Economics 250a Lecture 10
Search Theory 2

Outline
a) The Burdett-Mortensen equilibrium wage-posting model (from Manning)
b) Brief mention - Christensen et al (2005)
c) Flinn (2006) equilibrium search/matching model with minimum wage

References
Kenneth Burdett and Dale Mortensen (1998) "Wage Differentials Employer Size and Unemployment" *International Econ Review* 39, pp. 257-273.
    Alan Manning (2003). *Monopsony in Motion.* Princeton: PU Press.
    Bent Christensen, Rasmus Lentz, Dale Mortensen and George Neumann (2005). "On the Job Search and the Wage Distribution". JOLE 23 (January 2005), pp. 31-58
    Christopher Flinn (2006). "Minimum Wage Effects on Labor Market Outcomes Under Search, Matching and Endogenous Contract Rates" 74 (July 2006), pp. 1013-1062.

Manning's simplification of Burdett-Mortensen's wage posting model
    The BM model is an equilibrium search model in which atomistic employers each choose (or post) a wage, and employees search on the job. The model generates an equilibrium distribution of wage offers, F, an associated distribution of accepted wages G, an equilibrium rate of unemployment (or strictly speaking non-employment) u. Firms with higher wages are larger and have a lower attrition rate (because fewer workers are bid away). Firms with lower wages are smaller and have higher attrition. All wage choices yield the same profit. The model is an extension of a famous paper by Burdett (1978) which first proposed the idea of job ladders – on the job search by employed who are looking for a wage at a higher wage firm.
    Economists differ in their enthusiasm for the BM model and related wage posting models. Some argue that it does not make sense for firms to refuse to re-negotiate with workers who find a better-paying job (i.e., a firm that posts a single wage is committing to *no offer-matching*). The empirical importance of offer matching is an open subject for research.

    As a prologue, some evidence of the importance of wage posting comes from a survey by Hall and Krueger (NBER #16033, May 2010): they find that only about 30% of workers report there was some bargaining in setting the wage for their current job. The rate is especially low for blue collar workers (5%) but much higher for knowledge workers (86%).

    We will follow Manning's exposition (MM, chapter 2). Key notation:
    -M_w workers, all equally productive, each has non-work option b
    -M_f firms, each has constant productivity per worker p.
    - $M = M_f/M_w$

- each firm offers wage w for all its workers, and maximizes steady state profits

-$F(w)$ = distribution of wages across firms (to be determined)

-employed and nonemployed workers receive offers randomly (from F) at rate λ. A GE variant could make λ endogenous.

-employed workers leave for nonemployment at rate δ

Steady State Behaviors

a) As in our basic model with on the job seach, non-employed workers accept any job offer paying more than b. Employed workers accept any job paying more than their current wage.

b) Steady-state profits

$$\pi(w; F) = (p - w)N(w; F)$$

where $N(w; F)$ is the steady state level of employment at a firm paying w given F. We are going to assume π is the same for all firms, which means small firms have to pay lower wages and make higher profit per worker. Many researchers have argued this is the opposite of what we observe in the data.

c) Balancing flows: firm w has separation rate $s(w; F)$ and recruiting flow $R(w; F)$. So in steady state

$$s(w; F)N(w; F) = R(w; F).$$

d) (no spikes). If $0 < \lambda/\delta < \infty$, $F(w)$ has no spikes (atoms). Why? If there is a spike at some wage $w_0 < p$ then another firm could guarantee a higher level of profit by offering a wage $w_0 + \epsilon$. This has only slightly lower profit per worker but a discretely higher recruiting rate, contradicting the equal profit condition. (If $w_0 = p$ then a firm could make strictly positive profit by lowering its wage).

e) Separation rate for firm paying w is:

$$s(w; F) = \delta + \lambda(1 - F(w)).$$

f) Steady state non-employment. The outflow from non-emp is $\lambda u M_w$, the inflow is $\delta(1 - u)M_w$, balance gives:

$$u = \frac{\delta}{\delta + \lambda} \Rightarrow \frac{u}{1 - u} = \frac{\delta}{\lambda}$$

