Economics 201b
Spring 2009
Problem Set 1
Due Thursday April 2

1. Consider a two-person, two-good exchange economy. For each case below, assume the initial endowments are $\omega_a = (1, 3), \omega_b = (3, 1)$, draw the Edgeworth box for the economy, and indicate (no need to derive an analytical solution) the Pareto optimal allocations and the contract curve:

   (a) $U_a(x_{1a}, x_{2a}) = x_{1a}x_{2a}; \quad U_b(x_{1b}, x_{2b}) = x_{1b}x_{2b}$.

   (b) $U_a(x_{1a}, x_{2a}) = \min(x_{1a}, x_{2a}); \quad U_b(x_{1b}, x_{2b}) = \min(x_{1b}, x_{2b})$.

   (c) $U_a(x_{1a}, x_{2a}) = \max(4x_{1a}, 3x_{2a}); \quad U_b(x_{1b}, x_{2b}) = \max(4x_{1b}, 3x_{2b})$.

   (d) $U_a(x_{1a}, x_{2a}) = 2x_{1a} + x_{2a}; \quad U_b(x_{1b}, x_{2b}) = x_{1b} + 2x_{2b}$.

2. Consider a two-person, two-good exchange economy where the agents' utility functions are $U_a(x_{1a}, x_{2a}) = x_{1a}^3 x_{2a}$ and $U_b(x_{1b}, x_{2b}) = x_{1b}x_{2b}^3$, and the initial endowments are $\omega_a = (2, 2)$ and $\omega_b = (2, 2)$.

   (a) Draw the Edgeworth box for this economy. Find an analytical expression for the Pareto optimal allocations, and indicate the Pareto optimal allocations and the contract curve in the Edgeworth box.

   (b) Find the individual and market excess demand functions. Find the competitive equilibrium prices and allocations.

3. (**Kinked Preference**) Some recent models in decision theory are non-differentiable in nature. For example, popular models incorporating loss aversion in prospect theory, or ambiguity aversion as illustrated by Ellsberg Paradox, have kinked indifference curves. In this exercise we are going to take a reduced form of these preferences, and examine the implications of "kinkiness" on equilibrium prices and allocations, in our simplest 2×2 exchange economy.

   Consider an exchange economy with two goods and two consumers in which consumers' utility functions are $\forall i \in \{a, b\}$,

$$U(x_{1i}, x_{2i}) = \frac{2}{3}log(x_{1i}) + \frac{1}{3}log(x_{2i}) \qquad if \quad x_{1i} \leq x_{2i}$$
$$\frac{1}{3}log(x_{1i}) + \frac{2}{3}log(x_{2i}) \qquad if \quad x_{1i} > x_{2i},$$

   and

   (a) Suppose the initial endowments are $\omega_a = (3, 3), \omega_b = (3, 3)$. Draw the Edgeworth box for this economy. Find the Pareto optimal allocations. Verify that the initial endowment is an equilibrium allocation. Find the supporting equilibrium price(s).

(b) Now suppose the endowment is instead $\omega'_a = (4, 2), \omega'_b = (2, 4)$. Find the individual and market excess demand functions (notice that the utility function is not differentiable). Find the competitive equilibrium prices and allocations. Is there a unique equilibrium?

(c) Now suppose the endowments are instead $\omega''_a = (5, 1), \omega''_b = (2, 4)$. Repeat the exercise in (3b). (Note that the Edgeworth box is different in this case). Comment on how the kinkiness of preference affect the size of competitive equilibria.

4. (**Non-convex preference; more than 2 agents**)
Consider an exchange economy with two goods and m identical consumers, each with utility function $U(x_1, x_2) = \max(x_1, x_2)$, and initial endowment $w = (1, 1)$.

(a) What is the individual excess demand function for each consumer? Notice that the objective function is not quasi-concave.

(b) When $m = 2$, what are the competitive equilibria?

(c) When $m > 2$, what are the competitive equilibria?

(d) If $m = 2$, but the endowments are $w_a = (\frac{1}{2}, \frac{1}{2})$ and $w_b = (\frac{3}{2}, \frac{3}{2})$, what are the competitive equilibria?