Economics 201b
Spring 2009
Problem Set 5
Due Thursday April 30

1. Suppose there are $I$ consumers in an exchange economy with $L$ goods. Each consumer has a utility function parameterized by an exogenous parameter $r_i \in [0,1]$. For example, the parameter could indicate an elasticity of substitution. Thus, for each consumer $i$, $U_i : \mathbb{R}_+^L \times [0,1] \to \mathbb{R}$. Define $r \in [0,1]^I$ to be the vector of parameters for all consumers. In addition, each consumer has an initial endowment vector $\omega_i > 0$ (i.e. $\omega_{li} \geq 0\ \forall l$ and $\omega_{ki} > 0$ for some good $k$).

   Let each consumer's demand be a single-valued, continuous function on $(p, r_i) \in \Delta \times [0,1]$ (and as an aside, note that by doing so we are assuming that preferences are not strongly monotonic, else demand would not be defined for $p \in \Delta \backslash \Delta^o$. For example, it may be that consumers can achieve satiation in each good. Also note that, since existence of competitive equilibrium is not guaranteed, the correspondence may be empty-valued.) Let $\Phi(r)$ denote the exchange economy corresponding to the parameter vector $r \in [0,1]^I$.

   (a) Show that the correspondence

   $$E(r) = \{p \in \Delta : p \text{ is a competitive equilibrium price in the economy } \Phi(r)\}$$

   is upper hemi-continuous. (Recall that if $X \subset \mathbb{R}^M, Y \subset \mathbb{R}^N$ and $Y$ is compact, then the correspondence $f : X \to 2^Y$ is uhc if it has a closed graph.)

   (b) Suppose that for each $r \in [0,1)^I$ there is a competitive equilibrium in the economy $\Phi(r)$. Show that there is a competitive equilibrium when $r = (1, 1, ..., 1)$.

2. Recall the two-agent, two-good exchange economy of Problem Set #1, question 2. ($U_a(x_{1a}, x_{2a}) = x_{1a}^3 x_{2a}$ and $U_b(x_{1b}, x_{2b}) = x_{1b} x_{2b}^3$, and the initial endowments are $w_a = w_b = (2,2)$). Take any results shown in PS#1 as given, and normalize by $p_2 = 1$.

   (a) Show that this economy is regular.

   (b) Show that the index theorem holds in this economy. That is, having proven that the economy is regular, show that the sum of the index numbers of the competive equilibria of the economy is equal to one.

   (c) Suppose that the social planner is considering transferring some of the initial endowment of good 1 from agent $a$ to agent $b$. Denote the amount of the transfer by $t$. The planner is interested in how the imposition of a small transfer will affect $p_1^*$ in a competitive equilibrium. What is $\frac{dp_1^*}{dt}$ at the competitive equilibrium of this economy? That is, when the transfer is imposed, how will the equilibrium price of good 1 change?

(d) Now suppose the social planner is able to "create" a small additional amount of good 1; that is, $\bar{\omega}_1$ increases. The planner distributes this extra amount equally between the two agents. What is $\frac{dp_1^*}{d\bar{\omega}_1}$ at the competitive equilibrium of this economy?

3. Suppose we have exchange economy with I=2, L=2. We have $\omega_1 = (2, 1)$, $\omega_2 = (1, 2)$. Suppose

$$u(x_1) = (3x_{1,1}^{\gamma} + x_{2,1}^{\gamma})^{\frac{1}{\gamma}}, \qquad u(x_2) = (x_{1,2}^{\gamma} + 3x_{2,2}^{\gamma})^{\frac{1}{\gamma}}$$

Here $\gamma \in R$, $\gamma \neq 0$, and $\gamma < 1$. Normalize $p_2 = 1$.

(a) Calculate $\hat{z}(\hat{p})$. (Note here $\hat{p} = p_1$, $\hat{z}(\hat{p}) = z(p_1, 1)_1$).

(b) Verify directly that $\hat{p} = 1$ is an equilibrium price. Compute the index of this equilibrium price for $\gamma = -4$. If you get $index = -1$, what conclusions can you draw about the number of equilibria in this economy?

4. Consider an exchange economy with $L$ goods and $m$ consumers. Each consumer has a separable utility function of the form $U_i(x) = \sum_{\ell} v_i(x_{\ell})$ for a function $v_i : \mathbf{R}_{++} \to \mathbf{R}$ that is strictly increasing, strictly concave, differentiable, and satisfies $\lim_{c \to 0} v_i'(c) = \infty$. Each consumer has a strictly positive endowment $\omega_i \gg 0$.

(a) Suppose that the aggregate endowment of each good is constant, so that

$$\sum_i \omega_{i1} = \sum_i \omega_{i2} = \cdots = \sum_i \omega_{iL}$$

Show that the economy has at most one equilibrium.

(b) Suppose that

$$\sum_i \omega_{i1} > \sum_i \omega_{i2} > \cdots > \sum_i \omega_{iL}$$

Show that any competitive equilibrium price vector $p^*$ must satisfy $p_1^* < p_2^* < \cdots < p_L^*$.

(c) Suppose that $v_i(c) = \ln(c)$ for each $i$. Fix the aggregate endowment $\omega \in \mathbf{R}_{++}^L$. Show that the set of competitive equilibrium prices does not depend on the initial endowment allocation. That is, if $p^*$ is a competitive equilibrium price vector in this economy with initial endowment allocation $(\omega_1, \ldots, \omega_m)$, then $p^*$ is also a competitive equilibrium price vector for any other initial endowment allocation $(\omega_1', \ldots, \omega_m')$ such that $\sum_i \omega_i' = \omega$.