Economics 201B–Second Half

Lecture 14

Uncertainty, including GEI Model

- GEI (General Equilibrium, Incomplete Markets) Model studies situations in which markets are incomplete: there are restrictions on the allowed trades

- Six Models

V(a): DC   V(b): DI

↖   ↑

III: REE   IV: GEI

↖   ↗

II: A

↑

I: A-D

  - A-D: Arrow-Debreu complete contingent claims
  - A: Arrow securities
  - REE: Rational Expectations Equilibrium
  - GEI: General Equilibrium, Incomplete Markets
  - DC: Dynamic Completeness
  - DI: Dynamic Incompleteness

- Model I: Complete Set of Arrow-Debreu Contingent Claims
  - L physical goods (umbrellas, houses)
  - States

    $$s = 1, \ldots, S$$

    States reflect the set of possible situations (rainy, sunny, my house burns, doesn't burn)
  - At time 0, don't know which state s will prevail at time 1, but do know the set of states and the probability distribution over the states. Trade "contingent commodities" (an umbrella if it rains, a house if my house burns). You buy a piece of paper (security) which you can turn in for an umbrella if it rains, and is worthless otherwise.
  - At time 1, the state s is revealed. You turn in your state s securities for goods, the securities for the other states are worthless, there is no trade, consumption occurs.
  - Formally, a state-contingent commodity bundle is

    $$(x_{11}, \ldots, x_{L1}, x_{12}, \ldots, x_{L2}, \ldots, x_{1S}, \ldots, x_{LS}) \in \mathbf{R}_+^{LS}$$

    where

    $$(x_{1s}, \ldots, x_{Ls})$$

    is the consumption if state s occurs.

- Point: Create LS goods, just like an economy with no uncertainty. The Existence and Welfare Theorems and their proofs hold verbatim provided
  - There is a market for each of the LS state-contingent commodities; and
  - Either
    - Everyone knows the true probability distribution over states (no private information); or
    - Everyone agrees on the probability distribution over states, and everyone maximizes expected utility with respect to these common beliefs, but the common beliefs may be wrong; or
    - there is private information, but people don't update their beliefs based on the prices. In this case, Pareto Optimality is in the *ex ante* sense: at date 0, there is no reallocation of the contingent claims which makes each agent better off given the agent's date 0 beliefs.
- What's wrong with Model I? Too many markets. We don't see this many markets in practice, perhaps due to transactions costs.

- Model II: Arrow Securities

  - For each state s, there is a security which pays 1 unit of account in state s and zero in other states. Securities are in zero net supply (the sum of the holdings over agents is zero).
  - Alternatively, we could have the state s security pay off 1 unit of a numeraire good in state s.
  - Time 0: trade securities
  - Time 1:
    * State s is revealed.
    * The state s security pays 1 unit of account, so some agents get a positive dividend, others a negative dividend, and the net dividend is zero. The securities for other states are worthless.
    * A state s spot market opens, on which the L physical goods are traded; consumer's budget is given by the value of the consumer's endowment plus securities dividends.
  - Either
    * Everyone knows the true probability distribution over states (no private information); or
    * Everyone agrees on the probability distribution over states, and everyone maximizes expected utility with respect to these common beliefs, but the common beliefs may be wrong; or

    * there is private information, but people don't update their beliefs based on the prices. In this case, Pareto Optimality is in the *ex ante* sense: at date 0, there is no reallocation of the contingent claims which makes each agent better off given the agent's date 0 beliefs.

- **Theorem 1** *Walrasian Equilibrium exists. If, at time* 0*, agents correctly anticipate the spot prices in each state at time* 1*, the Equilibrium is Pareto Optimal.*

    * Doesn't require that agents correctly anticipate the state, just that they correctly anticipate what the spot prices will be in each state.

- Problems:

    * Correct anticipation of spot prices is very strong (simply knowing the set of possible states is strong!)

    * Having a security for each state is strong

        · High tansaction costs

        · Moral hazard: effort may affect probabilities

        · Symmetric information is implausible

        · If information is asymmetric, assuming that people don't try to draw inference from prices is implausible.

      - Partial Fix: Efficient Markets Hypothesis. If market prices securities correctly, then it is as if there were symmetric information.

- Model III: Rational Expectations Equilibrium (REE) (Radner)
  - Information asymmetric
  - People *do* infer information from prices.
  - Kreps' Example: REE need not exist.
    - $S = 2, L = 2$; the two states are equally likely.
    - Agent 1 knows s at date 0; Agent 2 has no information, other than that the probability of each state is 0.5.
    - Agents trade goods at time 0 for consumption at time 1. Equivalently, there are two securities, security 1 pays 1 unit of good 1 at time 1 and security 2 pays 1 unit of good 2 at time 1.
    - With full information, the market-clearing price is the same in the two states (but the consumptions are different).
    - Agent 1's demand depends in a nonzero way on the state.

