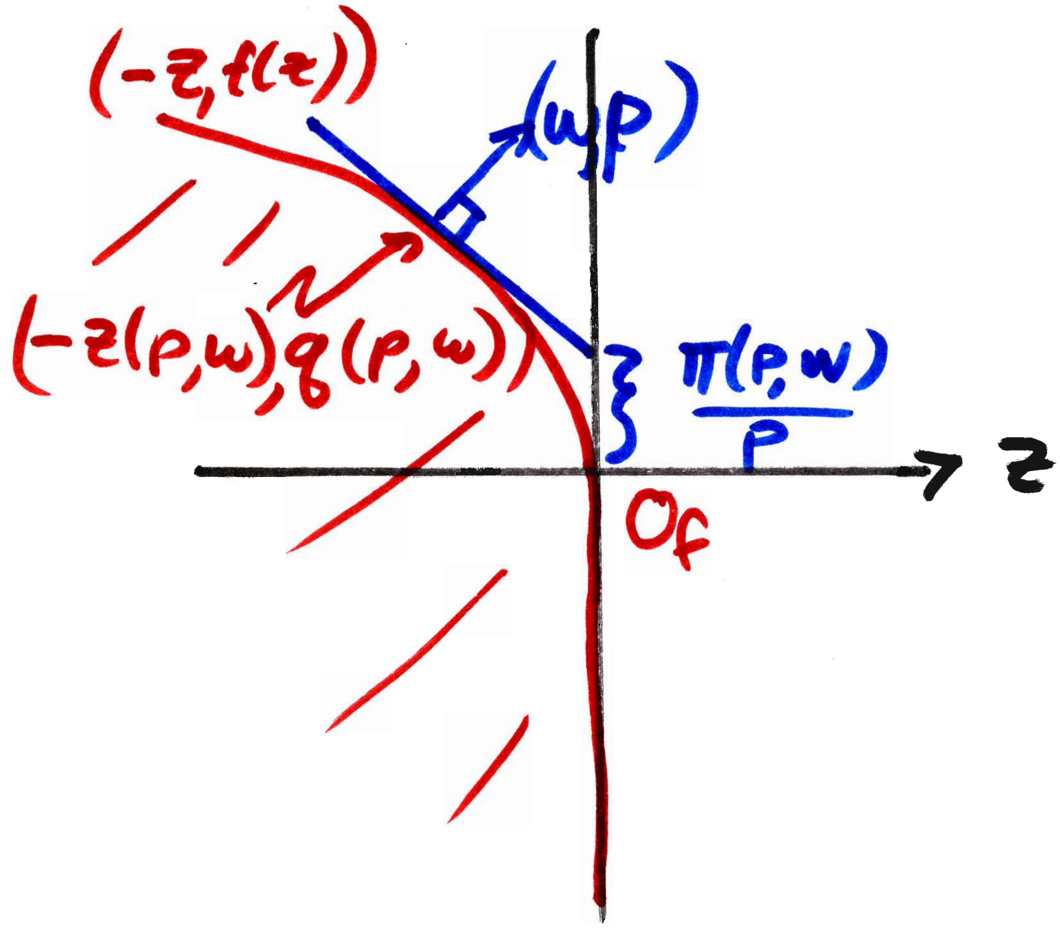
Economics 201B–Second Half

Lecture 4, 3/18/10

The Robinson Crusoe Model: Simplest Model Incorporating Production

- 1 consumer
- 1 firm, owned by the consumer
- Both the consumer and firm act as price-takers (silly in this model, but it shows how equilibrium operates)
- 2 goods:
  - Leisure x_1, endowment $\bar{L}$ (24 hours per day)
  - Consumption goood x_2 bananas, endowment $= 0$
- p: price of bananas
- q: quantity of bananas produced by firm
- w: wage rate = price of labor
- Production function $f(z)$: z units of labor produces $q = f(z)$ bananas. We assume f is strictly concave; first gather low-hanging bananas, then start climbing trees to gather, then tend plants to increase yield
- Firm's profit: $pq - wz$. Note that profit is a linear function of (q, z), the vector of inputs and outputs, whether or not (q, z) is feasible. The firm maximizes profit over the feasible set, taking prices as given
  - Labor demand $z(p, w)$ chosen to maximize

