\mathrm{D}}(\mathrm{t})$ ). Suppose that Mr. Friday's method of adjusting expected yam sales $\mathrm{Y}^{*}(\mathrm{t})$ gives an equation of motion $d Y^{*}(t) / d t=\beta\left(Y_{D}(t)-Y^{*}(t)\right)$, where $\beta$ is a small positive adjustment rate. This says that expected sales adjust adaptively toward yam demand at a rate that is proportional to their difference. From our previous analysis, we had $\mathrm{Y}_{\mathrm{D}}=\max (12 \mathrm{w}, \pi), \mathrm{Y}_{\mathrm{S}}=\min \left(3 / \mathrm{w}, \mathrm{Y}^{*}\right)$, and $\pi=3 / 2 \mathrm{w}$ when $\mathrm{Y}^{*}$ $\geq 3 / \mathrm{w}$, and $\pi=\mathrm{Y}^{*}-\mathrm{w}\left(\mathrm{Y}^{*}\right)^{2} / 6$ otherwise. Substituting these expressions into the equations of motion gives us two nonlinear differential equations,

$$
\begin{aligned}
& d w / d t=\alpha\left\{\begin{array}{ll}
3 / w-\max (12 w, 3 / 2 w) & \text { if } w \geq 3 / Y^{*} \\
Y^{*}-\max \left(12 w, Y^{*}-w\left(Y^{*}\right)^{2} / 6\right) & \text { if } w<3 / Y^{*}
\end{array}\right\} \\
& d Y^{*} / d t=\beta\left\{\begin{array}{cl}
\max (12 w, 3 / 2 w)-Y^{*} & \text { if } w \geq 3 / Y^{*} \\
\max \left(12 w-Y^{*},-w\left(Y^{*}\right)^{2} / 6\right) & \text { if } w<3 / Y^{*}
\end{array}\right\}
\end{aligned}
$$

With a little analysis, you can verify that solutions to this system of differential equations converge to one of the equilibria we have identified."
"Right," said Robinson, "seems obvious to me. Just looking at these equations makes me hungry.. I think it is time for a dinner break."
"Oh no, Robinson, not so fast," said Keynes. You are really going to have to pay attention here and work through the cases. There are seven in all, depending on what inequalities are satisfied, and one can see how things move in each case. The table below spells it out."

| Case | Inequalities | $\mathrm{dw} / \mathrm{dt}$ | $\mathrm{dY} * / \mathrm{dt}$ |
| :---: | :---: | :---: | :---: |
| 1 | $\mathrm{w}<3 / \mathrm{Y}^{*}, \mathrm{w}<\mathrm{Y}^{*} / 12, \mathrm{w}<\mathrm{Y}^{*} /\left(12+\left(\mathrm{Y}^{*}\right)^{2} / 6\right)$ | $\alpha \mathrm{w}\left(\mathrm{Y}^{*}\right)^{2} / 6>0$ | $-\beta \mathrm{w}\left(\mathrm{Y}^{*}\right)^{2} / 6<0$ |
| 2 | $\mathrm{w}<3 / \mathrm{Y}^{*}, \mathrm{w}<\mathrm{Y}^{*} / 12, \mathrm{w}>\mathrm{Y}^{*} /\left(12+\left(\mathrm{Y}^{*}\right)^{2} / 6\right)$ | $\alpha\left(\mathrm{Y}^{*}-12 \mathrm{w}\right)>0$ | $-\beta\left(\mathrm{Y}^{*}-12 \mathrm{w}\right)<0$ |
| 3 | $\mathrm{w}<3 / \mathrm{Y}^{*}, \mathrm{w}>\mathrm{Y}^{*} / 12$ | $\alpha\left(\mathrm{Y}^{*}-12 \mathrm{w}\right)<0$ | $-\beta\left(\mathrm{Y}^{*}-12 \mathrm{w}\right)>0$ |
| 4 | $\mathrm{w} \geq 3 / \mathrm{Y}^{*}, \mathrm{w}>\mathrm{Y}^{*} / 12 \quad(\Rightarrow \mathrm{w}>1 / 2)$ | $\alpha(3 / \mathrm{w}-12 \mathrm{w})<0$ | $\beta\left(12 \mathrm{w}-\mathrm{Y}^{*}\right)>0$ |
| 5 | $\mathrm{w}>1 / 2, \mathrm{w}<\mathrm{Y}^{*} / 12 \quad\left(\Rightarrow \mathrm{Y}^{*}>6\right)$ | $\alpha(3 / \mathrm{w}-12 \mathrm{w})<0$ | $\beta\left(12 \mathrm{w}-\mathrm{Y}^{*}\right)<0$ |
| 6 | $\mathrm{w}>3 / \mathrm{Y}^{*}, \mathrm{w}>1 / \sqrt{ } 8 \quad\left(\Rightarrow \mathrm{Y}^{*}>6\right)$ | $\alpha(3 / \mathrm{w}-12 \mathrm{w})>0$ | $\beta\left(12 \mathrm{w}-\mathrm{Y}^{*}\right)<0$ |
| 7 | $\mathrm{w}>3 / \mathrm{Y}^{*}, \mathrm{w}<1 / \sqrt{ } 8 \quad\left(\Rightarrow \mathrm{Y}^{*}>6\right)$ | $\alpha 3 / 2 \mathrm{w}>0$ | $\beta\left(3 / 2 \mathrm{w}-\mathrm{Y}^{*}\right)<0$ |

