

THE RELATIONSHIP BETWEEN FIRM SIZE AND FIRM GROWTH IN THE US MANUFACTURING SECTOR*

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Using panel data on the publicly traded firms in the US manufacturing sector in the recent past, I find that most of the change in employment at the firm level in any given year is permanent, that year-to-year growth rates are largely uncorrelated over time or with prior characteristics of the firm, and that there is almost no measurement error. Gibrat's Law is weakly rejected for the smaller firms in my sample and accepted for the larger firms. This finding remains when I control for the effect of selection (attrition) on estimates obtained from this sample.

I. INTRODUCTION

THE PRESENT paper is a first step in an investigation of the dynamics of firm growth in the US manufacturing sector during the recent past.¹ It updates work by earlier researchers on the relationship between firm size and growth using a more comprehensive dataset and modern econometric techniques to attempt to correct for some of the problems in estimating such a relationship. The question addressed here is "Do small to medium-sized publicly traded manufacturing firms grow faster than large ones?" If they do, is it because of the way they are selected into our sample, or because of a difference in the rate and direction of innovative activity, or simply because the economy is finite and diminishing returns set in eventually? I do not claim to be able to distinguish clearly among all these alternatives, or even that only one must be true, but I do explore the implications of each for the data.

Stochastic models of firm growth have been subjected to two kinds of empirical tests: the first posits a growth model that is stationary over time and then looks at the implications of this model for the equilibrium size distribution of firms. Various authors, beginning with Gibrat, have shown that the simplest version of a growth model, in which growth rates are

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independent of size, generates a log normal size distribution, albeit with an increasing variance over time. Mandelbrot [1963] provides a survey of this and other models, in which he shows the conditions under which the equilibrium size distribution is a stable Pareto distribution. Boundary conditions on exit and entry are required in order to achieve a stable distribution in most cases. Size distributions have been investigated empirically by Simon and coworkers (e.g., Ijiri and Simon [1977], Simon and Bonini [1958]), Quandt [1966], and Hart and Prais [1956]. Typically, the size distribution conforms fairly well to log normal, with possibly some skewness to the right. The power of this kind of test is low, since the relationship of growth rates to size is not explicitly investigated. However, several of the existing theories, such as those of Lucas [1978] and the stochastic theory of Simon and Bonini have as their main implication these static distributions.

The other approach to empirical work in this area is to investigate the relationship of growth rates and size in a panel of firms. This work is exemplified by Hymer and Pashigian [1962] and Mansfield in the sixties, and the more recent work by Birch, Armington and Odle [1982], and Evans [1985] using the Dun and Bradstreet files (as cleaned by Brookings Institution for the Small Business Administration) and by Evans using Fortune 500 firms. Except for Evans, none of these researchers attempted to correct econometrically for biases induced by sample selection and measurement error. One of the purposes of this paper is to investigate whether such biases have an appreciable effect on the results.

The first econometric problem is the phenomenon of regression to the mean: If the dependent variable in question is the growth rate, measured as size in a final period less size in an initial period, and the independent variable is size in the initial period, measured with error, then firms that have transitorily low size due to measurement error will on average seem to grow faster than those with transitorily high size. With yearly observations from a panel of firms it is possible to control for this kind of random measurement error with instrumental variable techniques (see Griliches and Hausman [1985]).

The second problem is somewhat more serious: Measuring growth with a panel of firms requires that data on size will be available for every firm in both the beginning and the end period. But small firms that have slow or negative growth are more likely to disappear from the sample than are large firms, yielding the well-known problem of sample selection bias. In addition, some of the most rapidly growing and successful small firms may not be present at the beginning of the period, which will produce biases in the other direction. In section IV of the paper I present estimates of a model that attempts to control for sample selection.

The plan of the paper is as follows: First I describe the data and present preliminary results on the role of measurement error in the size-growth

relationship. This is followed by an exploration of the time series behavior of employment growth, with the issue of selection bias set aside temporarily. Then I develop an econometric model of sample attrition and discuss the problems that arise in estimating such a model in the presence of heteroskedasticity and the absence of adequate information to identify separately the probability of firm survival. Finally, I investigate the relationship of investment, both in physical capital and research and development, to firm growth, using the sample selection model to control for attrition. The main conclusion of the paper is that the previously observed negative relationship between size and growth for smaller firms is robust to corrections for selection bias and heteroskedasticity, although this conclusion is clouded by the difficulties of separating nonlinearity from selection bias in the presence of size-related heteroskedasticity.

II. DESCRIPTION OF THE DATA

In this paper I confine my analysis to data on publicly traded manufacturing firms, drawn from the Compustat files. These data cover approximately ninety percent of the employment in the manufacturing sector in 1976, although they account for only about one percent of the firms in this sector.² Thus the study is really about the relationship of growth and size across firms that have already reached a certain minimum size, large enough to require outside capitalization. We would argue that these are the firms of interest, since their relative sizes are the main determinants of concentration in most markets. (This argument applies mainly to the manufacturing sector, where there are almost no privately held firms of any size.)

The universe from which I draw my sample consists of 1778 firms in the manufacturing sector in 1976³ (see Bound *et al.* [1984] for further description of these data). I considered two different panels selected from this universe: all the firms with employment data from 1972 through 1979, and all the firms with employment data from 1976 to 1983. The first set maximizes the sample size, since the basic universe of firms is as of 1976, while the second has the advantage that it begins in the year in which the sample was chosen and hence suffers from selection bias in only one direction. There are 1349 firms in

² The total number of employees in our 1976 cross-section is 16.7 million, possibly including some foreign employees. The total domestic manufacturing employment reported by the Bureau of Labor Statistics for the same year is 19 million. The number of enterprises in the Census of Manufacturers in 1977 is approximately 300,000.

³ An earlier draft of this paper was based on a sample of 2577 firms, which included all the firms on the Compustat full coverage file that were in the manufacturing sector. Further investigation of the full coverage sample has revealed that it is unsuitable for a study of growth and sample selection due to exit: many of the firms on the file are not really publicly traded and most do not have valuation information. In addition, many of them file for a year or two after a public offering, and then suspend filing because there are less than 300 shareholders on record. This is not really an exit, but we no longer can obtain data on the firm.

the first sample and 1098 in the second; 962 firms are in both samples. The remainder of the firms either entered or exited the sample during the period. A few firms participate in a merger of equals and become in effect a new firm; these observations are treated as exits (and the new entity as an entry).⁴

The overall growth rate of employment in this sample was about 2.9 percent in the 1972–79 period and 0.8 percent in the 1976–83 period. There are substantial differences across the industries, with the so-called “high tech” industries (drugs, computing equipment, communication equipment, and scientific instruments) typically growing more rapidly throughout both periods. (Details are available from the author on request.)