$$pq - wz = pf(z) - wz$$



(−z, f(z))
(w, p)
(−z(p, w), q(p, w))
π(p, w) / p
z
O_f

taking p, w as given. First order condition

$$pf'(z) - w = 0$$

- Output $q(p, w) = f(z(p, w))$
- Profit $\Pi(p, w) = pq(p, w) - wz(p, w)$

- Consumer owns firm, so receives the profit. Crusoe's budget constraint is

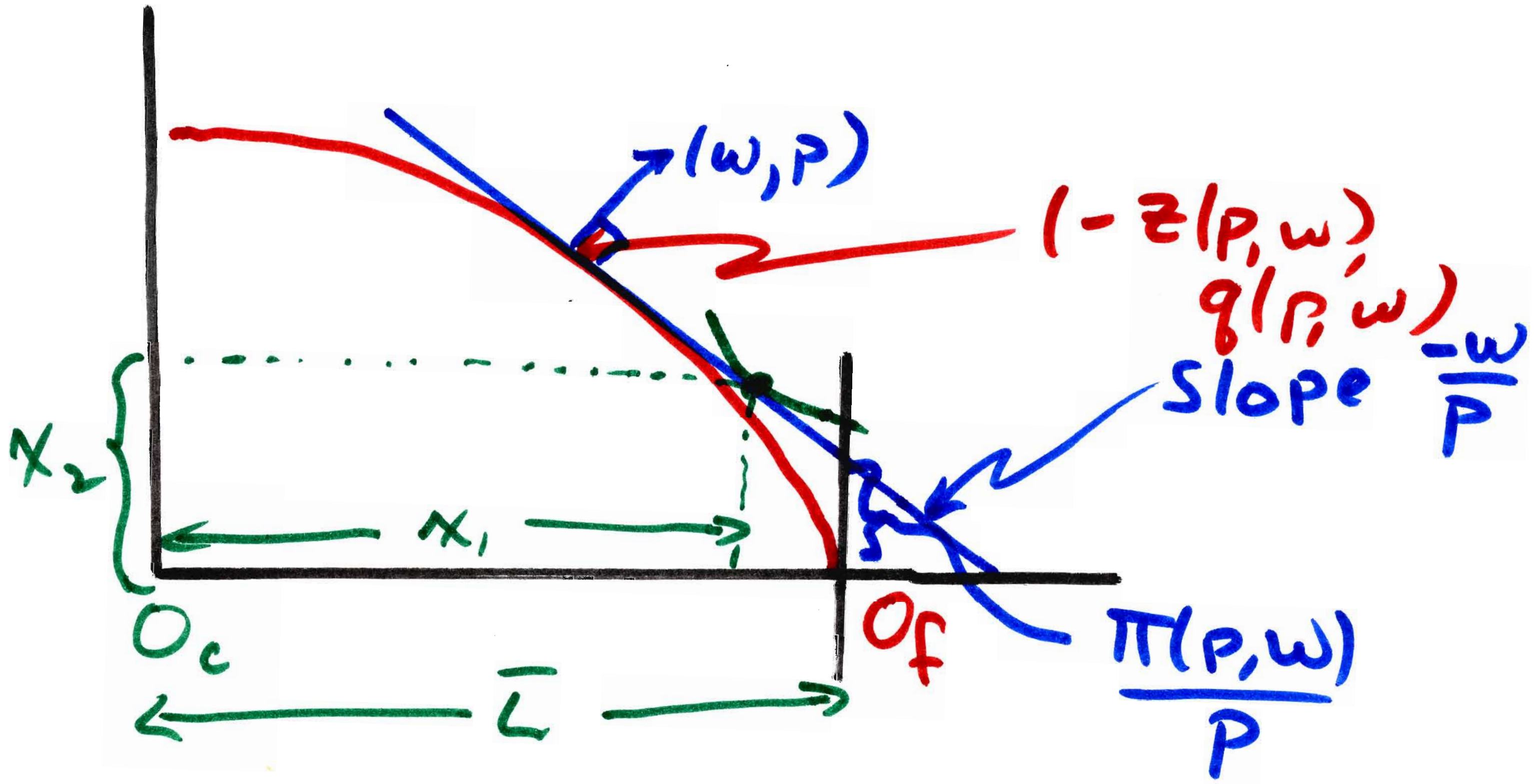
$$px_2 \leq w(\bar{L} - x_1) + \Pi(p, w)$$