"Figure 19, called a phase diagram, identifies these cases with regions in $\mathrm{Y}^{*}$-w space, and gives the directions of motion. The point E where $\mathrm{w}=1 / 2$ and $\mathrm{Y}^{*}=6$ is the equilibrium that is best for Robinson, and the straight line from the origin to $E$ is the locus of equilibria that are constrained by expected sales $\mathrm{Y}^{*}$. This line, continued past E , is the locus of points where $\mathrm{dY} * / \mathrm{dt}=0$. Above this line, $\mathrm{dY}^{*} / \mathrm{dt}>0$, and below this line, $\mathrm{dY}^{*} / \mathrm{dt}<0$. The line segment from the origin to E , and continuing horizontally at $\mathrm{w}=1 / 2$, is the locus of points where $\mathrm{dw} / \mathrm{dt}=0$. Above this, $\mathrm{dw} / \mathrm{dt}<0$, and below this, $\mathrm{dw} / \mathrm{dt}<0$. The region corresponding to each case is labeled, and the direction of motion in each region is indicated. In cases $1-3$, the slope of the direction of movement is $-\alpha / \beta$, a constant which is large in magnitude if Mr. Friday adjusts more slowly than the market manager. Assume that $\beta / \alpha<12$. Then, there is a path contained in regions 1 and 2 , defined by $\alpha\left(\mathrm{Y}^{*}-6\right)=\beta(\mathrm{w}-1 / 2)$, along which the equations of motion take the process to E . Any path starting from a point in regions