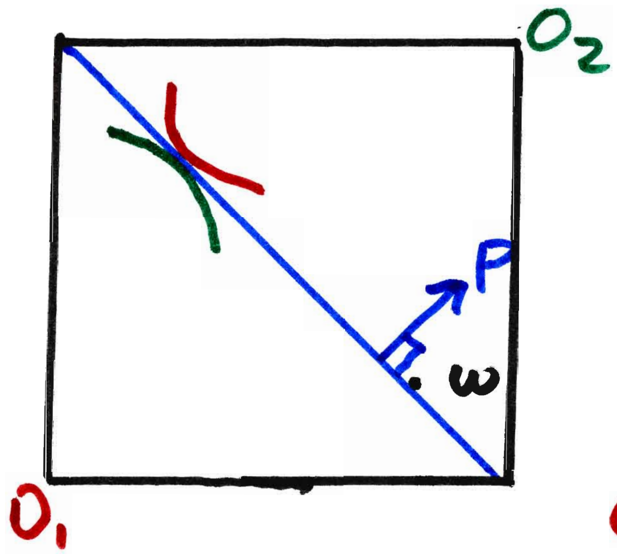
        - Suppose at equilibrium agent 2 does not learn the state. Then agent 2's demand must be the same at both states, while agent 1's demand is different in the two states, hence markets can't clear in both states.
        - Suppose at equilibrium agent 2 does learn the state. Then the prices are the same in the two states, so agent 2 can't learn the state.
        - Hence, there is no REE.
    - REE exists generically.

- Model IV: General Equilibrium, Incomplete Markets (GEI), 2 periods:
    - Go in a different direction from REE
    - Symmetric Information
    - 2 periods, S states, $L = 1$ one good
    - J securities, $J < S$, paying off in spot market units of account or a single numeraire good; like Arrow securities, but we don't restrict them to pay off in only one state.
    - **Theorem 2** *Walrasian Equilibrium exists.*

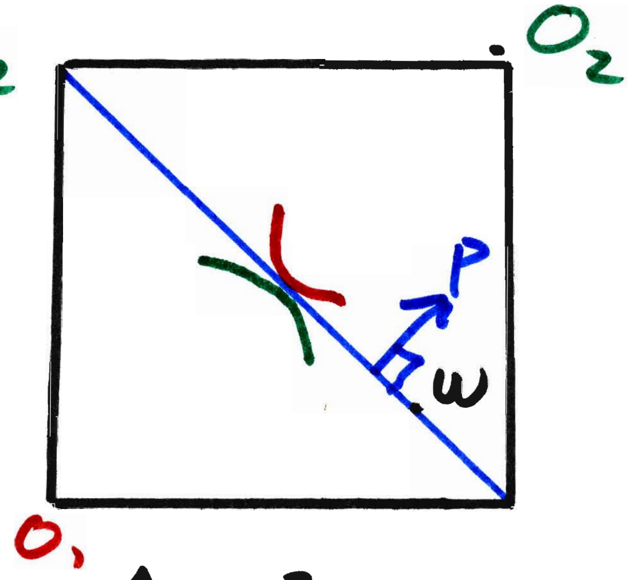
      Proof: Almost same as in Arrow-Debreu case.■

