Problem Set #1
(due 2/20/01)

1. Consider an individual who wishes to invest initial wealth, W, to maximize the utility of terminal wealth one period hence. The investor's problem consists of two portfolio choice decisions; first, how much of this wealth to place in bonds, which yield a certain return equal to i, and how much to invest in stocks, which yield a stochastic return with mean $r > i$ and variance s^2; second, how to distribute these assets between a taxable account and a tax-deferred account. Interest on bonds held in the taxable account (TA) is taxed at rate t, while returns to equity are taxed at rate at, where $0 < a < 1$. Assets placed in the tax-deferred account (TDA) receive an immediate deduction at rate t, with all returns (including the initial investment) taxed at rate t when withdrawn in the next period. An amount up to $V < W$ may be placed initially in the tax deferred account.

    A. Suppose that the individual is risk-neutral. Derive the optimal portfolio, in terms of the amounts of debt and equity held in each account. Now derive the optimal portfolio for an individual who is infinitely averse to risk.

    B. Show that, regardless of the individual's risk aversion, it will never be optimal to hold stocks in the TDA and bonds in the TA at the same time. (Hint: prove by contradiction that such a portfolio allocation would permit the investor to achieve a higher mean portfolio return and the same portfolio variance.)

    C. Now, suppose that the tax treatment of assets in the TDA were changed, so that investors received no deduction initially and paid no tax on subsequent withdrawals, but were still permitted to place up to an amount V in the TDA. How would this change alter optimal portfolios?

2. Suppose that a risk-neutral investor faces a tax rate of c on long-term capital gains and a tax rate of $t > c$ on long-term capital losses. The investor has an asset purchased a long time ago for P_0 that is now worth P_1, and must decide whether to sell the asset now and reinvest the proceeds for one more period, or to continue holding the asset for one more period. The period is long enough so that even the new investment under the first option would qualify for capital-gains treatment. The rate of return for the remaining period will be the same in either event – stochastic with mean r. The return r can be negative, but always exceeds the ratio $(P_0 - P_1)/ P_1$.

    A. Derive a necessary and sufficient condition for the investor to realize the capital gain now, expressed in terms of some critical value of the ratio $R = P_1/P_0$, say R^*.

    B. How is the value of R^* derived in part A. affected by an increase in t? in c? Explain each of these effects. (Hint: for c, consider the case in which t and c are initially equal.)

C. Now, suppose that the capital gains tax rate is different in the two periods. How does this change your answer to part A?

D. Suppose that the capital gains tax rates initially are the same in the two periods, and that the government lowers the rate c only in the first period. Based on the expression derived in part C., what impact does this have on R^*? Using your result, what can you say about the impact on the level of realizations in the current period? on the present value of government revenue?

3. The U.S. Tax Reform Act of 1986 (TRA86) lowered marginal tax rates on corporate and ordinary individual income, but raised tax rates on capital gains. Also, the reduction in marginal rates was greatest among individuals in the highest marginal tax brackets. These changes led to conflicting predictions regarding changes in corporate debt-equity ratios.

For both parts of this question, assume that TRA86 reduced the corporate tax rate from .46 to .34, that corporate interest payments were fully deductible at the corporate tax rate, and that individual interest income both before and after the reform was taxed as ordinary income. Assume that the applicable individual tax rate on all income from equity is the effective capital gains tax rate. Before the reform, each individual's effective capital gains tax rate was lower than that individual's ordinary tax rate for two reasons. First, the ability to defer gains (and avoid tax entirely at death) reduced the effective tax rate to .25 times the ordinary tax rate; second, only 40 percent of realized gains were actually subject to tax, the other 60 percent being excluded from taxation. The reform maintained the benefit of deferral but eliminated the 60 percent exclusion. Assume that investors require equal after-tax returns on debt and equity to hold both assets, and can hold only non-negative quantities of either asset.

A. First suppose that all individuals are identical, and that TRA86 reduced the representative individual's ordinary tax rate from .3 to .2. The representative firm chooses its debt-value ratio to minimize its cost of capital which, in addition to the usual expression based on the costs of debt and equity, includes a tax-deductible transaction cost term that is proportional to the square of the debt-value ratio b and equal to zero at $b=0$. If the interest rate does not change with the reform, what happens to the debt-value ratio?

B. Now assume that there are no transaction costs associated with borrowing, and that individual tax rates differ. Those individuals with a tax preference for debt hold only debt, and those with a tax preference for equity hold only equity. The amount of equity in the corporate sector equals the wealth of individuals with a tax preference for equity. TRA86 lowered the top individual tax rate from .5 to .33. What should have happened to the corporate debt-value ratio?