- Walrasian equilibrium prices are (p^*, w^*) such that markets clear:

$$\begin{aligned} x_2(p^*, w^*) &= q(p^*, w^*) \text{ (banana market)} \\ z(p^*, w^*) &= \bar{L} - x_1(p^*, w^*) \text{ (labor market)} \end{aligned}$$

- In the previous diagram showing the firm's problem, lines perpendicular to the price vector (w, p) (note this is labelled incorrectly as (p, w) in MWG) are isoprofit lines. Any two points on a given isoprofit line yield the same profit; this is true whether or not the point on the isoprofit line is a feasible production. In particular, if we consider the isoprofit line through the firm's profit-maximizing point on its production set, the x_2 *intercept of this line must be* $\frac{\Pi(p,w)}{p}$.

- The previous diagram superimposes the consumer's problem on the firm's problem. If $x_1 = \bar{L}$ (Crusoe gets no labor income), Crusoe's income is $\Pi(p, w)$, so Crusoe can purchase $\frac{\Pi(p,w)}{p}$ bananas, so $\left(\bar{L}, \frac{\Pi(p,w)}{p}\right)$ lies in Crusoe's budget frontier. *The isoprofit line through the firm's profit-mazimizing production is the consumer's budget frontier!*



(w, p)
(−z(p, w), q(p, w))
Slope −w/p
x_2
x_1
O_c
O_f
$\bar{L}$
π(p, w)/p

- The previous diagram does not show an equilibrium configuration. For market clearing, require

$$\begin{aligned} x_2(p^*, w^*) &= q(p^*, w^*) \text{ (banana market)} \\ z(p^*, w^*) &= \bar{L} - x_1(p^*, w^*) \text{ (labor market)} \end{aligned}$$