Kreps' Example

Full Information

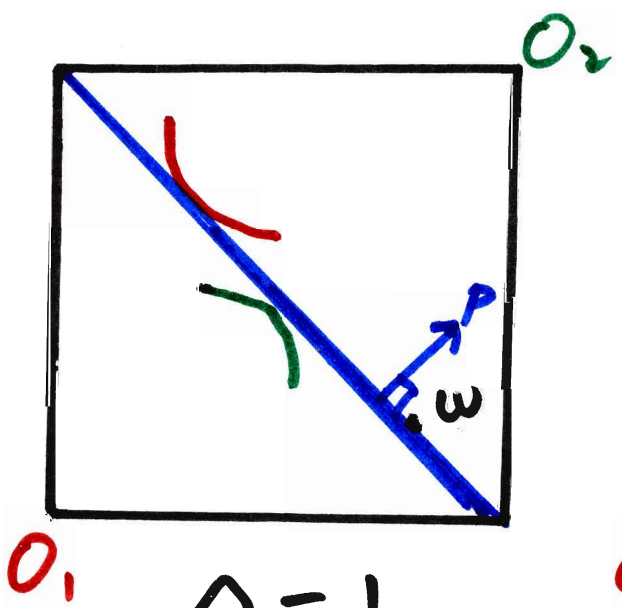


$\Delta = 1$

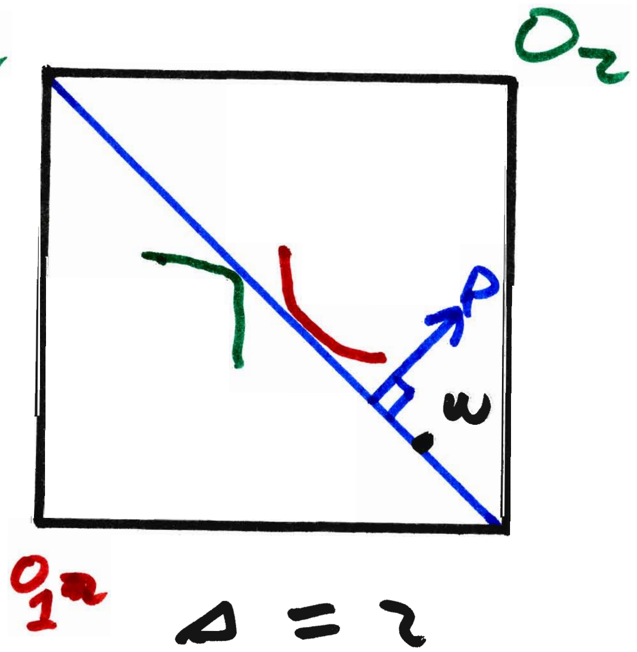


$\Delta = 2$

1 informed, 2 uninformed



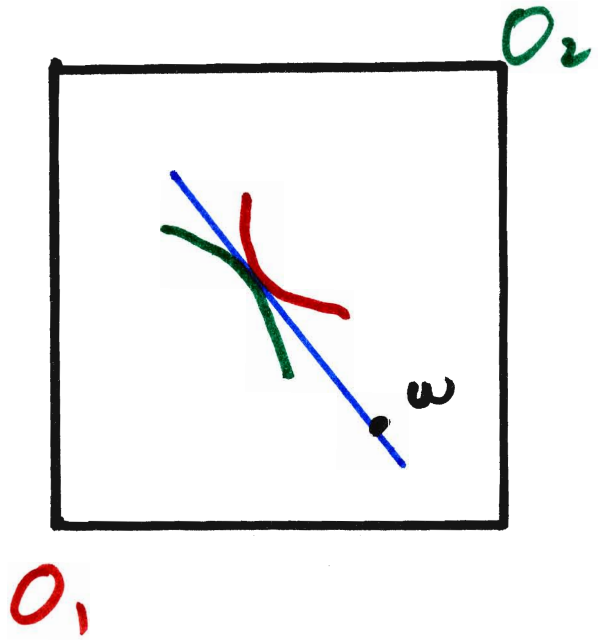
$\Delta = 1$



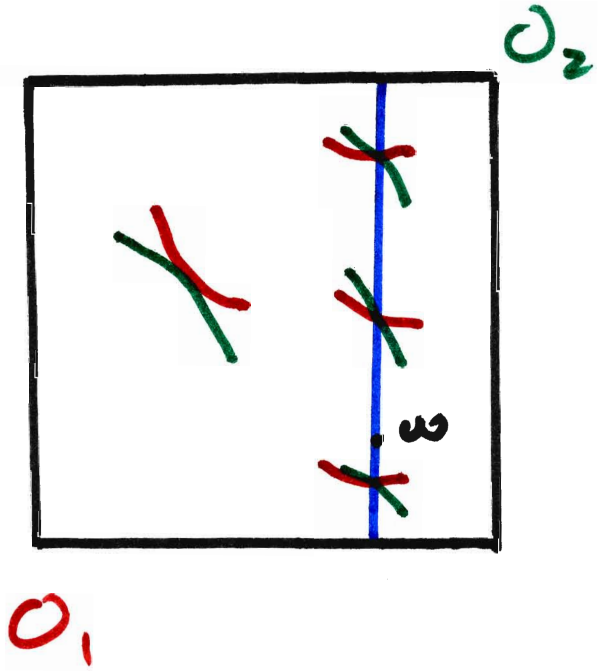
$\Delta = 2$

- *The First and Second Welfare Theorems are false!* At one level, this is obvious.
  - ∗ The restriction on trade means you can't equate MRS's
  - ∗ A given Pareto Optimum need not be in the set of allocations that can be reached by the given markets, so Second Welfare Theorem fails.
  - ∗ Example: One physical good ($L = 1$), two states ($S = 2$).
    - · With two Arrow securities, get both Welfare Theorems.
    - · With just one Arrow security (pays off 1 unit of account in state 2), you get autarky.
    - · Note in the example that the autarkic equilibrium is "Constrained Pareto Optimal." There is no other allocation that can be achieved by the given markets that makes both agents better off.
  - ∗ More Interesting Example, Edgeworth Cube: $I = 2$, $L = 1$, $S = 3$, $J = 2$.
    - · The securities are Arrow securities for states 1 and 2; no security for state 3.
    - · Set of Pareto Optima is a 1-dimensional manifold (a curve from one corner of the Cube to the opposite corner)
    - · Equilibrium is constrained to lie in the plane parallel to the $x_1 - x_2$ plane through the endowment.

