Economics 201b
Spring 2009
Problem Set 3
Due Thursday April 14

1. Consider a production economy with a finite set of firms $j = 1, \ldots, J$. Define the aggregate production set Y to be $Y = \{y = \sum_{j=1}^{J} y_j | \quad y_j \in Y_j, \forall j = 1, \ldots J\}$. As in the Arrow-Debreu economy defined in lecture 4, we assume each Y_j is a closed and non-empty subset of R^L. We say Y satisfies *free entry* if $Y + Y \subset Y$. A production set Y_j satisfies *constant returns to scales* (CRS) if $y \in Y_j \Rightarrow \alpha y \in Y_j$, for all $\alpha > 0$.

   (a) Prove or give a counter example to the claim: if each firm's production set Y_j satisfies CRS, then Y satisfies free entry. If you are giving a counter example, can you add additional assumption(s) on Y_j to make it true? Prove your modified claim if this is the case.

   (b) Again, prove or give a counter example to the claim: free entry imply CRS.

   (c) Show that if Y satisfies free entry and $0 \in Y_j, \forall j$, then in any competitive equilibrium all firms must make zero profits.

2. Consider the 2×2 economy discussed in section 3. There are two production factors: labor (L) and capital (K), with aggregate endowment $(\bar{L}, \bar{K})$, and there are two commodities: E and G, with zero endowment. There are two firms, the first firm (also called E) produces only commodity E with production function $E = F_E(L_E, K_E) = L_E^{\frac{1}{3}} K_E^{\frac{2}{3}}$, and the second firm (also called G) produces only commodity G, with production function $G = F_G(L_G, K_G) = L_G^{\frac{1}{3}} K_G^{\frac{2}{3}}$. There are two agents a and b in the economy, each has utility function $U_a(E_a, G_a) = E_a^{\frac{1}{3}} G_a^{\frac{2}{3}}$, and $U_b(E_b, G_b) = E_b^{\frac{1}{3}} G_b^{\frac{2}{3}}$ (agents derive utility only from output goods but not production factors). Agent a is endowed with ownership $\theta_{aE} = \frac{3}{4}, \theta_{aG} = \frac{3}{4}$, and $\omega_{aK} = \frac{1}{5}\bar{K}, \omega_{aL} = \frac{1}{2}\bar{L}$.

   (a) Finish what was discussed in the section, find the set of competitive equilibria in this economy.

   (b) Find the set of Pareto optimal allocations in this economy (derive it analytically by solving the problem (P)).

   (c) Check if the First Welfare Theorem holds in this economy by comparing your results in (a) and (b). Are the assumptions of the Second Welfare Theorem satisfied? If not, does it matter?

   (d) Without redoing the calculations, find the competitive equilibria when the shareholdings are reversed, and justify your finding.

3. **Non-convex technology**: Consider the Robinson Crusoe economy we discussed in class. Now we have convex production function.

$$f(z) = z^2$$
$$u(x_1, x_2) = x_1 x_2$$
$$x_1 = \bar{L} - z; \quad \bar{L} = 1$$

   (a) Find the set of Pareto Optima (Hint: draw a picture).

   (b) Find the set of Walrasian equilibria.

   (c) Does the first welfare theorem hold? How about the second welfare theorem? Explain with a picture.

4. The Minkowski's Separating Hyperplane Theorem provides weak separation between a convex set and a point outside the set. In general, when the convex set is also closed, we can get a stronger separation result: for X a non-empty, closed and convex subset of R^L, and a point $z \notin X$, there exists a vector $p \in R^L/\{0\}$, such that *p strictly separates* z and X, i.e. , $p \cdot z < \alpha = \inf p \cdot X$, for some $\alpha \in R$. This is Theorem 1.24 in de la Fuente (p241).

   (a) A *half space* is a set of the form $\{x \in R^L : p \cdot x \geq c\}$ for some $p \in R^L, p \neq 0$. Apply the strict separation theorem above to show that any closed, convex $B \subset R^L$ equals the intersection of all the half-spaces that contain it.

   (b) Show that if set B is closed but not convex, then there exists some $x \notin B$ that cannot be strictly separated from B.

5. In this question you are required to fill in some easy steps in the proof of the Second Welfare Theorem, which we left as an exercise.

   (a) If set B_i is convex $\forall i$, then $B = \sum_i B_i$ is also convex.

   (b) If there exists $p^* \neq 0$ such that $\inf p^* \cdot B \geq 0$, then $\inf p^* \cdot B = \sum_{i=1}^{I} \inf p^* \cdot B_i$.