PROBLEM SET I

Simon C. and Blume L. (1994) Ex: 13.12, 16.1, 16.2, 16.4 & Sundaram R. (1996) Ch 7.8:10

1. Write the following quadratic forms in matrix form:
   a) $x_1^2 - 2x_1x_2 + x_2^2$
   b) $5x_1^2 - 10x_1x_2 - x_2^2$
   c) $x_1^2 + 2x_2^2 + 3x_3^2 + 4x_1x_2 - 6x_1x_3 + 8x_2x_3$

2. Determine the definiteness of the following symmetric matrices:
   a) $\begin{pmatrix} -3 & 4 \\ 4 & -6 \end{pmatrix}$
   b) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$
   c) $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{pmatrix}$
   d) $\begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 5 \\ 3 & 0 & 4 & 0 \\ 0 & 5 & 0 & 6 \end{pmatrix}$

3. How many *k*th order principal minors will an $n \times n$ matrix have for each $k \leq n$?

4. Let $Q = x^T Ax$ be a quadratic form in R^n. By evaluating *Q* on each of the coordinate axes in R^n, prove that a necessary condition for a symmetric matrix to be positive definite (positive semi-definite) is that all the diagonal entries be positive (non-negative). State and prove the corresponding result for negative definite (negative semi-definite) matrices.
   Prove, by example, that this necessary condition in not sufficient.

5. Let $f : R^n \to R$ be concave, *A* an $n \times m$ matrix and $b \in R^n$. Consider the function $h : R^m \to R$ defined by:
   $h(x) = f[Ax + b] \qquad x \in R^m$
   Is *h* concave? Why or why not?