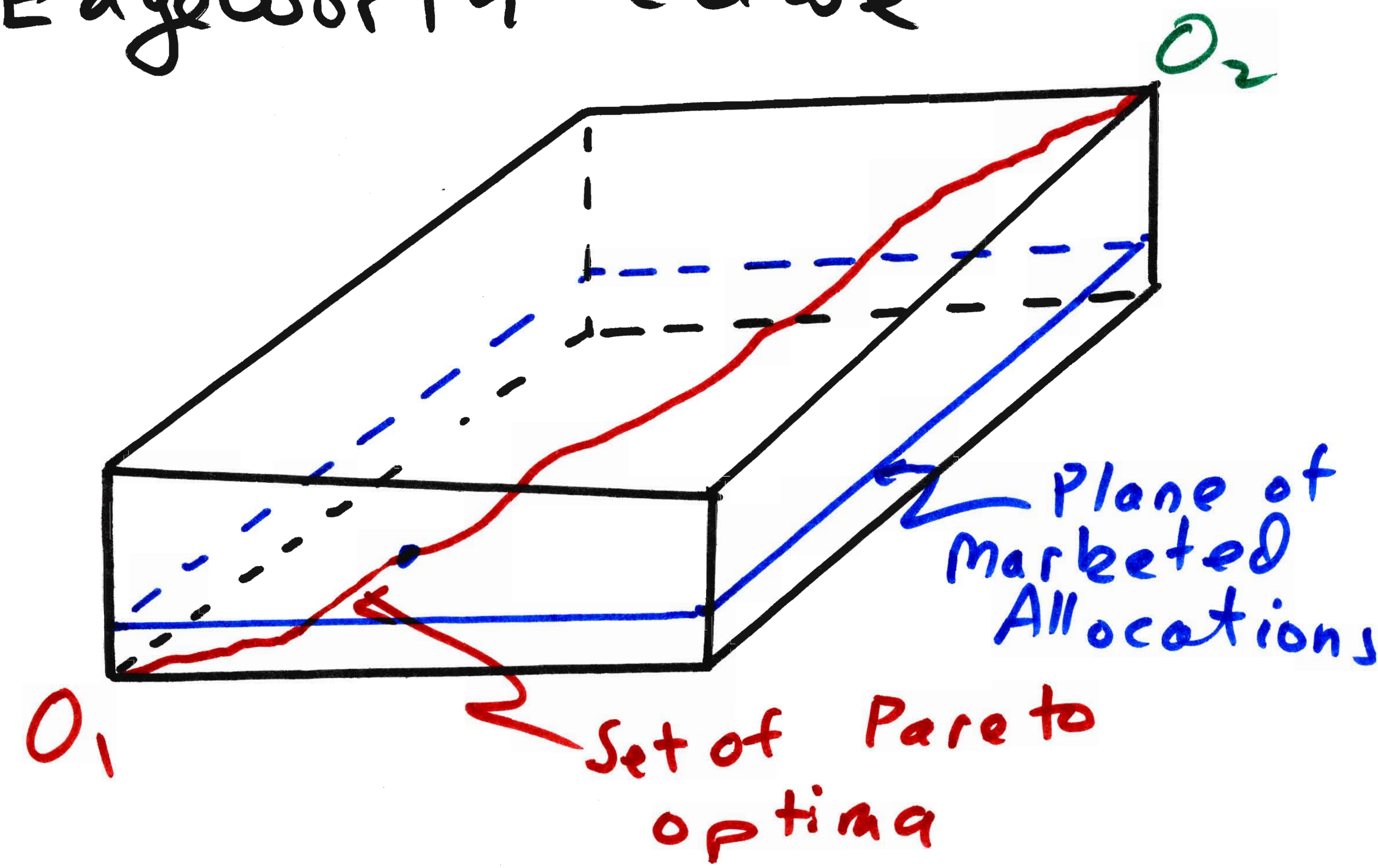
Two Arrow Securities



One Arrow Security



Edgeworth Cube



In Marketed Plane