LECTURE / DISCUSSION

Efficiency

Efficiency

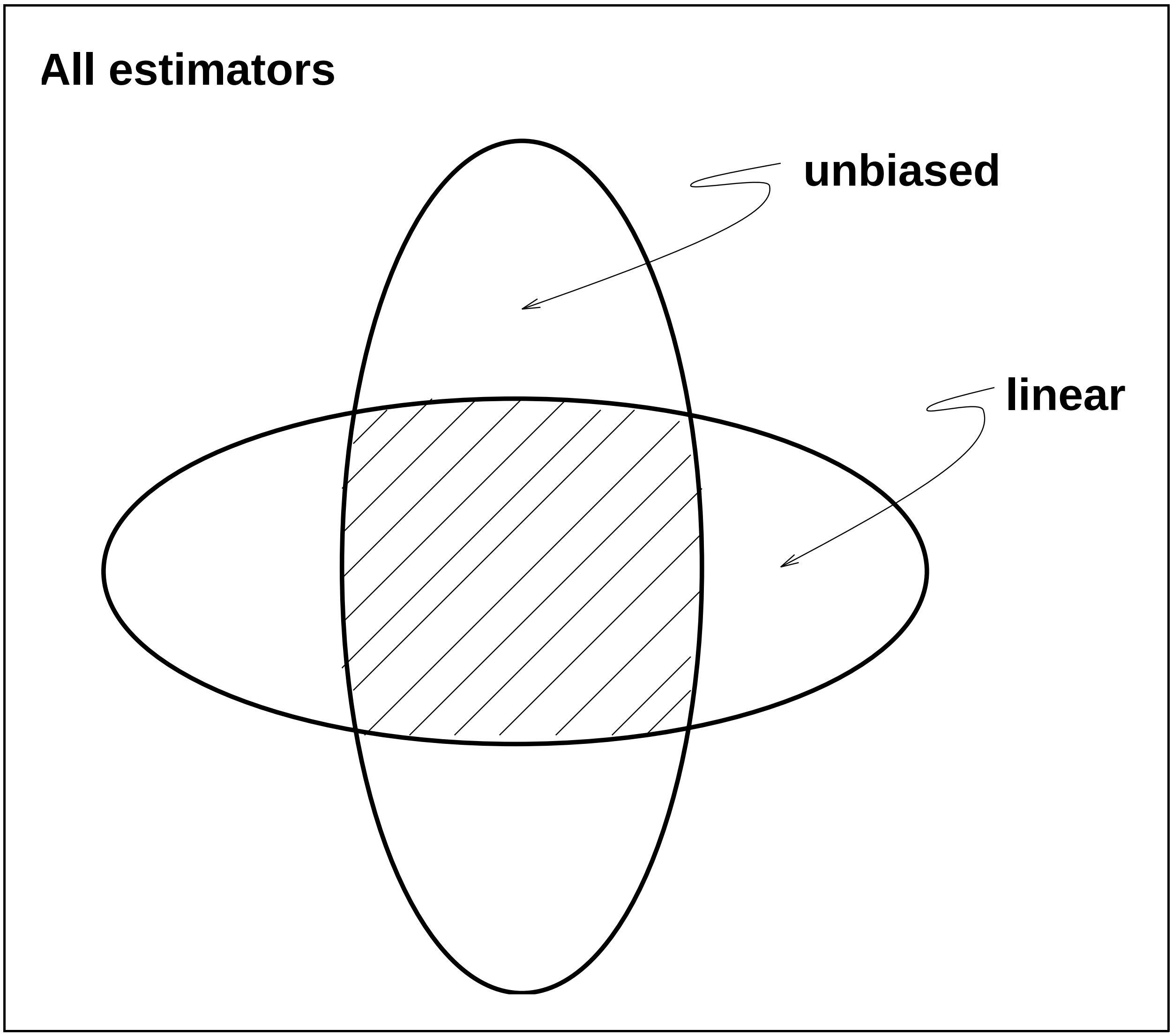
I. Compare two unbiased estimators $\hat{\beta}_1$ and $\hat{\beta}_2$.

$\hat{\beta}_1$ is **more efficient** than $\hat{\beta}_2$ if $V(\hat{\beta}_1) < V(\hat{\beta}_2)$.

II. An estimator is said to be **efficient** within a class if it has lower variance than all other estimators in the class.

Gauss-Markov Theorem

OLS is efficient in the class of unbiased, linear estimators.



OLS is BLUE--best linear unbiased estimator.

Linear Estimators

$$\tilde{\beta} = \sum c_n y_n$$

Recall OLS $\hat{\beta} = \dfrac{\sum x_n y_n}{\sum x_n^2}$.

Let $c_n = \dfrac{x_n}{\sum x_n^2}$.

Then:

$$\text{OLS } \hat{\beta} = \sum c_n y_n = \sum \left(\frac{x_n}{\sum x_n^2} \right) y_n$$

$$= \frac{\sum x_n y_n}{\sum x_n^2}$$

So: OLS $\hat{\beta}$ is a linear estimator.