We first consider the possible role of measurement error in biasing a regression of changes on levels. A simple model of Markov growth with errors in variables would look like

$$(1) \quad y_t = X_t + w_t$$

$$(2) \quad X_t = X_{t-1} + u_t$$

with w and u uncorrelated white noise errors, X unobserved (“true” size) and y the observed logarithm of employment. Under this model, the true relationship between change in size and its level is

$$(3) \quad E(\Delta X_t | X_{t-1}) = 0$$

but the estimated relationship will be

$$(4) \quad E(\Delta y_t | y_{t-1}) = -(\sigma_w^2 / (\sigma_X^2 + \sigma_w^2)) y_{t-1}$$

where σ_w^2 and σ_X^2 are variances of the error term and the unobservable size respectively. For these data, the ratio of the variance of growth rates (within a firm) to the total variance of log employment is approximately six percent. Under the simple Markov model presented above, the variance of growth rates is the sum of twice the measurement error (σ_w^2) plus the variance of the disturbance u ; if we make the most extreme assumption that all the variation is measurement error so that $0.06 \approx 2\sigma_w^2 / (\sigma_X^2 + \sigma_w^2)$, then the largest negative value we would expect for this coefficient is -0.03 (for year-to-year growth rates). If we compute annual growth rates over a longer period, this estimate of the coefficient would be divided by the number of years over which the employment change is computed.

In Table I, I present the results of a simple regression of growth on size for the two samples, with and without individual industry effects. The coefficient of $\log E_{72}$ in a regression of the annual growth rate from 72 to 79 is -1.14% ,

⁴This is obviously an inadequate treatment of an interesting aspect of growth in the manufacturing sector, but it involves a relatively small number of firms, and it is beyond the scope of the present paper to model major merger activity. We hope to explore the extent to which this kind of growth impacts on our estimates in future work.

TABLE I
GROWTH RATE REGRESSIONS

Dependent Variable	Annual Growth Rate in Percentage Terms			
	72-79 OLS	73-79 OLS	73-79 OLS	73-79 Inst. Var.
1972-1979: 1349 Firms				
Intercept	4.0(0.2)	2.9(0.3)	2.9(0.3)	3.0(0.3)
Logarithm of Size in Year	-1.14(0.15) 72	-0.98(0.14) 72	-0.92(0.14) 73	-0.99(0.14) 73
Standard Error	8.4	9.4	9.5	9.5
Intercept		20 Industry Dummies		
Logarithm of Size in Year	-1.09(0.13) 72	-0.95(0.14) 72	-0.90(0.14) 73	-0.97(0.14) 73
Standard Error	7.6	8.7	8.7	8.7
F-statistic for Industry Dummies	7.20	6.36	6.44	6.44
Dependent Variable	Annual Growth Rate in Percentage Terms			
	76-83 OLS	77-83 OLS	77-83 OLS	77-83 Inst. Var.
1976-1983: 1098 Firms				
Intercept	1.89(0.30)	1.07(0.32)	1.06(0.33)	1.13(0.33)
Logarithm of Size in Year	-1.06(0.15) 76	-0.99(0.16) 76	-0.93(0.16) 77	-1.00(0.16) 77
Standard Error	8.6	9.2	9.2	9.2
Intercept		20 Industry Dummies		
Logarithm of Size in Year	-0.89(0.16) 76	-0.79(0.17) 76	-0.72(0.17) 77	-0.80(0.17) 77
Standard Error	8.3	8.9	8.9	8.9
F-statistic for Industry Dummies	4.83	5.24	5.30	5.30

Note: Figures in parentheses are standard errors.

while that of $\log E_{76}$ in the growth rate regression is -1.06% . That is, doubling a firm's size decreases its annual growth rate by about eight tenths of one percent. In the remainder of the table I try to correct for possible measurement error bias in this relationship, but it remains remarkably stable. First, the model of pure random walk with measurement error would predict an estimated coefficient of zero in the regression of the growth rate on size in the period preceding that from which the growth rate is measured, whereas in column 2 we obtain an estimate only slightly smaller in absolute value than in the previous regression. Second, in the last column we regress the growth rate on size at the beginning of the period using size one year prior to the beginning of the period as an instrument, since by assumption it is uncor-

related with the measurement error, but this regression yields almost the same result as the ordinary least squares estimate in column 5.

The addition of industry dummies does not change the coefficients much, although they are significant at conventional levels (the 1% critical level for the F-statistic is about 2.2). Since this pattern held for most of the results reported in this paper, we have not presented estimates with industry dummies in the rest of the tables; they almost invariably were moderately significant but had little or no effect on the other coefficients. The study of interindustry differences in these data appears to be warranted but is beyond the scope of this paper.

My tentative conclusion is that uncorrelated errors of measurement in employment cannot be responsible for more than about ten percent of the observed negative relationship between size and growth. From the fall in absolute value of the size coefficient in the growth equation in going from column 1 to columns 2 or 4 in Table I, we can infer a value for $\sigma_w^2/(\sigma_x^2 + \sigma_w^2)$ of approximately 0.0015, or one tenth of a percent. Since the variance of the logarithm of employment is approximately 2.7 and the mean is around 0.8, this does correspond to a standard deviation of about fifteen percent of the level of employment in any one year, a not inconsiderable amount. It is simply that a measurement error of this magnitude is swamped by the large variance in size across our firm population, and it introduces very little bias in the estimating equation.

Repeating the exercise using sales produced much the same result in the growth-size regressions, although the standard deviation of the measurement error in this case could be about half again as large. For the instrumental variable estimates corresponding to column 4 of Table I, the size coefficient was -0.90 (0.21).

III. THE TIME SERIES BEHAVIOR OF EMPLOYMENT GROWTH

Since I rejected the simplest random walk with measurement error model in the previous section, the approach I take in this section is to try to discover what time series model *will* adequately describe the data, so that I can specify more fully the way in which firm growth deviates from Gibrat's Law. In an appendix to the unpublished version of this paper (available from the author on request), I present the results of a time series analysis of the three different panels of firms drawn from my sample. In this section I interpret the results of that analysis.

The fact that the coefficient of lagged size in the growth rate equation in Table I was always negative, even with a measurement error adjustment, suggests that the simple random walk model I was considering should be modified to include an autoregressive component. This is because the model in equation (2) has a coefficient on lagged size equal to unity, while the estimates imply a coefficient on lagged size which is somewhat less than one.