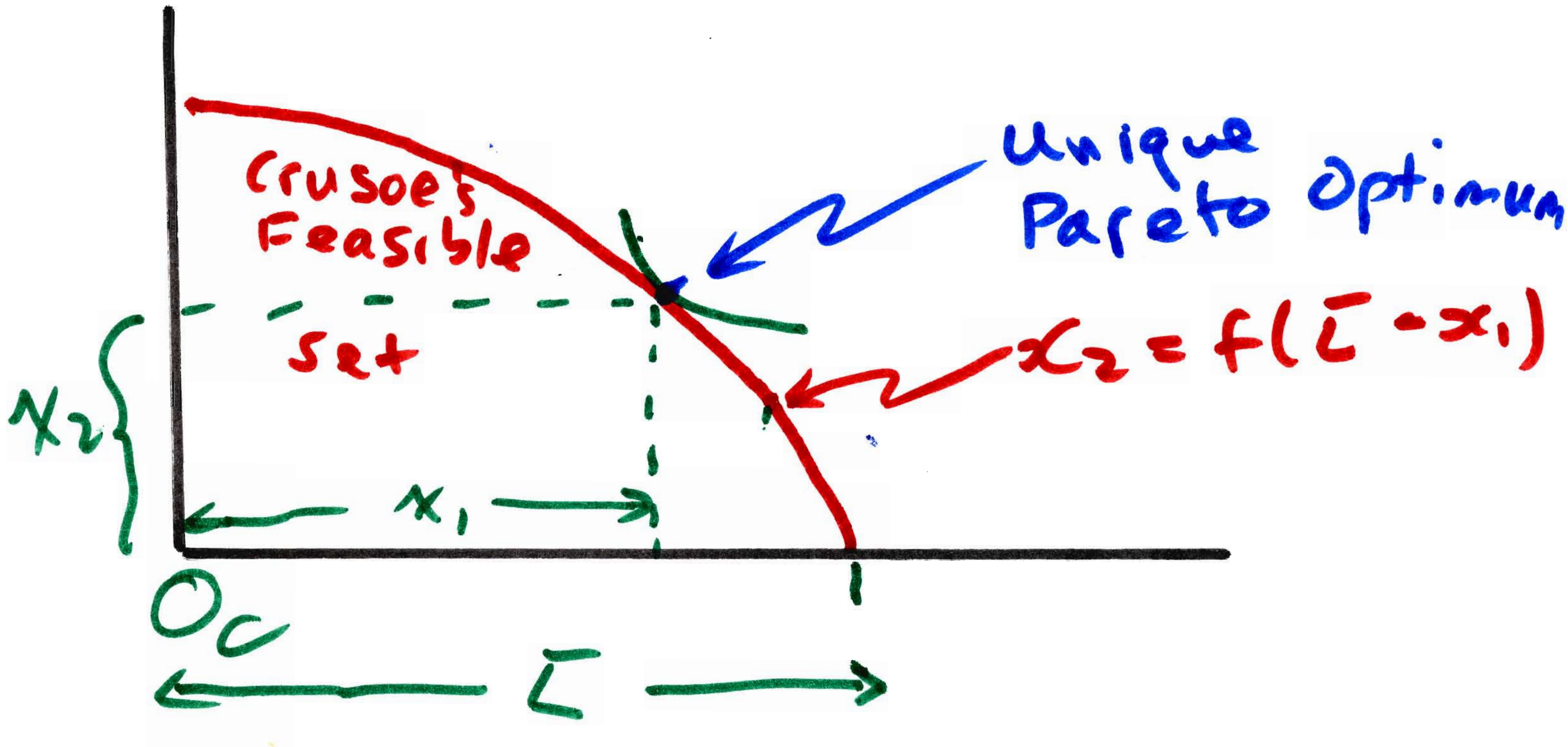
  Markets clear if and only if the *firm's profit-maxizing point and Crusoe's demand point coincide*. In the diagram, Crusoe is supplying less labor than the firm is demanding, and Crusoe is consuming fewer bananas than the firm is selling

- In the following diagram, we dispense temporarily with the firm and look at the consumer's overall problem, in which Crusoe applies the technology directly without going through the structure of the firm

- Notice that Crusoe's feasible set is just given by the production technology, so the feasible set is nonlinear; it is not a "budget set;" each point in the feasible set is a feasible consumption for Crusoe.

- What consumption would Crusoe choose? The economy has a *unique(!) Pareto optimum*, given by the point of tangency between the feasible set and Crusoe's indifference curve.

- *Second Welfare Theorem*: If we choose (p^*, w^*) such that (w^*, p^*) is perpendicular to the common tangent at the Pareto Optimum, then firm's profit-maximizing production and Crusoe's demand point coincide, so the unique Pareto Optimum is a Walrasian Equilibrium (without transfers); that's the Second Welfare Theorem.