Gauss-Markov Theorem

If:

$$Y_n = \alpha + \beta X_n + \varepsilon_n$$

- $E(\varepsilon_n \mid X_n) = 0$
- ε_n independent
- ε_n homoscedastic

Then: OLS is BLUE.

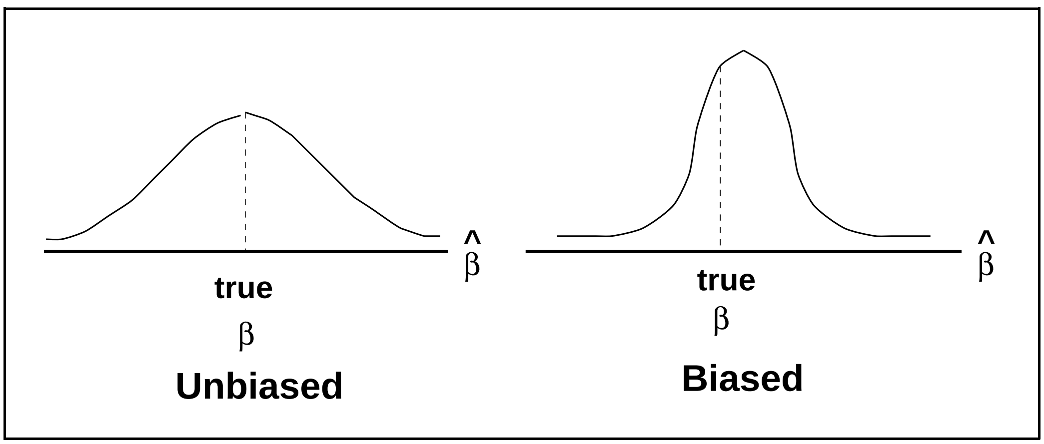
Review

$E(\varepsilon \mid x) = 0 \Rightarrow$ OLS is unbiased

$E(\varepsilon \mid x) = 0$
ε_n independent $\Rightarrow$ OLS is BLUE
ε_n homoscedastic

Caveats

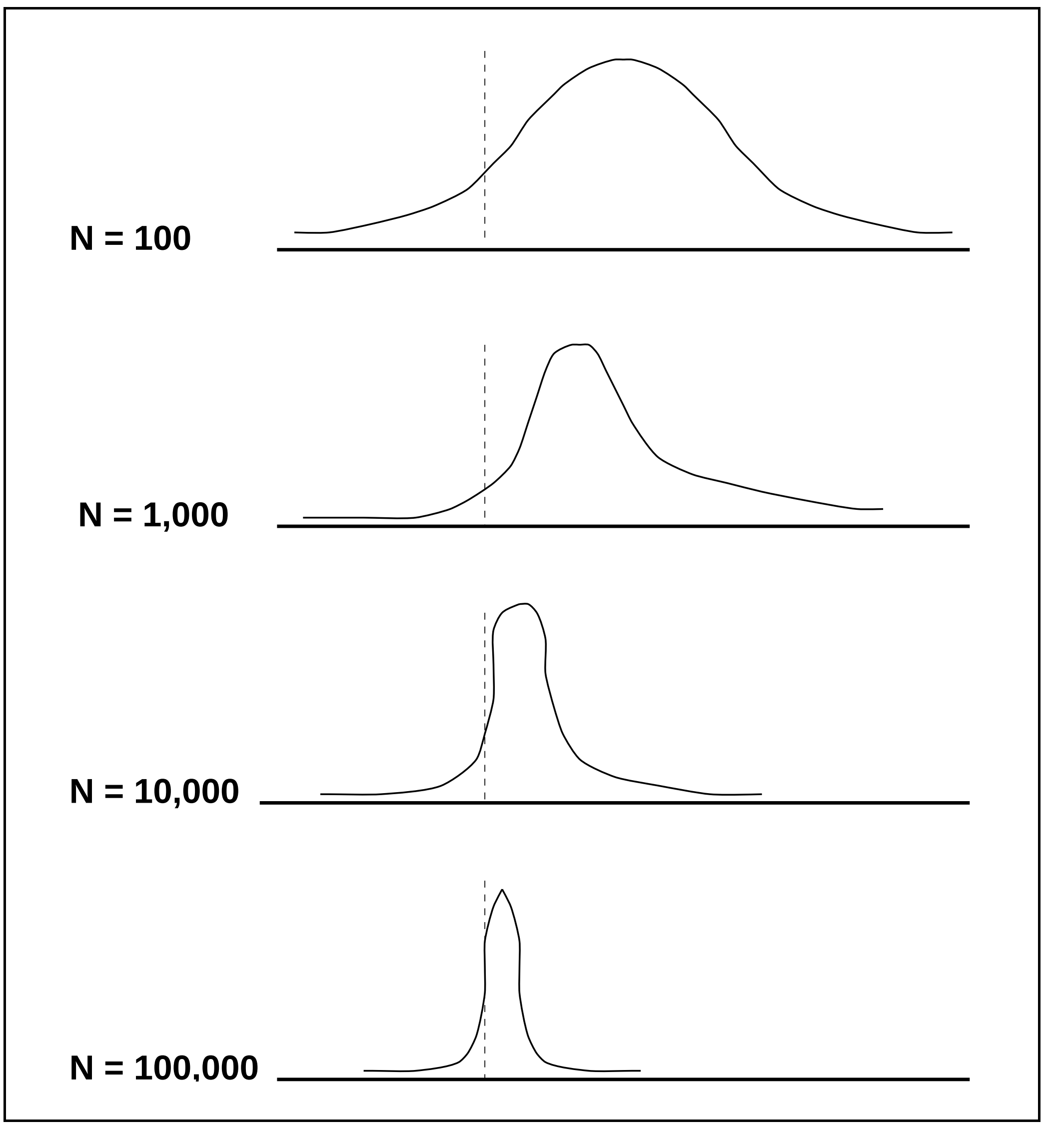
- There might be unbiased nonlinear estimators that are "better"--i.e., that have smaller variance.
- There might be biased estimators that are "better."