This expanded model can be written as⁵

$$(5) \quad X_t = \beta X_{t-1} + u_t \quad Eu_t^2 = \sigma_u^2 \quad EX_{t-1}u_t = 0$$

$$(6) \quad y_t = X_t + w_t \quad Ew_t^2 = \sigma_w^2 \quad Eu_s w_t = 0, \forall s, t$$

I have allowed the variance of employment growth to vary from year to year since the observed variances change considerably over time. The model above is equivalent to a standard ARMA(1, 1) model, but the latter is valid over a larger parameter space; this property turns out to be important. The ARMA(1, 1) model is written as

$$(7) \quad (1 - \alpha L)y_t = (1 - \mu L)\varepsilon_t, \text{ where } \varepsilon_t \text{ is white noise}$$

whereas the AR(1) model with measurement error, (3), implies

$$(8) \quad (1 - \beta L)y_t = u_t + (1 - \beta L)w_t = u_t + w_t - \beta w_{t-1}$$

If the disturbances are normally distributed, it is easy to show that the two models are equivalent with

$$\alpha = \beta$$

$$(1 + \mu^2)\sigma_\varepsilon^2 = \sigma_u^2 + \sigma_w^2(1 + \beta^2)$$

$$\sigma_w^2 = (\mu/\beta)\sigma_\varepsilon^2$$

However, the measurement error model requires that σ_w^2 be positive, which imposes the constraint that μ and α are of same sign in the ARMA(1, 1) model and restricts the parameter space. When I estimate the ARMA(1, 1) model using these data, the constraint is *not* satisfied, which implies a slightly negative σ_w^2 .

Estimates for the models in this section were obtained by maximum likelihood under the assumption that the disturbances are homoskedastic and normal. The method is described in somewhat more detail in Hall [1979]. MaCurdy [1981] has shown that these estimates are consistent even if the disturbances are not multivariate normal (as seems likely in this case), although the estimated standard errors are no longer correct. Estimates for the model in equation (8) are shown in the top half of Table II. They are quite stable across the periods and are consistent with the *IV* estimates in Table I, since they imply a coefficient of $100(\beta - 1) = -1.0$ percent in the growth rate equation together with a slightly *positive* measurement error bias (of the order of 0.001, or one tenth of a percent).

Since the time series analysis in the appendix shows that an ARMA(2, 1) model fits the data significantly better than an ARMA(1, 1) model, we also explore what Leonard [1984] calls a "mean-reverting" model, which is familiar from the investment literature as a flexible accelerator model. This

⁵ Throughout this section, I have suppressed the *i* subscript (indicating the firm) on y , X , w , u , and ε for simplicity.

TABLE II
TIME SERIES BEHAVIOR OF LOG EMPLOYMENT

	1972-79	1976-83	1972-83
Autoregressive Model with Measurement Error ^a			
β	0.991 (0.001)	0.990 (0.002)	0.991 (0.001)
Var. of Meas. Error (σ_w^2)	-0.0018 (0.0004)	-0.0036 (0.0005)	-0.0027 (0.0012)
Var. of Shock (σ_u^2) ^c	0.0368 (0.0015)	0.0426 (0.0023)	0.0406 (0.0174)
Adjustment Cost Model ^b			
λ	1.745 (0.147)	0.553 (0.097)	0.878 (0.051)
Var. of Meas. Error (σ_w^2)	0.057 (0.005)	0.015 (0.004)	0.029 (0.002)
Var. of Shock (σ_u^2) ^c	0.032 (0.001)	0.057 (0.006)	0.075 (0.019)

Notes: ^a These estimates are derived from the ARMA(1, 1) estimates which were based on the covariance of the levels of log employment described in Appendix A. See the text for a definition of the model and an explanation of the negative variance estimate.

^b These estimates are derived from the ARMA(1, 1) estimates which were based on the covariance of the first differences of log employment described in Appendix A.

^c These are derived for a representative estimate of σ_u^2 (about 0.035). They actually will change slightly each year.

model rests on the idea that the number of employees is a kind of stock that is not instantaneously adjustable at zero cost to the firm. For a firm with constant returns to scale that has quadratic adjustment costs, there is a linear relationship between employment changes and the current and desired levels of employment:

$$(9) \quad \Delta y_t = (1 - \lambda)(y_t^* - y_{t-1}) + w_t, \text{ or}$$

$$(10) \quad (1 - \lambda L)y_t = (1 - \lambda)y_t^* + w_t$$

The time series process implied by this model depends on what is assumed about the process generating the desired level of employment. If $y_t^* = y_t^*$ for all t , (7) becomes

$$(11) \quad (1 - \lambda L)y_t = \alpha_t + w_t, \quad \text{where } \alpha_t = (1 - \lambda)y_t^*$$

Because of the short panel, this cannot be estimated consistently in levels, so I write it in first differences:

$$(1 - \lambda L)\Delta y_t = (1 - L)w_t$$

This is an ARMA(1, 1) process with μ constrained to be equal to one.

It seems more reasonable to assume that the desired level of employment is a Martingale process, since we might expect that the target size evolves as the firm receives random shocks each year involving demand, cost, and so forth. This implies

$$(12) \quad (1 - L)y_t^* = u_t \quad \text{with } Eu_t^2 = \sigma_u^2$$

and the process becomes

$$(13) \quad (1 - \lambda L)\Delta y_t = (1 - \lambda)u_t + (1 - L)w_t$$

which is equivalent to an ARMA(1, 1) process (4) with both α and μ free.⁶ Since the estimated μ for this model is not unity, it is easy to reject the first version (constant target size). The estimates for the second version are shown in the bottom of Table II. They are unstable across the time periods. The estimates for the first period do not make much sense in the context of this model since they imply that the firm adjusts its size away from the desired level of employment ($1 - \lambda = -0.745$).

I conclude the following from this time series analysis of employment growth: (1) The failure of Gibrat's Law to hold is not due to serially uncorrelated measurement error in the size variable, but rather to the fact that there is very slight tendency for large firms to become smaller and small firms to become larger; this tendency is stable over time (the autoregressive coefficient of 0.991 in all three columns of Table II). (2) The variance in growth rates across firms changes significantly from year to year (the relationship of this finding to macroeconomic effects deserves investigation in further work). (3) A simple adjustment cost model (in which employment is not adjusted freely from year to year due to such effects as labor hoarding, etc.) fits the data fairly well, but produces extremely unstable parameter estimates over the relatively short time period which I am studying. The estimates imply that adjustment costs rose between the earlier (1972 to 1979) and later periods (1976 to 1983), so that the response of firm employment to surprises in optimal size is somewhat dampened in the later sample.

