

**PROBLEM SET 9 (due Dec. 2)**

The following problems are from Jehle-Reny, Exercise 9.5:

1. #9.5

2. #9.8, with symmetric independent private values uniformly distributed on  $[0,1]$

3. #9.10

4. #9.11

5. Suppose an ascending bid auction for a single item where two bidders  $i = 1,2$  have values uniformly distributed on  $[0,c]$ , given  $c$ , but  $c$  is a common unknown that has an exponential distribution,  $P(c>t) = \exp(-t)$ . Suppose each bidder knows these distributions, and is rational in his beliefs about the values of the other player. Show that despite this, this auction will display the “winner’s curse” in which the winner will use information from his rival’s bids to update his beliefs, and will on average overbid, so that the expected payoff to the winner from the auction is negative.

6. Suppose a two-bidder asymmetric independent private value first-price sealed-bid auction in which the first bidder’s value distribution has CDF  $G_1(v) = v^\alpha$  and the second bidder’s value distribution has CDF  $G_2(v) = v^\beta$ , where  $0 < v < 1$ ,  $\alpha \neq \beta$  and  $\alpha, \beta > 0$ . Show that in this problem, despite the asymmetry in the value distributions, there is a Nash equilibrium in which each bids his conditional second value.

7. Suppose a two-bidder asymmetric independent private value first-price sealed bid auction in which the first bidder’s value is 400, the second bidder’s value is either 300 or 800, each with probability one-half, the seller has an announced reservation price of 300, and the auction rules are that in the case of ties, then among those tied, the item goes first to the first bidder, second to the second bidder, and last to the seller. Show that in this case, strategies in which each bidder bids his conditional second value is not a Nash equilibrium.