These relationships holds in any steady state "flow" model and are useful to remember.

g) Distribution function of wages across workers (i.e., fraction of workers earning less than some wage w is:

$$G(w; F) = \frac{\delta F(w)}{\delta + \lambda(1 - F(w))} = \frac{F(w)}{1 + \frac{\lambda}{\delta}(1 - F(w))} < F(w) \quad (*)$$

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To prove this: consider the set of jobs with $\tilde{w} \leq w$. The size of the pool of workers in these jobs is $(1 - u)G(w)M_w$ (by definition of $G(w)$).

- net entry to this set is from the pool of unemployed. The inflow is $uM_w\lambda F(w)$ since a share $\lambda F(w)$ of unemployed get an offer from b to w.

- net exit from this set has two parts: job destruction flow=$\delta(1-u)G(w)M_w$ and exit to higher wage jobs= $\lambda(1 - u)G(w)(1 - F(w))M_w$.

- equating inflow and outflow we get:

$$u\lambda F(w) = \delta(1-u)G(w)+\lambda(1-u)G(w)(1-F(w)) = (1-u)G(w)(\delta+\lambda(1-F(w)))$$

Simplifying yields $(*)$. Note that $(*)$ implies as $\lambda \to 0$ $G \to F$.

h) Flow of recruits to the firm. A firm that pays w gets new workers from 2 sources. It gets a share of the non-employed who received an offer that does not depend on its wage: the flow rate from the non-employed pool is

$$\lambda u \frac{M_w}{M_f} = \frac{\lambda u}{M} \quad \text{where } M = \frac{M_f}{M_w}.$$

It also gets a share of all those who are employed at a wage less than w. The flow rate from this group is

$$(1 - u)\lambda G(w; F)\frac{M_w}{M_f} = \frac{(1 - u)\lambda G(w; F)}{M}$$

Thus

$$\begin{aligned} R(w; F) &= \frac{\lambda}{M}(u + (1 - u)G(w; F)) \\ &= \frac{\lambda}{M}\left(\frac{\delta}{\delta + \lambda} + \frac{\lambda}{\delta + \lambda}\frac{\delta F(w)}{\delta + \lambda(1 - F(w))}\right) \\ &= \frac{\lambda\delta}{M}\left(\frac{1}{\delta + \lambda(1 - F(w))}\right). \end{aligned}$$

Finally, using $sN = R$ and substituting we get

$$N(w; F) = \frac{R(w; F)}{s(w; F)} = \frac{\lambda\delta}{M[\delta + \lambda(1 - F(w))]^2}$$

Thus a firm that pays a higher wage will have more employees. Note that in this model:
$$R(w; F) = \frac{\lambda\delta}{M}\frac{1}{s(w; F)}$$

which implies that the elasticity of the recruiting rate with respect to the wage is just the negative of the elasticity of the separation rate with respect to the wage. Manning (p. 97) presents another derivation of this relation, which has to hold quite generally.

We are finally ready for the last step: substituting N into the equation for profits we get

$$\begin{aligned} \pi(w; F) &= (p-w)N(w; F) \\ &= \frac{\lambda\delta(p-w)}{M[\delta + \lambda(1 - F(w))]^2} \end{aligned}$$

In equilibrium all offered wages give the same level of profit, and no other possible wages yield higher profit. To solve for the equilibrium level of profit, BM (and Manning) show that the lowest wage offered in equilibrium is b. The basic point of the proof is that if the lowest equilibrium wage offered is above b, then a firm at this position could lower its wage and get the same flow of recruits (all of whom are coming from non-employment) and have a lower wage - so there would be a contradiction. Using this fact, we get

$$\pi(w; F) = \pi(b; F) = \frac{\lambda\delta(p-b)}{M[\delta + \lambda]^2} = \frac{u(1-u)(p-b)}{M}$$