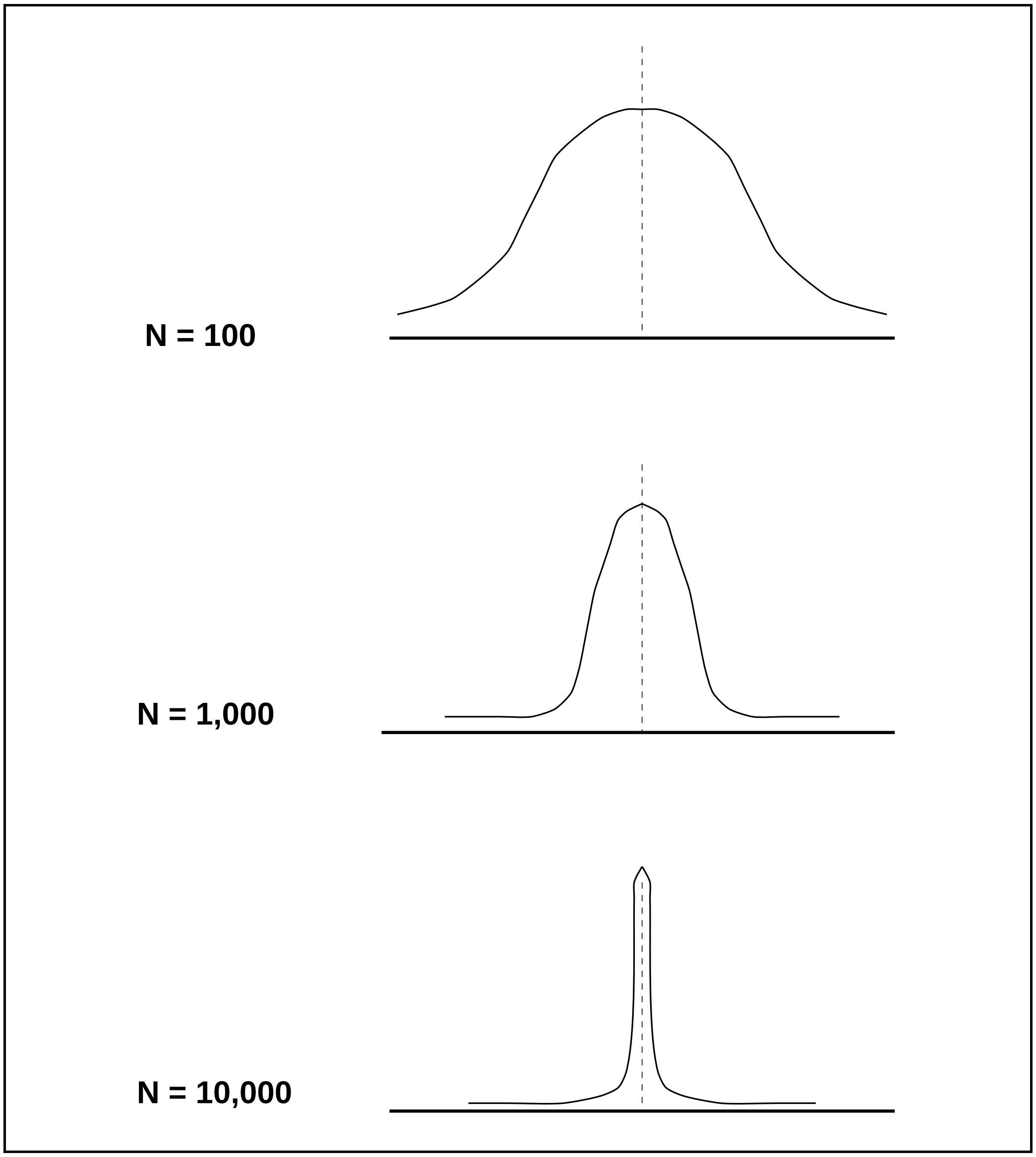
Consistency

Consistent estimator: its distribution converges on the true β as sample size increases.

Biased but consistent estimator:



Unbiased and consistent estimator:



An unbiased estimator is consistent if its variance decreases as sample size increases.

OLS is unbiased and consistent.