IV. CORRECTING FOR SAMPLE ATTRITION

In obtaining the previous time series results, I used a balanced sample of firms, ignoring the possible biases introduced by entry into and exit from the sample during the time period. In this section I explore the consequences of sample attrition on the estimates of a growth equation. My sample is drawn from the universe of Compustat firms in 1976. Hence there is selection in both directions: small, fast-growing firms may exist in 1976 but not in 1972, and

⁶ This can be shown in the same way we showed the equivalence of (4) and (5). Since the order of the AR part is the same, $\alpha = \lambda$, and we have

$$2\sigma_w^2 + (1 - \alpha)^2\sigma_u^2 = \sigma_\varepsilon^2(1 + \mu^2)$$

$$\sigma_w^2 = \mu\sigma_\varepsilon^2$$

which implies

$$\sigma_u^2 = \sigma_\varepsilon^2(1 - \mu)^2/(1 - \alpha)^2$$

where α , μ , σ_ε^2 are the parameters of the ARMA(1, 1) model.

TABLE III
GROWTH RATE REGRESSIONS WITH SELECTION CORRECTION
1778 FIRMS

Dependent Variable	OLS and Probit		Sample Selection	
	Annual Growth Rate			
	76-79	76-83	76-79	76-83
<i>Number of Firms</i>	1551	1184	1551	1184
<i>Intercept</i>	5.54 (0.40)* (0.33)	1.81 (0.34) (0.29)	5.54 (**) (0.69)	1.69 (0.33) (1.05)
$\text{Log}E_{76}$	-1.06 (0.18) (0.18)	-0.92 (0.16) (0.15)	-1.06 (**) (0.18)	-0.91 (0.15) (0.17)
<i>Standard Error</i>	11.7	8.6	11.7	8.6
Dependent Variable	Probability of Survival			
<i>Intercept</i>	1.10 (0.041) (0.041)	0.352(0.034) (0.034)	1.10 (**) (0.041)	0.352(0.034) (0.034)
$\text{Log}E_{76}$	0.057(0.021) (0.024)	0.104(0.019) (0.019)	0.057(**) (0.024)	0.104(0.019) (0.020)
$\hat{\rho}$	-	-	0.0 (**) (0.204)	0.022(0.011) (0.197)
<i>Log of Likelihood</i>	446.3	105.1	446.3	105.1
χ^2 for Squared and Cubic Size Terms (DF = 4)			60.4	32.8
LM Test for Heteroskedasticity (DF = 2)	60.6	43.0	60.6	43.2

* The first set of numbers in parentheses are heteroskedastic-consistent standard error estimates and the second set are ordinary estimates.

** The HS-consistent standard errors are not computable since the maximum likelihood estimate of ρ is exactly zero.

some firms exit in the years after 1976. Here I consider the consequences of exit for estimates of the growth rate-size relationship in these data.⁷

Because I wish to focus on the effects of exit conditional on a complete population of firms in the initial period, I use growth rates based in 1976 for the results in this and subsequent sections of the paper. Two periods are considered: three year growth rates from 1976 to 1979 and seven year growth rates from 1976 to 1983. The comparison allows us to obtain some idea of the stability of the results over time. All growth rates are actually measured in annual terms in order to make the coefficients comparable. The first two columns of Table III (top portion) show the results of a simple growth rate on

⁷ An earlier draft of this paper considered the bias induced by entry between 1972 and 1976 using a truncated Tobit model (see, for example, Maddala [1983, pp. 176-177]), but I found the estimates to be extremely imprecise due to the fact that identification of the parameters in the selection equation come only from the probability term in the denominator of the likelihood function, so I have omitted these estimates here.

size regression using these two periods. The size coefficient is approximately the same as those in Table I, about -1.0 percent per unit change in the logarithm of employment. I now consider whether this can arise because of the attrition of smaller firms during the period.

In order to measure the growth rate, I required that data be available for the firm in both the beginning and the ending period. Even if I was able to draw a sample of firms that are representative of the population in the initial period, by the time I reached the final period, the smaller and more slowly growing firms are those most likely to have dropped out of the sample.

In concrete terms, let y_i be the growth rate of the i^{th} firm, and let y_i and \tilde{y}_i be the initial and final period logarithm of size. Then the observed growth rate is

$$(14) \quad \Delta y_i = \tilde{y}_i - y_i = \gamma_i + u_i$$

where u_i is an i.i.d. random variable, with $E(y_i u_i) = E(y_i u_i) = 0$. The probability that a firm will survive to the end of the period so that Δy_i is observed is given by a Probit model, with the latent variable a function of such firm characteristics as industry or beginning of period size. Thus the model is a standard generalized Tobit model of the form

$$(15) \quad \begin{aligned} \Delta y_i = y_{1i} &= X_i \beta + v_{1i} && \text{if } y_{2i} > 0 \\ y_{1i} &\text{ not observed} && \text{if } y_{2i} < 0 \\ y_{2i} &= Z_i \delta + v_{2i} \end{aligned}$$

with a covariance matrix

$$(16) \quad E v v' = \begin{bmatrix} \sigma_1^2 & \rho \sigma_1 \\ & 1 \end{bmatrix}$$

where I have normalized the residual variance of the unobserved latent variable y_{2i} to be unity. A discussion of this model and its estimation by the method of maximum likelihood is given in Griliches, Hall and Hausman [1978].

It is an implication of this model that a regression run on only the observed data will have the property

$$(17) \quad \begin{aligned} E(\Delta y_i | \Delta y_i \text{ observed}) &= X_i \beta + E(v_{1i} | \Delta y_i \text{ observed}) \\ &= X_i \beta + \rho \sigma_1 \Lambda(Z_i \delta) \end{aligned}$$

where $\Lambda(\cdot)$ denotes the inverse Mills ratio $\phi(\cdot)/\Phi(\cdot)$ and $\phi(\cdot)$ and $\Phi(\cdot)$ denote the normal density and cumulative distribution respectively. That is, if the disturbances in the two equations are positively (negatively) correlated, the estimated growth rates will be biased upward (downward). In this case, if size is a predictor of survival, then $\Lambda(Z_i \delta)$ will be higher for small firms than for large, and the bias will be correspondingly larger for these firms. Note that if the disturbances of the survival equation and the growth rate equation are

uncorrelated ($\rho = 0$), no bias will result even though size may still be a predictor of survival.

