

Economics 201b  
Spring 2009  
Problem Set 4  
Due: Thursday April 23

**Exercise 1** Recall the “Robinson Crusoe” economy, discussed in the lecture with one consumer and one firm. Consumer’s utility function is quasi-concave, continuous and monotone. Production function is convex. Answer briefly following questions (one or two sentences should be enough, graphs are encouraged):

- a) Can there be Walrasian equilibrium in this economy with non-zero production?
- b) Will your answer to the question above change if there are fixed costs involved? What if production function exhibits increasing returns to scale?

**Exercise 2** This problem asks you to think about some properties of correspondences. Let  $X \subseteq \mathbb{R}^n$  and  $Y \subseteq \mathbb{R}^m$ .

- a) Illustrate graphically an example of a correspondence  $\Psi : X \rightarrow 2^Y$  such that  $\Psi$  is closed valued but not uhc and, therefore, does not have a closed graph.
- b) Now, suppose  $\Psi$  does not have a closed graph. Illustrate graphically an example of a correspondence  $\Psi : X \rightarrow 2^Y$  such that  $\Psi$  is not uhc despite the fact that  $Y$  is compact.

**Exercise 3** Recall the conditions for Kakutani’s fixed point theorem. Let  $A \subset \mathbb{R}^L$  be nonempty, compact, and convex. Then  $f : A \rightarrow 2^A$  has a fixed point if  $f$  is nonempty-valued, convex-valued, closed-valued and uhc. Illustrate graphically how we can fail to have a fixed point if the conditions are violated as follows (in each case, allow the other conditions to still hold):

- a)  $A$  is not bounded (and therefore not compact)
- b)  $A$  is not closed (and therefore not compact)
- c)  $f$  is not convex-valued
- d)  $f$  is not closed-valued
- e)  $f$  is not uhc

**Exercise 4** Consider a two good economy, and illustrate graphically four examples of functions  $z : \Delta^o \rightarrow \mathbb{R}^2$  which demonstrate that if any one of the conditions:

- a) continuity,

- b) Walras' Law,
- c) boundedness below ( $\exists x \in \mathbb{R}$  s.th.  $z(p) \geq x \forall p \in \Delta^\circ$ ), or
- d) the boundary condition (if  $p^n \rightarrow p \in \Delta \setminus \Delta^\circ$ , then  $|z_l(p^n)| \rightarrow \infty$ ) fails, then there may not be a solution to  $z(p) = 0$ .

That is, each function you draw should violate only one of the four conditions, and have the property that  $\exists p$  s.th.  $z(p) = 0$ .

**Exercise 5** In this problem, we will investigate what happens to the welfare and existence of equilibria in the presence of taxes. So consider an exchange economy with  $I$  consumers and  $L$  goods, in which each consumer has a utility function  $U_i : \mathbf{R}_+^L \rightarrow \mathbf{R}$  that is continuous, strictly quasi-concave and strongly monotone, and an initial endowment  $\omega_i$  which contains a positive amount of each good.

The government exogenously imposes a system of taxes such that consumption of each good is taxed at a rate that depends both on the consumer and the good being purchased. That is, if consumer  $i$  buys a quantity  $x_{li}$  of good  $l$ , then the consumer will pay a tax of  $t_{li}p_l x_{li}$ . The tax rates  $\{t_{ij}\}$  are set exogenously in this model by a tax authority and they are strictly positive. Each consumer also receives a lump-sum rebate  $R_i$  from the tax authority and in equilibrium, the total tax receipts must be rebated equally across the consumers in a lump-sum fashion.

- a) Write down the budget set for a typical consumer in this economy.

As you can see in this model with taxes, a competitive equilibrium is therefore a price vector  $p^*$ , an allocation  $x^* = (x_1^*, \dots, x_I^*)$  and rebates  $R = (R_1, \dots, R_I)$  such that all markets clear,  $x_i^*$  is a utility maximizing choice over budget set for each consumer  $i = 1, \dots, I$  with rebates  $R_i = \frac{1}{I} \sum_{i,j} t_{i,j} p_i^* x_{i,j}^*$ .

- b) For  $I = L = 2$ , illustrate graphically the concept of a competitive equilibrium with taxes in an Edgeworth Box. Show graphically that the First Welfare Theorem need not hold in this economy.
- c) Show that a competitive equilibrium with taxes exists.

*Hint: What is an equilibrium? What properties do the consumers' demands in this economy satisfy? Is there any powerful result that you can invoke to support your claim?*