Economics 201b
Spring 2010
Problem Set 1
Due Thursday March 18

A word about notation: throughout this course, we'll often be dealing with economies with multiple agents and multiple goods, which can make interpreting subscripts and superscripts somewhat difficult if we're not all using the same system. Thus, when describing an allocation of goods, please use notation such as x_{21} to indicate, for example, agent 1's allocation of good 2. That is, let the first subscript identify the good, and the second subscript identify the agent. Use notation such as x_2 to indicate, for example, agent 2's consumption set. Variables like p without a subscript will usually denote a price vector for the goods of the economy.

1. Consider a two-person, two-good exchange economy in which the utility functions are $U_i(x_{1i}, x_{2i}) = x_{1i}x_{2i}$ for $i = 1, 2$, and the initial endowments are $\omega_1 = (1, 3)$ for agent 1 and $\omega_2 = (3, 1)$ for agent 2.

   (a) Draw the Edgeworth box for this economy, indicating the set of Pareto optimal allocations. Also indicate those allocations which are both Pareto optimal and individually rational for both agents (contract curve).

   (b) Find an analytic expression for the set of Pareto optimal allocations and contract curve.

   (c) Show that any price vector $p = (p_1, p_2)$ where $p_1 = p_2$ along with the allocation (x_1, x_2) where $x_1 = x_2 = (2, 2)$ is an equilibrium.

   (d) Are there any other equilibria? (First find individual demand $D_i(p)$ for $i = 1, 2$ and market excess demand $E(p)$.)

   (e) Suppose we kept everything the same except changed the utility functions from $x_{1i}x_{2i}$ to $\min\{x_{1i}, x_{2i}\}$. What is the set of equilibria now? (Notice an agent with this utility function has preferences that are *not* strictly convex nor strongly monotone. However, preferences for many things that come in pairs essentially satisfy this relation.)

   (f) Suppose in addition we also changed ω_2 from $(3, 1)$ to $(4, 1)$ - notice how the set of equilibria changes compared to (e). What is the equilibrium price now? What are the equilibrium allocations? (Also draw a picture.)

2. Again, consider a two-person, two-good exchange economy with utility functions $U_1(x_{11}, x_{21}) = x_{11}x_{21}$, $U_2(x_{12}, x_{22}) = \min\{x_{12}x_{22}, 4\}$, and initial endowments $\omega_1 = (1, 4)$, $\omega_2 = (4, 1)$.

   (a) Indicate those allocations which are both Pareto optimal and individually rational for both agents.

   (b) Characterize all equilibria. Are any of these equilibrium allocations Pareto optimal? (Also draw a picture.)

3. Consider a two-person, two-good exchange economy with utility functions $U_1(x_{11}, x_{21}) = x_{11}$, $U_2(x_{12}, x_{22}) = x_{22} + \log x_{12}$, and endowments $\omega_1 = (2, 0)$ and $\omega_2 = (0, 2)$. Since agent 2's utility involves log, when the price of the second good is 0, agent 2's utility is undefined for all consumption bundles in her budget set. Thus for the purposes of this problem, we will assume that the price p_2 of the second good is always positive.

   (a) What are the offer curves OC_1 and OC_2? (Also draw a picture.)

   (b) Does there exist an equilibrium?

4. Consider a three-person, two-good exchange economy where $U_i(x_{1i}, x_{2i}) = x_{1i}x_{2i}$ for $i = 1, 2, 3$, and $\omega_1 = (6, 1)$, $\omega_2 = (1, 6)$, $\omega_3 = (1, 1)$. Now consider the exact allocation $x_1 = (3, 3)$, $x_2 = (3, 3)$, $x_3 = (2, 2)$.

   (a) Show that this allocation is Pareto optimal.

   (b) Notice that each agent i *strictly* prefers the consumption set x_i to their endowment ω_i. Despite this, give an argument (not involving prices or equilibrium) as to why it might be unlikely for our exchange economy to end up with the exact allocation (x_1, x_2, x_3).