3 and 4 below this path will follow a straight line to an equilibrium in the line segment from the origin to E. There is also a path dividing region 4 that goes directly to E . If you really exercise your calculus, Robinson, you can get an expression for the points in this path for $\mathrm{w}>1 / 2$. Paths starting to the right of this in region 4 cross into region 5, and in that region curve down to E . Paths starting to the left of this in region 4 cross into region 3, and from there move along a line to an equilibrium on the line segment connect the origin and E .
 Finally, paths starting in regions 5, 6, and 7 curve down to E. The overall picture then is that when Mr. Friday's initial expectations are high enough, the process will converge to E , but the lower the initial $\mathrm{Y}^{*}$, the larger the initial w needed to get on a path that leads to $E$ rather than a less desirable equilibrium. Are you following all this?"
"Oh, this really clears things up," replied Robinson, "and I can hardly wait until after dinner when I can get started on the incredibly tedious job of working out the path dividing region 4 into the starting points that reach $E$ and those that reach less satisfactory equilibria."
"I appreciate your animal spirits," said Keynes. "I think you have the makings of a graduate student in economics."
"Be still my heart," said Robinson. "Are we ready for the part about feeding the body now?"
"Almost," said Keynes, "I know you have been upset about reaching equilibria with $\mathrm{Y}^{*}<6$ that are much worse for you than your optimum at E. However, I should point out that you are locating there voluntarily, since you are working all you want to at the low prevailing wage.. There is no disequilibrium in the labor market causing involuntary unemployment."
"You can't imagine how much better it makes me feel to know I am not being coerced into starvation," said Robinson. "Tell me this. If expectations are so important, and have such potential to screw things up, why don't designers of market mechanisms pay more attention to them?"
"They do now," said Keynes, "but for a long time economists were barking up several wrong trees. One pack that includes not only Walras, but also Adam Smith and Milton Friedman, is enchanted by the ability of markets to allocate resources. They observe correctly that in most circumstances there are no practical resource allocation mechanisms that work better than markets, but then go further and argue that intervention in poorly functioning markets almost always makes
things worse. There is some wisdom in this position, given the economic skills of most politicians, but a careful analysis shows that there are a number of circumstances under which markets break down and remedies are available. Economies have a lot in common with animals - the forms of economic organization that survive are those that work reasonably well and are mostly self-repairing, but occasionally judicious treatment can speed recovery from an illness, and minimize discomfort. The fact that there are quack practitioners and bogus treatments out there should not blind us to the efficacy of good medicine.
"A second pack of economists that includes most of my early interpreters observed correctly that sometimes markets produce resource allocations that are clearly out of whack, such as in the U.S. and European economies in the Great Depression in the 1930's. To explain these situations, they concentrated on sticky wages and interest rates that failed to clear the markets for labor and for savings and investment, leading to involuntary unemployment and excessive saving. While these disequilibrium phenomena can be important in the short run before prices adjust fully and an economy settles down, or even in the longer run if things are too chaotic, in most cases one would expect the participants in the economy to look for "daylight" where they can put together some combination of trades that work to their advantage, and these economic incentives would push markets toward equilibrium. The failure of the early mathematical expositions of my theory to account for incentives for price adjustments toward equilibrium is a cross that I must bear."
"Wait," said Robinson, "what's sauce for the goose is sauce for the gander. If it is so plausible that economic incentives will break down disequilibrium caused by sticky prices, why wouldn't economic incentives also undo Mr. Friday's fixation on expected yam sales? It's true that he turned down my offer to barter more labor for more yams at a point where $Y^{*}$ was less than 6 , but maybe he would have seen the light if I had suggested replacing him with a more responsive management."
"Coming from you, that's a surprisingly intelligent observation," said Keynes. "It's true that Mr. Friday had to be unusually stupid to turn down a barter offer that was so obviously mutually advantageous. One might expect that the pressures of competition would eventually make it impossible for Mr. Friday to indulge in his psychosis, since eager young MBA would step in and offer Robinson a better deal. In your simple Robinson Crusoe economy where the advantages of barter are obvious to all parties, this is what we would expect to happen. However, in a more complex economy, where Robinson worked for the Crusoe Aircraft Engine Company, a barter in which Robinson would work additional hours in exchange for a few bags of Crunchy Granola would involve many parties, including air frame manufacturers, air lines, travel agents, junketing Granola executives and employees, and so forth. It is easier to imagine in this circumstance that such a multilateral barter would be hard to organize and carry off. Robinson would have a difficult time setting it up, and it is quite plausible that Crusoe Aircraft Engine Company could fail to see how putting Robinson to work a few more hours a week would guarantee an increase in the demand for aircraft engines. In any case, an economic outcome on a South Seas island that appears only because of a transparent business psychosis may occur in a complex economy when businesspeople are perfectly 'rational', just not sufficiently omniscient to see all the possibilities for multilateral barter in the economy. Some economists argue that expectations are sufficiently rational so that this could never happen, that economic entrepreneurs are always eager to broker such deals. However, the evidence is that they are too optimistic, it does not always happen.
"In more complex economies, there are additional difficulties introduced because the participants in the economy do not all have the same information. For example, if Robinson offers to barter more labor for more yams, Mr. Friday might turn the offer down because he cannot be sure whether Robinson will be productive enough to make the barter pay off. Given this uncertainty about Robinson's characteristics, Mr. Friday might even suspect that the fact that Robinson is offering to barter is itself a signal that he is a poor worker who cannot find enough employment in the regular labor market. This can both cause markets to break down, and derail the direct barters that might be offered when such failures occur. This can happen even if the participants all have perfectly rational expectations; asymmetry of information is enough to lead to problems."
"So, what's the bottom line?" asked Robinson. "Are there ways to fix up the Walrasian mechanism so that expectations do not lead it to non-optimal allocations?"
"There are," said Keynes,"Clearly, you run into difficulty when Mr. Friday's expectations on maximum sales are too low. Therefore, the answer is to either keep those expectations high, or organize the multilateral barter that can overcome their limiting effects. I recommend that you establish a government which plays some direct role in your economy. When government works well, it can play the role of agent or deal-maker for the whole public, in effect putting together the multilateral barters that could overcome expectations and get to efficient resource allocation. In your situation, you would like the government to get Crusoe to produce more. You might, for example, introduce money, and run the printing presses to give yourself more purchasing power when output is low. Or you might have the government become a net purchaser of yams, which it could finance by running a deficit, and transfer the yams to you as an income supplement. Then Mr. Friday could count on government contracts to keep his sales high. Alternately, the government could give you more purchasing power through a deficit-financed wage subsidy, or through a legal minimum wage. You could force higher wages by collective labor action, going on strike and refusing to work unless you get a wage near $w=1 / 2$. You might even nationalize Crusoe, Inc. ur adopt an 'industrial policy' in which government gets directly involved in firm management, and in this manner force Mr. Friday to abandon the use of an expected sales criteria. To understand how these things would work, you would need to study the role of money, taxes, and government transfers and spending on goods and services in an economy. It may be tricky to institute these actions in such a way that Mr. Friday really expects them to increase yam demand, and not just be a wash with the same purchasing power just routed through different pockets. And in the situation where the economy would on its own get to the optimal allocation E despite Mr. Friday's expectations, some of the interventions I have mentioned could now prevent that and make things worse, the nightmare of Smith, Walras, and Friedman. Why, I could write a book about the possibilities open to you."
"You have, sir," chorused A.J. and H.V.
"Why so I have," said Keynes. "Well, then, read the book, Robinson. For now, let's go find a few yams to take care of your empty stomach."
"Best idea I've heard in months," replied Robinson, and off they trouped to the digging ground.