Finally, then, once can solve for $F(w)$. The results are:

$$b \le w \le p - \left(\frac{\delta}{\delta + \lambda}\right)^2 (p - b)$$

$$\begin{aligned} F(w) &= \frac{\delta + \lambda}{\lambda}\left(1 - \sqrt{\frac{p-w}{p-b}}\right) \\ G(w) &= \frac{\delta}{\lambda}\left(\sqrt{\frac{p-b}{p-w}} - 1\right) \\ E[w] &= \frac{\delta}{\delta + \lambda}b + \frac{\lambda}{\delta + \lambda}p \end{aligned}$$

Christensen et al. (2005)

CLMN use a variant of the BM setup, and estimate some of the underlying parameters using a relatively simple database from Denmark that contains wages earned by workers at each firm in 12-month period, the "exit rate" of workers from each firm, and the number of people hired at the firm who were previously not working. The twist is that now each worker chooses a search intensity. In general, search intensity will be decreasing with the current wage. They do not try to "explain" how higher and lower wage firms can co-exist.

a) They use the wage distribution for people who were hired from non-employment to estimate the distribution $F(w)$.

b) they use the exit hazard rate of workers at firms with different wages to infer the (normalized) seach intensity $\lambda(w)$. As in the baseline BM model, net exit from a firm that pays a wage w has two components: job destruction

which is independent of wages at rate δ; and exit to higher wage jobs, at rate $\lambda(w)(1 - F(w))$. Thus the exit hazard is

$$d(w) = \delta + \lambda(w)(1 - F(w))$$

The "hard work" in the paper is to get an expression for $\lambda(w)$, which depends on the optimal choice of search intensity by a worker who is receiving a wage w and gets offers from a distribution $F(w)$. This is derived as follows.

Let $W(w)$ denote the value function for a job paying wage w, and let U denote the value of unemployment. The continuous time Bellman equation is:

$$rW(w) = max_s w - c(s) + \lambda_0 s \int (max[W(x), W(w)] - W(w))\, dF(x) + \delta(U - W(w))$$

Note that the net arrival rate of offers is $\lambda_0 s$ where λ_0 is some scale factor. Re-organizing this can be written as

$$W(w) = max_s \left(\frac{w - c(s) + \delta U + \lambda_0 s \int max[W(x), W(w)] dF(x)}{r + \delta + \lambda_0 s} \right)$$

Next, using the envelope theorem we can show that:

$$W'(w) = \frac{1}{r + \delta + \lambda_0 s(w)(1 - F(w))}$$

Finally, the FOC for optimal search (using integration by parts) is:

$$
\begin{aligned}
c'(s) &= \lambda_0 \int_w^\infty [W(x) - W(w)] dF(x) \\
&= \lambda_0 \int_w^\infty [W'(x)(1 - F(x)) dx \\
&= \lambda_0 \int_w^\infty \frac{(1 - F(x)) dx}{r + \delta + \lambda_0 s(w)(1 - F(w))}
\end{aligned}
$$

CLMN assume $c(s)$ is a convex function with constant elasticity, so $c'(s) = c_0 s^{1/\gamma}$ so this solves out to

$$\lambda(w) \equiv \lambda_0 s(w) = K \left[\int_w^\infty \frac{(1 - F(x)) dx}{r + \delta + \lambda(w)(1 - F(w))} \right]^\gamma$$

(where K depends on c_0 and λ_0). This is a functional equation for $\lambda(w)$.

So CLMN actually estimate the exit hazard $d(w)$ and given $F(w)$ solve for (δ, K, γ) assuming r is known.

Flinn's search-matching model with a minimum wage

In the macro search literature it is common to assume that the motive for search is not the job ladder phenomenon, but rather the search for a better idiosyncratic "match". Flinn presents a search-matching model that can be used to model the effect of the minimum wage. The model also illustrates how to make the job arrival rate endogenous via a "vacancy creation" equation.

Notation:

-θ = value of the match, d.f. $G(\theta)$; otherwise all workers and firms homogenous

-ρ = discount rate

-η = job destruction rate

-b = flow utility while searching

-λ = arrival rate of offers – will be endogenized later

-no on the job search.