In the last two columns of Table III I present the results of estimation of the growth rate equations for the two periods 1976–1979 and 1976–1983 using maximum likelihood estimation of the sample selection model (the model described by equations (15) and (16)). In neither period was the sample selection correction significant; the estimated ρ is essentially zero, and the coefficients do not change between the ordinary least squares and sample selection estimates. In the case of the first period, the selection model is close to being unidentified since ρ is zero and the standard errors are not really computable. The bottom half of the table shows the estimates for the equation describing the probability of survival. We can see that survival is significantly positively related to size (a t-statistic of about 5 in column 2, for example), but because the estimated ρ is zero, this does not bias the coefficients of the growth rate equation. What this means is that the variation in growth rates across firms which remains after controlling for size is uncorrelated with the probability of survival. Big surprises in growth rates, either positive or negative, do not seem to be related to survival, at least in a way that is detectable when we look at the entire manufacturing sector.

I should note that the lack of correlation between the survival and growth rate equations holds even though I have excluded a quadratic size term from the model, so that the spurious collinearity with a Mills ratio term that the inclusion of powers of size might induce is not the problem.⁸ In the table I show the $\chi^2(4)$ statistic for the inclusion of quadratic and cubic terms in both equations in the presence of correlated sample selection. There is evidence of nonlinearity in the relationship of growth and size, and in the next section I attempt to disentangle this nonlinearity from size-related heteroskedasticity coupled with sample selection.

I note in passing that adding industry dummies improved the explanatory power of the survival equation (from 76 percent correct to 84 percent correct) but did not change the size coefficient in the growth rate equation very much. The conclusion is that selection bias of this simple kind does not seem to account for the negative relationship between growth and size.

V. CORRECTING FOR HETEROSKEDASTICITY

It is well known that estimates of limited dependent variable models are not robust to departures from normality or heteroskedasticity of the disturbances.

⁸ Because the variables in the growth rate equation and the survival equation are the same, the Mills' ratio term shown in equation (17) is just a nonlinear function of the X_i 's (size). If a quadratic size term is included in the growth rate regression, then the estimated ρ might be insignificant even though there was selection; this would happen because the Mills' ratio was collinear with size and size squared. See Griliches, Hall and Hausman [1978] or Maddala [1983, p. 271], for a further discussion of this point.

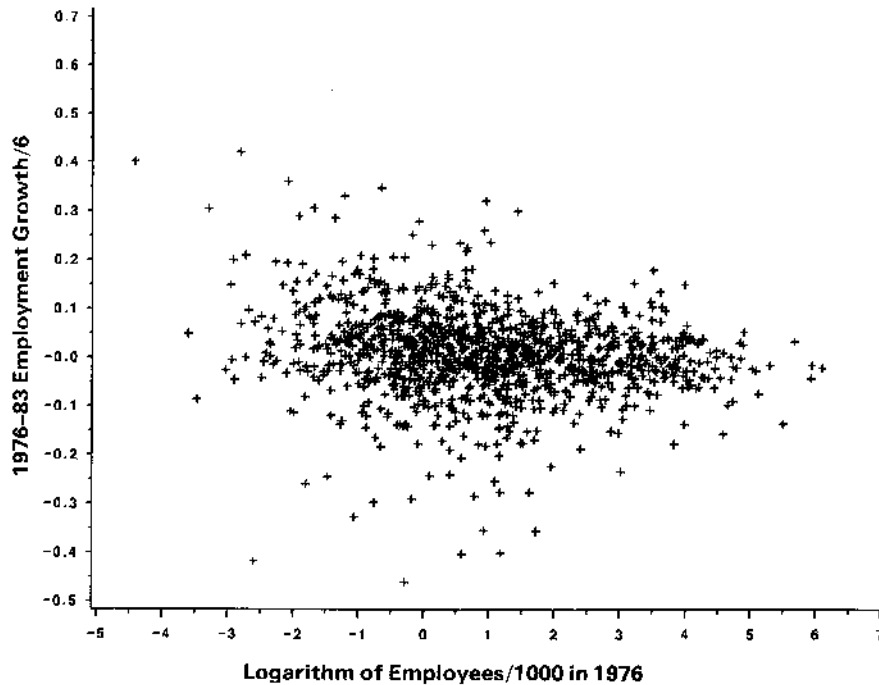


Figure 1
1976-1983 Growth Rate versus Size in 1976.

This seems likely to be a problem here from the evidence of the plot in Figure 1, which suggests that the variance of growth rates is size-related. To check this, I use a version of a Lagrange Multiplier test due to Poirier and Rudd [1983] in order to test for heteroskedasticity in the generalized Tobit model. This test consists of regressing a function of the squared residuals and the estimated correlation coefficient from the sample selection model on the variables of the model. The value of the test statistic when the heteroskedasticity is modelled as a function of size and size squared is shown in columns 3 and 4 of Table III.⁹ The null hypothesis of homoskedasticity is decisively rejected in favor of size-related heteroskedasticity.

If I were willing to maintain that the error in the selection (survival) equation was homoskedastic and normally distributed, it would be possible to compute consistent estimates of the coefficients of the growth equation and their standard errors by including the estimated Mills ratio in the regression

⁹The test statistic is almost the same as that given in the first two columns, which is not surprising given the low estimated value of ρ . We would not generally expect the statistics to be the same if ρ were significantly different from zero, however.

TABLE IV
GROWTH RATE REGRESSIONS WITH CORRECTIONS FOR HETEROSKEDASTICITY AND SELECTION
1778 FIRMS

Dependent Variable	Annual Growth Rate			
	76-79	76-79	76-79	76-83
Intercept	4.51 (0.45)	6.79 (0.45)	5.73 (0.76)	6.37 (0.57)
LogE ₇₆	-0.53 (0.16)	-1.52 (0.29)	-1.54 (0.29)	-2.12 (0.30)
(LogE ₇₆) ²		0.16 (0.07)	0.64 (0.14)	0.16 (0.07)
(LogE ₇₆) ³			-0.10 (0.03)	
Slope (E = 700)	-0.53 (0.16)	-1.64 (0.35)	-2.03 (0.37)	-2.23 (0.34)
Slope (E = 17 000)	-0.53 (0.16)	-0.78 (0.20)	-0.14 (0.29)	-1.39 (0.21)
Standard Error (Wid.)	10.7	11.1	10.9	9.65
			8.32	
Dependent Variable	Probability of Survival			
Intercept	1.26 (0.05)	1.22 (0.05)	1.26 (0.06)	0.33 (0.04)
LogE ₇₆	-0.95 (0.02)	-0.27 (0.05)	-0.28 (0.05)	-0.04 (0.03)
(LogE ₇₆) ²		0.07 (0.02)	0.04 (0.03)	0.04 (0.01)
(LogE ₇₆) ³			0.005 (0.007)	0.0001 (0.005)
$\hat{\rho}$	0.16 (0.10)	-0.50 (0.11)	-0.41 (0.22)	-0.74 (0.06)
Log Likelihood	603.2	619.7	625.6	188.9
LM Test for Heteroskedasticity (DF = 2)	0.17	0.07	0.08	1.85
			2.17	1.95