[^0]:    ${ }^{1}$ With a little help from Axel Leijonhufvud and Hal Varian.
    ${ }^{2}$ This schedule satisfies the formula $Y=[6(24-H)]^{1 / 2} .$.
    ${ }^{3}$ Robinson's preference elevation satisfies $u=\log (Y)+H / 12$.

[^1]:    ${ }^{4}$ Robinson's choice maximizes $u=\log (Y)+H / 12$ subject to the constraint $Y=[6(24-H)]^{1 / 2}$. Substituting the constraint, this problem is solved at $\mathrm{H}_{\mathrm{E}}=18$ and $\mathrm{Y}_{\mathrm{E}}=6$.

[^2]:    ${ }^{5}$ Walras was not actually from the Paris school, but when dealing with simple folk avoided raising the possibility that French scholars could live outside Paris.

    $$
    { }^{6} \mathrm{Y}=[6 \mathrm{~L}]^{1 / 2} .
    $$

[^3]:    ${ }^{7}$ Crusoe, Inc. seeks to maximize $\pi=[6 \mathrm{~L}]^{1 / 2}-\mathrm{wL}$. This problem is solved at $\mathrm{L}=3 / 2 \mathrm{w}^{2}$, yielding $\mathrm{Y}=3 / \mathrm{w}$ and $\pi=3 / 2 \mathrm{w}$.