Crusoe's Feasible Set
Unique Pareto Optimum
$x_2 = f(\bar{L} - x_1)$
x_2
x_1
O_C
$\bar{L}$

- *First Welfare Theorem*: If (p^*, w^*) is a Walrasian Equilibrium Price, then firm's profit-maximizing point and Crusoe's demand point coincide, (p^*, w^*) supports this common point, so it is Pareto Optimal

Arrow-Debreu Economy

- L commodities, indexed by $\ell = 1, \ldots, L$
  - I consumers, indexed by $i = 1, \ldots, I$
  - Consumption sets $X_i \subseteq \mathbf{R}^L_+$
  - Endowments $\omega_i \in \mathbf{R}^L_+$
  - Preference relations $\succeq_i$ on X_i, assumed complete and transitive
  - Social endowment

$$\begin{aligned} \bar{\omega} &= \sum_{i=1}^{I} \omega_i \\ &= (\bar{\omega}_1, \ldots, \bar{\omega}_L) \end{aligned}$$

- J firms, indexed by $j = 1, \ldots, J$
  - Production Sets $Y_j \subset \mathbf{R}^L$ assumed closed and nonempty
  - Shareholdings: Consumer i owns share θ_{ij} of firm j,

$$\sum_{i=1}^{I} \theta_{ij} = 1 \text{ (for each } j)$$

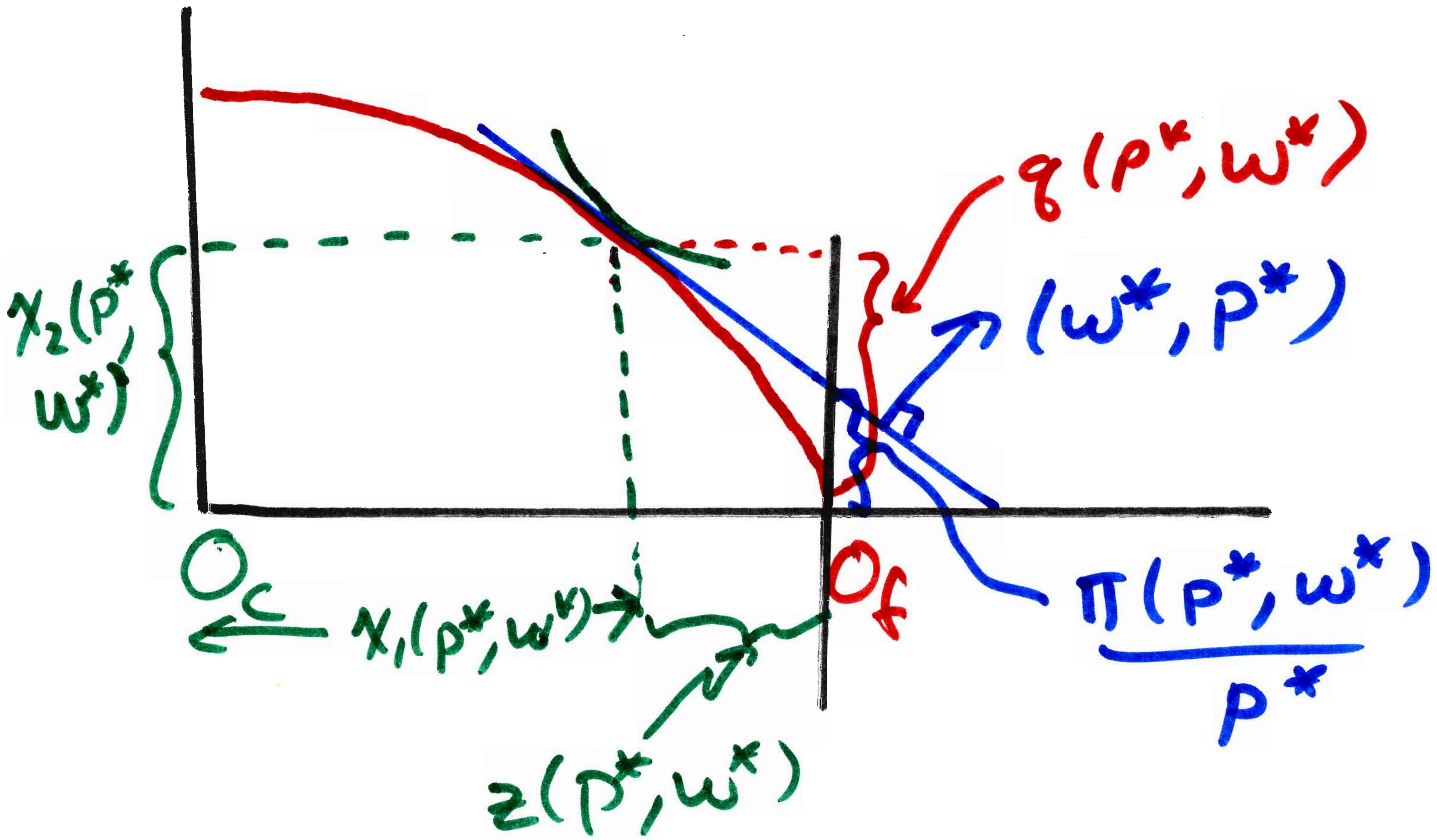
- *Income Transfer*: An income transfer is $T \in \mathbf{R}^I$ such that