Start by assuming no minimum wage:

-firm profit if employing a worker with match θ at wage $w = \theta - w$, value to firm $= \frac{\theta - w}{\rho + \eta}$.

-value functions for worker $V_n, V_e(w)$ if searching, unemployed

-reservation wage w^* will satisfy $V_n = V_e(w^*)$

-Bellman equations for a worker:

$$
\begin{aligned}
\rho V_n &= b + \lambda \int_{w^*} (V_e(w) - V_n) f(w) dw \\
(\rho + \eta) V_e(w) &= w + \eta V_n
\end{aligned}
$$

Where $f(w)$ is the density of wages. Some manipulations establish:

$$
\begin{aligned}
\rho V_n &= w^* \\
V_e(w) - V_n &= \frac{w}{\rho + \eta} - \frac{\rho}{\rho + \eta} V_n = \frac{w - w^*}{\rho + \eta}
\end{aligned}
$$

What happens when a searching worker meets a firm and the value of the match is θ? Assume they conduct Nash bargaining: choose a wage to maximize

$$
(V_e(w) - V_n)^\alpha \left(\frac{\theta - w}{\rho + \eta} \right)^{1-\alpha}
$$

$$
= \left(\frac{w - w^*}{\rho + \eta} \right)^\alpha \left(\frac{\theta - w}{\rho + \eta} \right)^{1-\alpha}
$$

which leads to a split the rents model with worker share α :

$$
w = w^* + \alpha(\theta - w^*)
$$

The lowest match that will ever be considered is $\theta = \theta^* = w^* = \rho V_n$. So we can also write the wage when the match value is θ as:

$$
w = w^* + \alpha(\theta - \rho V_n).
$$

Now from the relations above:

$$V_e(w) - V_n = \frac{w - w^*}{\rho + \eta} = \frac{\alpha(\theta - \rho V_n)}{\rho + \eta}$$

Finally we can rewrite the expression for V_n :

$$
\begin{aligned}
\rho V_n &= b + \lambda \int_{w^*} (V_e(w) - V_n) f(w) dw \\
&= b + \lambda \int_{\theta^*} \frac{\alpha(\theta - \rho V_n)}{\rho + \eta} dG(\theta) \\
&= b + \frac{\lambda \alpha}{\rho + \eta} \int_{\rho V_n} (\theta - \rho V_n) dG(\theta)
\end{aligned}
$$

which can be solved for ρV_n, given $b, \lambda, \alpha, \rho, \eta$ and $G(\theta)$. Notice that this creates a distribution of wages from G.

Add a minimum wage

With a minimum wage m the worker's value of search is $V_n(m)$, which we will have to solve for. As before assume there is a rent-splitting wage process. Then ignoring the minimum wage the wage when the value of the match is θ would be:

$$w = \alpha\theta + (1 - \alpha)\rho V_n(m).$$

Define $\widehat{\theta}$ as the value such that

$$m = \alpha\widehat{\theta} + (1 - \alpha)\rho V_n(m)$$

For $\theta > \widehat{\theta}$ the minimum wage is not a problem. But for a range of lower values the minimum is binding. Assuming $\widehat{\theta} > m$ there is a range of $\theta's$ that are efficient (ie, have match value at least as big as the minimum) but under the ordinary wage model would be paid less than the minimum. Flinn assumes these matches are consumated and the wage is set to m, generating a spike at the minimum wage. Very nice idea! The picture is here:

The value of unemployment is now:

$$
\rho V_n(m) = b + \frac{\lambda}{\rho + \eta} \int_m^{\widehat{\theta}} (m - \rho V_n(m)) dG(\theta) \tag{1}
$$

$$
+ \frac{\lambda \alpha}{\rho + \eta} \int_{\widehat{\theta}} (\theta - \rho V_n(m)) dG(\theta) \tag{2}
$$

The presence of the minimum wage creates a wedge between $V_n(m)$ and $V_e(m)$. The lowest-wage job is now more valuable than unemployment (whereas

in a standard model the job that is just acceptable has the same value as continuing to search). This is an interesting feature of the model to think about.

