Econ. C103, 2003 Daniel McFadden

Final Exam

(Each question counts 20 points. This exam should take 20 minutes per question)

1. Consider the following simultaneous-move strategic-form game:

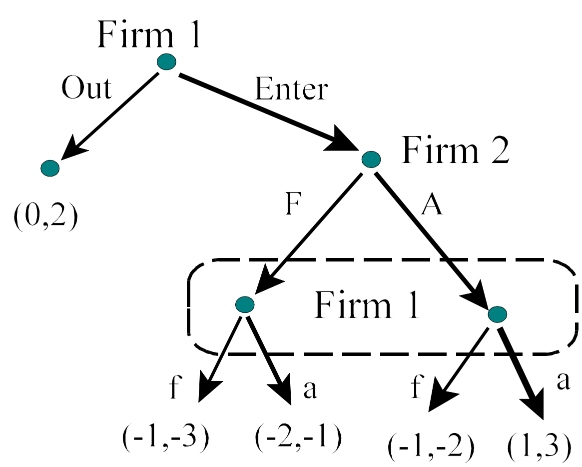
		Player 2		
		A	B	C
Player 1	U	(1,1)	(-1,0)	(-1,-1)
	D	(-1,-2)	(1,0)	(1,1)

Find all the Nash equilibria (pure and mixed) of this game.

2. Firm 1 has to decide whether to enter a market in which Firm 2 is the incumbent. If Firm 1 enters, then the firms have to decide simultaneously whether to fight (F or Firm 2/f for Firm 1) or accommodate (A for Firm 2/a for Firm 1), with the payoffs given in the graph below.

(A) Suppose there can be no communication between the firms before the game. Find all the subgame-perfect Nash equilibria, pure or mixed.

(B) Suppose Firm 2 can legally pre-commit itself to fight if Firm 1 enters, say by contracting with each of its customers that it will strictly beat any price posted by Firm 1. How does this change the game, and the subgame-perfect Nash equilibria?



EXAM CONTINUES ON OTHER SIDE

3. Two firms F′ and F″are said to be <u>Cournot duopolists</u> if they produce a common product in quantities q′ and q″ at a marginal cost $m > 0$, and then face price $p = A - B(q' + q'')$ in the market, where $A > m$ and $B > 0$ are constants known to both firms.

(A) Find a Nash equilibrium when the two firms choose quantities simultaneously.

(B) How would the Nash equilibrium change if firm F′ first chooses q′, and then knowing this, firm F″ chooses q″? Which firm gains and which loses relative to (A)?

4. Consider a first-price sealed bid auction of a single object with two bidders j = 1,2 and no reservation price. Bidder 1's $v_1 = 2$, and bidder 2's valuation is $v_2 = 5$. Both v_1 and v_2 are <u>known</u> to <u>both</u> bidders. Bids must be in whole dollar amounts. In the event of a tie, the object is awarded by a flip of a fair coin. Is there a Nash equilibrium? What is it? Is it unique? Is it efficient?

5. Suppose an exchange economy has J consumers and 2 goods. Consumer j has a utility function of the form $u_j = z_j + t_j \cdot v_j$, where z_j is consumption of the first commodity, which is divisible, with each consumer having an initial endowment of one unit, and t_j is consumption of the second commodity, which can be consumed only in integer units, with consumer 1 having an endowment of one unit and all others having an endowment of zero. The v_j are values of the indivisible good which are drawn independently for each consumer from the uniform distribution on [0,1]. Each consumer knows her own v_j, but knows only that the v's of all others are drawn from the uniform distribution..Suppose an auction mechanism is used to assign the indivisible good, with consumer 1 participating as a possible buyer as well as the seller. Suppose this mechanism leads to a resource allocation that is in the core of the economy.

(A) From the properties of the core, prove that the allocation produced by this auction must have resulted in the indivisible good going to the consumer with the highest v_j, with a payment by the buyer no less than the second highest v_j, and no greater than the highest v_j.

(B) Discuss the relationship of this result to the revenue equivalence theorem of auction theory.