All standard error estimates are heteroskedastic-consistent estimates; they are the same as the conventional estimates to two digits. The weights are inversely proportional to size and size squared (see the text for an explanation of the heteroskedasticity correction).

and using White's formula for heteroskedastic-consistent standard errors (Olsen [1980]). However, in spite of the fact that the probit disturbance appears to be homoskedastic, this assumption seems unwarranted here, since the selection equation itself arises from much the same process as generated the heteroskedastic disturbances in the regression equation. A more promising avenue to explore would be the modelling of the heteroskedasticity in some simple fashion depending on size.

Accordingly, I constructed a simple model for the variance σ_i^2 of the disturbances in the growth equation by regressing the estimated residuals squared on size and size squared in the initial period. A typical regression of this sort had a negative coefficient on size and a small positive coefficient on size squared. The predicted standard deviation of the growth rate disturbance fell from about 17 percent for small firms to seven percent for the larger firms. I assumed that the heteroskedasticity in the selection equation is proportional to that in the growth equation, and used these estimated σ_i^2 as weights in both equations to induce approximate homoskedasticity of the disturbances. Note that this procedure performs the estimation of the model in two stages, and the maximum likelihood estimates are no longer fully efficient, but are conditional on the model chosen for σ_i^2 . It would be possible, but difficult due to the high nonlinearity involved, to estimate this new model by maximum likelihood by including the model for σ_i^2 explicitly in the denominator of the residual functions, but I have chosen not to do this in order to simplify the estimation.

The results of this procedure are shown in Table IV. Focusing for the moment on columns 1 and 4, which are comparable to the sample selection estimates in the previous table, we can see that the size coefficient has fallen by one half, and the estimate of ρ is now positive, but insignificant. The LM test for heteroskedasticity of the disturbances of the weighted model no longer rejects after the weighting has been performed. However, the results now show that size has an opposite effect on the probability of survival during the two periods, which seems highly unlikely, given that one sample is a subset of the other. This turns out to be due to a combination of the weighting scheme used and the nonlinearity of the probit index with respect to size; it shows how sensitive this type of estimate can be to weighting.

Because of this problem and because my goal in performing this test in the sample selection setting was to sort out the different effects of size-related heteroskedasticity, size-related sample attrition, and nonlinearity in the relationship of growth and size, in the other columns of Table IV I present estimates of the growth rate equation with quadratic and cubic size terms. Note first that the LM test statistic is still insignificant, so heteroskedasticity of a size-related kind is not a problem here. In the growth equation, the quadratic term is significant in both periods, and the cubic term is significant only in the first. The estimates for the probit equation imply a probability of survival which is roughly constant (about 0.88 in 1979 and 0.65

in 1983) until a size of around 10 000 employees and then rises fairly quickly to near one. This is consistent with the observed survival rates.

In both periods, the estimated ρ is quite negative. The fact that the estimate of ρ is robust to the order of the polynomial expansion of the size equation is evidence that the Mills ratio term is not simply proxying for some higher order function of size (in fact, a quartic does not enter the growth equation significantly in the presence of quadratic and cubic terms). However, a negative correlation between the disturbances of the growth equation and the survival equation does call into question the basis for my original model of exit from the sample, since it seems to imply that firms that grow faster than predicted by their size are more likely to exit from the sample, holding size constant. I will explore this puzzle further in the next section when I look at the reasons for exit from the sample.

VI. SAMPLE ATTRITION AS A FUNCTION OF TOBIN'S Q

The preceding discussion highlights a problem with the generalized Tobit approach to sample selection correction. Many previous researchers have pointed out that in the absence of exclusion restrictions in the selection equation the identification in such sample selection models comes through the nonlinearity of the Mills' ratio—i.e., the exact functional form of the disturbance distribution function (see, e.g., Bound *et al.* [1984], or Maddala [1983]). In principle, as we add higher order terms to the regression equation, these terms become increasingly collinear with the Mills ratio variable, which itself can be approximated by a polynomial expansion in the Z 's. When there are additional variables in the selection equation, this collinearity disappears and it becomes possible to include nonlinear terms without necessarily having them proxy for the selection bias correction.

However, when correcting for selection related to size in a growth equation, it is extremely difficult to think of variables which belong in an equation describing the probability of survival, but not in the growth equation. One possible avenue to pursue is a more explicit modelling of the reasons for exit, about which we have some information. Of the 1778 firms in 1976, 225 exit from the sample by 1979 and another 369 exit by 1983. Both Compustat and the CRSP files (which include many but not all of these firms) contain a code giving the reason for deletion when the data for the firm is removed from the file. Using these codes, Addanki [1985] and I, in parallel work, were able to establish that approximately sixty percent of the firms were dropped due to merger or acquisition, eight percent because of bankruptcy or liquidation, and the remainder for reasons unknown. The last category includes smaller firms, many of which were probably acquired.

We hypothesize that a firm will be acquired and disappear from the sample when the existing assets of the firm are not being employed in an optimal way; a prospective buyer is willing to buy the firm at the current stock price in the

hopes of producing an above average return on the stock by redeploying the assets in some way. That is, the probability of a firm's being acquired is a function of the average Tobin's Q for the firm, the ratio of the market value to the book value of the assets. The market value is assumed to be the current capitalized value of the future earnings potential of the firm's assets. The higher is Q , the less likely that the firm will be acquired and disappear from our sample. This is a fairly crude story, which leaves unexplained why the market is undervaluing the assets in this way; it simply posits that if they are undervalued, an opportunity exists for a potential purchaser. Of course, if the stock market is perfectly efficient, the Q for these firms should be driven up on the expectation that they will be acquired in the future. Therefore it is somewhat surprising that this variable turns out to be a fairly good predictor of survival, somewhat better than the pure size variable we have been using.