Equilibrium

Now we are going to 'endogenize' λ. We have some additional notation and assumptions

- participation equation: $Q(\rho V_n)$ – people decide to enter labor force based on ρV_n

-ℓ = fraction of workers who participate (either work or search)

$$\ell = Q(\rho V_n(m))$$

-$\widetilde{u}$ = total number (mass) of unemployed

-v = total number (mass) of vacancies

-$k = \widetilde{u}/v$ = ratio of searchers to job openings

-$m(\widetilde{u}, v)$ = matching function = flow rate of matches. Standard c.r.s. assumption on $m(.)$:

$$m(\widetilde{u}, v) = vq(\widetilde{u}/v) = vq(k)$$

for some increasing function $q(.)$. With this assumption we get the arrival rate of offers (to workers) is:

$$\lambda = \frac{m(\widetilde{u}, v)}{\widetilde{u}} = \frac{q(k)}{k}$$

and the job filling rate is:

$$\frac{m(\widetilde{u}, v)}{v} = q(k).$$

What determines v? Assume firms can create a vacancy for cost ψ. The expected value of a vacancy is

$$\rho V_v = -\psi + q(k)(1 - G(m))(J - V_v)$$

where J = the expected profits of a consumated match. If we assume $V_v = 0$ (vacancies are created until the net profit is 0), we get

$$\psi = q(k)(1 - G(m))J \tag{3}$$

$$\Rightarrow \quad \frac{1}{v} = \widetilde{u}q^{-1}\left(\frac{\psi}{J(1 - G(m))}\right). \tag{4}$$

For a given amount of unemployment and values for ψ and J this gives the amount of vacancies created.

What is J? For any given match the firm's expected discounted profit is $J(\theta)$, where

$$(\rho + \eta)J(\theta) = (\theta - w(\theta)) + \eta V_v.$$

With $V_v = 0$, we get

$$
\begin{aligned}
J(\theta) &= \frac{\theta - w(\theta)}{\rho + \eta} \\
&= \frac{\theta - m}{\rho + \eta} \text{ if } \theta \leq \widehat{\theta} \\
&= (1 - \alpha)\frac{(\theta - \rho V_n(m))}{\rho + \eta} \text{ if } \theta > \widehat{\theta}
\end{aligned}
$$

Thus

$$
J = E[J(\theta)|\theta \geq m].
$$

Given $V_n(m)$ we can find J.

Finally, what is $\widetilde{u}$? Recall that in a model where U unemployed people have a job finding rate of f and $E = L - U$ employed people have a job-losing rate of s that in steady state $s(L - U) = Uf$, implying that the steady state unemployment rate is $u = U/L = f/(f + s)$. In this model the job loss rate is η and the job finding rate is $\lambda(1 - G(m)) = \frac{q(k)}{k}(1 - G(m))$. So the unemployment rate is

$$
u = \frac{\eta}{\eta + \frac{q(k)}{k}(1 - G(m))}.
$$

If the size of the labor force is ℓ then

$$
\widetilde{u} = u\ell = \frac{\eta}{\eta + \frac{q(k)}{k}(1 - G(m))}Q(\rho V_n(m)) \tag{5}
$$

So now we are ready to discuss the equilibrium. The primitives are

$$
\rho, b, \eta, \alpha, G(.), Q(.), q(.), \psi, m
$$

The endogenous variables are

$$
\ell, u, v, \rho V_n(m)
$$

So note that in contrast to the 'partial equilibrium' case, we now have to solve for u, v. Flinn notes that there is a simple recursive algorithm:
1. choose a value for λ
2. using equation 4 (above) solve for $x = \rho V_n(m)$
3. given x find $\ell = Q(x)$, and also solve for J
4. using equation 7 (above) solve for $\widetilde{u}$
5. using equation 6 (above) solve for v
6. this generates a new value of $\lambda = q(\frac{\widetilde{u}}{v})/(\frac{\widetilde{u}}{v})$.