$$\sum_{i=1}^{I} T_i = 0 \text{ (Budget Balance)}$$

- *Budget set*:

$$B_i(p, y, T) = \left\{ x \in X_i : p \cdot x \leq p \cdot \omega_i + \sum_{j=1}^{J} \theta_{ij} p \cdot y_j + T_i \right\}$$

  Note the budget set depends on prices, the income transfer, *and on the firms' production decisions*



$q(p^*, w^*)$
(w^*, p^*)
$x_2(p^*, w^*)$
O_c
$x_1(p^*, w^*)$
O_f
$\frac{\pi(p^*, w^*)}{p^*}$
$z(p^*, w^*)$

- *Demand*:

$$D_i(p, y, T) = \left\{x \in B_i(p, y, T) : \forall_{x' \in B_i(p,y,T)}\ x \succeq_i x'\right\}$$

- An *allocation*

$$(x, y) = (x_1, \ldots, x_I, y_1, \ldots, y_J)$$

is a specification of $x_i \in X_i$ $(i = 1, \ldots, I)$ and $y_j \in Y_j$ $(j = 1, \ldots, J)$; the allocation is *feasible* if

$$\sum_{i=1}^{I} x_i = \bar{\omega} + \sum_{j=1}^{J} y_j$$

Notice that this is a vector equation (one equation for each of the L goods) and that we require *equality*. The set of feasible allocations is denoted by A

- *Walrasian Equilibrium with Transfers*: In the Arrow-Debreu economy, a Walrasian Equilibrium with Transfers is a 4-tuple (p^*, x^*, y^*, T) such that

  1. $T \in \mathbf{R}^I$ is an income transfer. We don't put an $*$ on T because T is *not* determined endogenously by market-clearing

  2. p^* is a price, i.e. $p^* \in \mathbf{R}^L$ (don't require $p \in \mathbf{R}^L_+$)

  3. for $j = 1, \ldots, J$, $y_j^* \in Y_j$ and

     $$\forall_{y_j \in Y_j}\ p^* \cdot y_j^* \geq p^* \cdot y_j \text{ (price-taking profit maximization)}$$

  4. $$x_i^* \in D_i(p^*, y^*, T) \text{ (price-taking preference maximization)}$$

5. (x^*, y^*) is a feasible allocation, i.e.

$$\sum_{i=1}^{I} x_i^* = \bar{\omega} + \sum_{j=1}^{J} y_j^* \text{ (market-clearing)}$$

- *Pareto Optimality*: A feasible allocation (x, y) is

  – Pareto Optimal if there is no other feasible allocation (x', y') such that

$$\begin{aligned} x_i' &\succeq_i x_i \ (i = 1, \ldots, I) \\ x_i' &\succ_i x_i \text{ (some i)} \end{aligned}$$

  – weakly Pareto Optimal if there is no other feasible allocation (x', y') such that

$$x_i' \succ_i x_i \ (i = 1, \ldots, I)$$

Note that the firms' profits or "preferences" are not taken into account; only the welfare of the consumers matters. But of course the production technology does play a role in determining whether an allocation and a proposed Pareto improvement are feasible.