Economics 201b
Spring 2009
Problem Set 7
Due: Thursday May 14, in Oleksa's mailbox on the 6-th floor of Evans

Exercise 1 *Consider a two consumer exchange economy with the following identical preferences and endowments:*

$$u(x_1, x_2) = \max\{\min\{3x_1, x_2\}, \min\{2x_1, 3x_2\}\}, \quad (\omega_1, \omega_2) = (\frac{7}{3}, \frac{7}{3}).$$

a) *Draw the indifference curves for one of the consumer. Are this consumer's preferences convex?*

b) *Draw the Edgeworth box for this economy, denoting Pareto set, individually rational and core allocations.*

c) *Now suppose we have an economy of $I \in \mathbb{N}$ identical consumers with $I \geq 2$, each of which has the same preferences is as the consumers described above and endowments are $(\omega_1, \omega_2) = (1 + 2\alpha, 3 - \alpha)$, where $\alpha \in (0, 1)$. Find a necessary and sufficient condition for α that must be satisfied for there to exist a Walrasian equilibrium of this economy. Show that as I increases, the set of $a \in (0, 1)$ that satisfy the condition you found increases in size.*

d) *Explain in a few sentences how these results relate to Theorem 5 in the (revised) Lecture Notes 12. That is, relate your above results to the fact that, in this economy, we can show that $\forall a \in (0, 1)$, $\exists p^* >> 0$ and $x_i^* \in D_i(p^*)$ such that*

$$\frac{1}{I} \sum_l \max\{(\sum_i x_i^* - \sum_i \omega_i)_l, 0\} \leq 2\sqrt{\frac{L}{I}} \max\{||\omega_i||_1 : i = 1, ..., I\}$$

Compute an explicit bound for $\frac{1}{I} \sum_l \max\{(\sum_i x_i^* - \sum_i \omega_i)_l, 0\}$ for the economy in question.

Exercise 2 *Consider an exchange economy with 4 consumers and L goods. Each consumer has a continuous, strongly monotone, strictly quasiconcave utility function. Consumers are divided into 2 types, a and b, and each consumer of type t has the same utility function U_t and the same endowment ω_t for types $t = a, b$. There are two consumers of each type.*

a) *Show that in any competitive equilibrium allocation, all consumers of the same type consume the same bundle.*

b) *Show that in any core allocation consumers of the same type must be indifferent, i.e., if $(x_a^1, x_a^2, x_b^1, x_b^2)$ is a core allocation, then $x_a^1 \sim_a x_a^2$ and $x_b^1 \sim_b x_b^2$ (where x_t^n denotes the consumption vector of the n-th consumer of type t).*

Exercise 3 *In this exercise, we will look at (slightly more complex) core allocations in economy with production. There are three consumers and three goods. Consumer i is initially endowed with one unit of good i. Each consumer's utility function is $U(x_1, x_2, x_3) = (x_1 x_2 x_3)^{\frac{1}{3}}$. Each consumer has access to the following constant returns to scale technologies: consumer is able to convert two units of good 1 to one unit of good 2, two units of good 2 to one unit of good 3, and two units of good 3 to one unit of good 1.*

a) *Consider the following production and consumption plans:*

   i. *a half of good 1 is converted to good 2 and then every good is distributed equally among consumers;*

   ii. *there is no production; $\frac{3}{5}$ units of every good is given to consumer 3 and the rest of every good is distributed equally between consumers 1 and 2.*

   For each of these two plans find a blocking coalition (and the way to block) or prove that the plan belongs to the core.

b) *Describe the core of this economy in a most explicit way.*

Exercise 4 *A very old legend tells us that one Indian rajah had three sons. Or four? Or five? Actually since the legend is very old, we are not very sure how many sons rajah had, but it definitely was a finite number. So, without any loss of generality lets say that rajah had N sons, with $N \in \mathbb{N}$. Before that great rajah died he told his sons that all his wealth equal to 1 he bequested to one of them.*

Waiting for last will of rajah to be announced, princes gathered in the great hall of a palace to decide how would they divide the bequest. Every price believed that the older sons were more likely to receive raja's wealth. Moreover, all believed that i-th oldest son would receive the wealth with probability $\frac{2(N-i+1)}{N(N+1)}$. Each price had Bernoulli's utility functions $U(w) = \ln w$.

a) *Compute Arrow-Debreu equilibrium.*

After long discussion, princes decided to trade securities. Each security of type k entitles the owner to $\frac{1}{N}-th$ share of the total bequest, if the it was received by one of the $k-th$ oldest prices. Princes agreed to trade securities of types $(1, 2, ..., K)$.

b) *Does the Arrow-Debreu equilibrium that you computed in (a) constitute Radner's equilibrium for an arbitrary K?*

Exercise 5 *In the following exercise you will asked to price by arbitrage a variety of assets. There are two dates. At date* 1 *there are three states; at date* 0 *there is trade in assets. There are two basic assets whose return vectors in current dollars are* $r_1 = (8, 4, 3)$ *and* $r_2 = (0, 0, 1)$*. The market prices of these assets are* $q_1 = 16$ *and* $q_2 = 1$*, respectively.*

a) *Suppose that one unit of a derived asset is described as "One unit of this asset confers the right to buy one unit of asset* 1 *at* 25% *of its spot value in period* 1 *(after the state of the world occurs). Write the return vector of this asset and price it.*

b) *The situation is the same as in* (a) *except that the asset is modified to read "One unit of this asset confers the right to buy one unit of asset* 1 *at* 25% *of its spot value in period* 1 *(after the state of the world occurs) provided the spot value is at least* 4*."*

c) *Suppose that the asset is as in* (b) *except that "at least* 4*" is replaced by "at least* 7*." Write down the return vector and argue that this asset cannot be priced by arbitrage with the available primary assets.*

d) *How would the analysis in* (c) *differ if you had in addition a riskless asset with a price equal to* 1*? (You do not need to compute the price explicitly.)*