The assets of a firm include more than the physical assets; in particular, we are interested in the value of the assets represented by the firm's technological position, or knowledge stock, as proxied by its R & D history. Thus we would like to use a Q variable that contains a measure of R & D stock as well as physical capital in the denominator. Following Hayashi [1982], Wildasin [1984] has shown that the market value of a firm that maximizes discounted cash flow using more than one stock of capital is given by a weighted sum of the value of the capital stocks:

$$(18) \quad V = \sum_{i=1}^n \lambda_i K_i$$

where K_i are the capital stocks in physical units and the λ_i are the shadow prices of these stocks, which depend on taxes, depreciation, and adjustment costs and are not necessarily equal over different kinds of stocks. Unfortunately, we do not have a measure of these shadow prices, so we do not know how to weight the physical assets and R & D stock appropriately in computing Q . Denoting the physical assets by A and the knowledge stock by R , we can write Q as

$$(19) \quad Q = \frac{V}{\lambda_1 A + \lambda_2 R} = \frac{V}{\lambda_1 A} \left(\frac{1}{1 + \gamma R/A} \right)$$

where γ is the ratio of the two shadow prices. Because the measured Q variable in these data exhibits a very longtailed distribution, which tends to give extreme weight to a few outliers, I chose to use the logarithm of the variable in the selection equation, so that the variable becomes

$$(20) \quad \log Q \approx \log(V/A) - \log(1 + \gamma(R/A))$$

I approximate $\log(1 + \gamma(R/A))$ by $\gamma(R/A)$ since I expect $\gamma(R/A)$ to be small, so that the variables actually used in the selection equation are $\log(V/A)$ and R/A . Firms with no R & D program have an R/A stock equal to zero; the inclusion of a separate dummy for these firms in the selecting equation had no

effect on the results. Because of work by Addanki [1985], who found that the valuation of a firm's R & D program at the time of acquisition differed depending on whether a firm was a patenter, I allowed for a separate coefficient on R/A for those firms that filed successful patent applications in 1976.

As a measure of the Q of physical assets, V/A , I use the total market value of the firm (common stock, debt, and preferred stock) divided by the sum of net capital stock, inventories, and other assets (including subsidiaries). The value of the components of both V and A have been adjusted for the effects of inflation using the methodology of Brainard, Shoven, and Weiss [1980]; the computations are more fully described in Cummins, Hall, Laderman, and Mundy [1984].¹⁰

The estimates for a probability of survival equation using these variables are shown in Table V, along with growth rate equations augmented by the two investment variables (these will be discussed later). What the probit equations show is that both the V/A and R/A variables are more important in predicting survival than the raw size variable, employment, although the R/A variable has a large standard error.¹¹ At the sample means these estimates imply that a doubling of employment increases the probability of survival to 1983 by 0.03, a doubling of Q increases the probability 0.10, and a doubling of R & D increases it 0.03 for non-patenters and 0.01 for patenters, *ceteris paribus*. Firms with a larger portion of their assets in R & D are less likely to disappear from the sample, while having patents makes them somewhat more likely to exit than firms with R & D and no patents. This last result is consistent with Addanki [1985], who found that the R & D expenditures of firms that have a record of patenting are more highly valued than those of firms that hold no patents.

The use of these variables to help predict the probability of survival has had some effect on the estimates of the growth rate equation. The size coefficient has increased substantially in absolute value over the estimates in columns 1 and 4 of Table IV, and the estimated correlation between the residuals of the selection equation and the growth rate equation is quite negative. The results are not sensitive to the exact specification of the selection equation. Inclusion of the Q variable as a predictor seems to be enough to produce a rather anomalous result: a firm that grows faster than predicted by its size and level of investment is somewhat *more* likely to exit from the sample, controlling for size and Q . This implies that the average

¹⁰ For comparison, I also used an unadjusted Q based solely on the raw numbers on the Compustat files. In 1976, the value of this Q was lower, and the dispersion less. The qualitative results of the Probit equation were unchanged, and the coefficients were more significant, suggesting that the process of adjusting for inflation bias also introduces more measurement error into the variable.

¹¹ The other variables in the growth equation were also included in the selection equation, but they had insignificant coefficients so the estimates reported do not include them.

growth rate for the smaller firms is underestimated and hence that the size coefficient in the growth rate equation is biased towards zero when we do not correct for selection.

VII. INVESTMENT AND FIRM GROWTH

This section reports on some descriptive regressions that relate the firm growth rates to the level of investment, both physical and R & D, in 1976. These results reported in this section are in no sense derived from a structural model; we are merely documenting the magnitude of the correlation between investment and growth in the manufacturing sector.

In Table V we have added three variables to the standard growth rate equation: the logarithm of capital expenditures in 1976, the logarithm of R & D investment in 1976, and a dummy equal to one for those firms who do no or negligible R & D. Both of the expenditure variables have been scaled by

TABLE V
GROWTH RATE REGRESSIONS WITH Q
1753 FIRMS

Dependent Variable	OLS and Probit		Sample Selection	
	Annual Growth Rate			
	76-79	76-83	76-79	76-83
<i>Number of Firms</i>	1529	1171	1529	1171
<i>Intercept</i>	6.68 (0.46)	2.59 (0.38)	8.38 (0.51)	6.78 (0.44)
$\text{Log}E_{76}$	-1.08 (0.16)	-1.14 (0.13)	-1.46 (0.18)	-1.94 (0.15)
$\text{Log}(I/E)_{76}$	1.26 (0.28)	1.22 (0.24)	1.41 (0.29)	1.36 (0.24)
$\text{Log}(R/E)_{76}$	1.31 (0.35)	1.33 (0.20)	1.26 (0.35)	1.09 (0.23)
$D(R = 0)$	-3.50 (0.62)	-2.30 (0.53)	-2.71 (0.63)	-0.80 (0.51)
<i>Standard Error (Wtd.)</i>	10.3	7.74	11.0	9.32
	Probability of Survival			
Dependent Variable	1979	1983	1979	1983
<i>Intercept</i>	1.18 (0.06)	0.26 (0.05)	1.10 (0.06)	0.20 (0.04)
$\text{Log}E_{76}$	-0.09 (0.02)	0.08 (0.02)	-0.06 (0.03)	0.08 (0.02)
$\text{Log}Q_{76}$	0.17 (0.07)	0.29 (0.05)	0.25 (0.07)	0.36 (0.04)
$(R/A)_{76} \cdot (\text{Patents} = 0)$	0.28 (0.49)	0.29 (0.41)	0.82 (0.52)	0.61 (0.36)
$(R/A)_{76} \cdot (\text{Patents} > 0)$	0.26 (0.42)	0.16 (0.35)	0.37 (0.34)	0.37 (0.35)
β	—	—	-0.61 (0.09)	-0.81 (0.03)
<i>Log Likelihood</i>	631.3	269.2	646.2	306.9
<i>LM Test for Heteroskedasticity (DF = 4)</i>			20.6	0.43

All standard error estimates are heteroskedastic-consistent estimates.

Estimates are obtained by maximum likelihood of the sample selection model with the disturbances weighted to correct for heteroskedasticity.

subtracting the logarithm of 1976 employment so that the total size effect still appears in the coefficient of $\log E_{76}$. The investment coefficients are quite substantial: at the mean level of investment for these firms, an increase of four million dollars in physical investment is associated with a one percent increase in the annual growth rate from 1976 to 1979, while it takes only two million dollars of R & D investment to achieve the same effect for those firms which have R & D programs. In the second period the effects are the same, which implies considerable persistence in the correlation of growth and investment. Firms that have no R & D program grow on average about one to two percent more slowly than those which do.

Earlier work in this area (Mansfield [1962] and Hymer and Pashigian [1962]) found that two results seem to hold when firm growth is examined over a large size range of firms: (1) the variance (in logarithms) is larger at the lower end of the size distribution, and (2) Gibrat's Law is closer to holding for large firms than for small. We have already seen that the first result holds in

TABLE VI
GROWTH RATE REGRESSIONS BY FIRM SIZE

Dependent Variable	Small Firms (N = 1023)		Large Firms (N = 730)	
	Annual Growth Rate			
	76-79	76-83	76-79	76-83
Number of Firms	832	604	697	567
Intercept	9.11 (0.63)	7.27 (0.61)	6.92 (0.96)	6.07 (0.93)
$\log E_{76}$	-1.58 (0.56)	-1.49 (0.49)	-0.94 (0.30)	-1.76 (0.28)
$\log(I/E)_{76}$	2.74 (0.43)	1.46 (0.38)	0.86 (0.33)	1.20 (0.30)
$\log(R/E)_{76}$	2.39 (0.38)	1.27 (0.37)	1.01 (0.35)	0.84 (0.27)
$D(R=0)$	-4.13 (0.88)	-1.72 (0.77)	-1.48 (0.81)	0.32 (0.68)
Standard Error (Wtd.)	11.6	6.59	7.50	5.63
	Probability of Survival			
Dependent Variable	1979	1983	1979	1983
Intercept	1.05 (0.07)	0.30 (0.07)	1.13 (0.17)	0.11 (0.18)
$\log E_{76}$	-0.19 (0.08)	-0.005 (0.08)	0.10 (0.07)	0.20 (0.07)
$\log Q_{76}$	0.16 (0.08)	0.38 (0.08)	0.56 (0.13)	0.75 (0.11)
$(R/A)_{76} \cdot (\text{Patents} = 0)$	1.48 (0.91)	1.07 (0.55)	-5.75 (2.52)	-4.05 (2.83)
$(R/A)_{76} \cdot (\text{Patents} > 0)$	0.45 (0.37)	0.52 (0.54)	-1.6 (0.97)	0.05 (0.93)
$\hat{\rho}$	-0.77 (0.06)	-0.80 (0.05)	-0.57 (0.10)	-0.83 (0.05)
Log Likelihood	313.0	292.9	593.4	507.8
LM Test for Heteroskedasticity (DF = 4)	0.60	0.05	0.49	1.97

All standard errors are heteroskedastic-consistent estimates.

Estimates are obtained by maximum likelihood of the sample selection model with the disturbances weighted to correct for heteroskedasticity.

this sample and the nonlinear estimates in Table IV suggest that the second one probably holds as well. To check this result I divided the sample into two size classes and reestimated the equations in Table V. The size cut I chose was 2500 employees in 1976. The median number of employees in 1976 is 2300 and the geometric mean is 2700 (based on the 1349 firms that survive from 1972 to 1979), so there are roughly equal numbers in each class for the observed samples.

A summary of the results for these two size classes is presented in Table VI; these estimates are also computed with corrections for heteroskedasticity and sample selection. The results for the larger firms are not especially different from those for the smaller firms, although they are somewhat attenuated. A noteworthy feature of the estimates is the substantial difference in the variance of the growth rates across the two samples: in 1979, the ratio of the mean variances (after weighting by weights normalized to be unity on the average) is about 0.4. The estimated investment coefficients are not that different from those for the whole sample, although they have larger standard errors. The finding that a dollar of R & D expenditures is a more important predictor of growth in the immediate future than is a dollar of expenditure on physical capital is robust across size classes: the ratio of the amount required to obtain an increase in annual growth rates of one percent is 1.6 for the smaller firms and three for the larger firms.

I can imagine two different interpretations of this finding. First, R & D might be more highly correlated with future success of the firm, both because it is (possibly) more forward looking, and because R & D expenditures at the firm level tend to be substantially less volatile over time than expenditures on physical capital (Hall, Griliches, and Hausman [1986]). Second, the actual rate of return to R & D expenditures may simply be higher than that to capital expenditures for the usual reason that such expenditures have more non-diversifiable risk, and investors require a higher return for holding the stock of R & D-intensive firms. There is room here for further work.

VIII. CONCLUSION

The goal of this paper was to investigate several econometric explanations that have been suggested for the finding of a negative correlation between firm size and growth and to lay some groundwork for a more careful modelling of firm dynamics. With respect to the size-growth relationship, we have negative results in the sense that neither measurement error in employment nor sample attrition can account for the negative coefficient on firm size in the growth rate equation. There are large random changes in employment at any one firm from year to year, but these changes are largely permanent and do not reflect a non-serially correlated measurement error. Substantial differences in the variance of growth rates across size classes was also observed with smaller firms having a variance at least twice as large.

With qualifications due to the difficulty of constructing an adequate model of sample attrition, it does appear that the smaller firms in the sample grow faster, with a four percentage point difference in annual growth rates between firms in the 25th and 75th percentiles in size. Because of the large element of randomness in growth rates across firms from year to year, however, this difference is not enough to cause firms to move very far in the size distribution over a ten year period.

On the whole, correcting for attrition bias had very little effect on these results; this should give us some confidence that further research on this type of panel of firms can be done, at least over fairly short time periods, without fear of large biases in the results due to those firms that exit from the sample. It needs to be emphasized that this conclusion applies to this particular set of firms, and will not necessarily hold true for firms in a very different sector or size range.

I have also found that the obvious systematic differences among firms, such as industry and the level of investment, do very little to reduce the variance of growth rates. The best I could do was a reduction in the standard error from 12.6 percent to 12.1 percent (this conclusion is based on the unweighted data, since it is difficult to interpret the standard error after weighting). At the firm level, year-to-year growth rates in employment are largely unpredictable by past characteristics of the firm.

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