Economics 250a Lecture 9 Search Theory – some basic models and applications

Outline

0) what is unemployment?

a) basic search model

b) continuous time variant

c) model with on-the-job search

d) the recent empirical literature

References

There are many sources for basic search theory. You can find good notes on the web by Randy Wright, Daron Acemoglu, etc. A few of the papers you should look at:

Dale Mortensen (1977). "Unemployment Insurance and Job Search Decisions" Industrial and Labor Relations Review 30, pp. 505-517

Dale Mortensen and Christopher Pissarides (1999). "New Developments in Models of Search in the Labor Market." In Ashenfelter and Card (eds) *Handbook* of Labor Economics. Amsterdam: Elsevier, volume 3B.

David Card and Dean Hyslop (2005). "Estimating the Effects of a Time-Limited Earnings Subsidy for Welfare-Leavers" *Econometrica*, 73, pp. 723-1770.

Other papers we'll mention:

N. Kiefer and G. Neumann (1979) "An Empirical Job Search Model with a Test of the Constant Reservation Wage Hypothesis". *JPE* 87, pp. 89-107.

David Card, Raj Chetty and Andrea Weber (2007). "Cash-On-Hand and Competing Models of Intertemporal Behavior: New Evidence from the Labor Market" *Quarterly Journal of Economics* 122, pp. 1511-1560.

Johannes Schmieder, Till von Wachter and Stefan Bender. (2012) "The Effects of Extended Unemployment Insurance over the Business Cycle: Evidence from Regression Discontinuity Estimates over Twenty Years." *QJE* 127(2). Available at von Wachter's web site.

0) What is unemployment?

Unemployment is conventionally defined as not working (in some specified period, like a week), but "actively" looking for work. This conception arose in the 1930's as statisticians at the WPA struggled with a "sensible" way to measure unemployment. In the US, unemployment is measured in the monthly Current Population Survey. Unemployed people are those over 16 who were not working in the survey week, but actively looked for work in the past month and were available for work. Note that "discourged workers" who are available for work but have given up looking are not counted as unemployed. The BLS publishes alternative unemployment rates that include these and other groups - marginally attached, and the "under-employed".

The figures at the end of the lecture show some "recent" data (1978+) on levels of unemployment and the median duration of the "spells" that are captured in progress in the labor force interview.

a) The simplest possible discrete-time search model

The leading theoretical explanation for unemployment is the neoclassical search model, developed in the 1960s-1970s (many years after the WPA's work).

A currently unemployed individual is infinitely lived, risk neutral, and has a discount factor $\beta = \frac{1}{1+r}$. Each period the individual receives one job offer which he/she can accept or reject. If a job is accepted it starts at the beginning of next period and lasts forever. There is no "on-the-job search". (We'll address that in the next model). Wages offers are drawn from a distribution F(w) with p.d.f. f(w). An individual who works at wage w has a flow utility of w – so we are implicitly normalizing the cost of work as 0. An unemployed individual receives a (flow) benefit b and has a flow utility of $b - \nu$. So ν is the relative cost of search, which will be negative if search is less onerous than working. Let V denote the value of an optimal plan from the current period. Clearly V does not depend on time (i.e., we have a stationary value function). Moreover, if an individual with a value V is offered a wage with

$$d.p.v.$$
 of job $= \frac{w}{1-\beta} > V$

then he/she should accept it. This means we can set up a simple Bellman equation:

$$V = b - \nu + \beta \int_0^\infty \max(V, \frac{w}{1 - \beta}) f(w) dw$$
$$= b - \nu + \beta \int_0^\infty \left(V + \max(0, \frac{w}{1 - \beta} - V) \right) f(w) dw$$

which implies that

$$V(1-\beta) = b - \nu + \beta \int_0^\infty \max(0, \frac{w}{1-\beta} - V) f(w) dw$$
$$= b - \nu + \frac{\beta}{1-\beta} \int_0^\infty \max(0, w - (1-\beta)V) f(w) dw$$

Define w^* by $V = \frac{w^*}{1-\beta}$. This is the lowest wage the individual will accept, or the "reservation wage". Plugging in and simplifying the integral we get:

$$w^* = b - \nu + \frac{1}{r} \int_{w^*}^{\infty} (w - w^*) f(w) dw$$
(1)

which can be re-stated as

$$w^* - (b - \nu) = P(w \ge w^*) \times \frac{E(w - w^*|w \ge w^*)}{r}$$

The l.h.s is the foregone income from rejecting an offer at $w = w^*$. The r.h.s is the value of waiting one more period and sampling again, in which case with

probability $P(w \ge w^*)$ a new acceptable wage is drawn, yielding added income $E(w-w^*|w\ge w^*)$ in each period into the future (hence the demoninator term r which converts to an annuity). Note that in this model each agent has a constant reservation wage w^* , regardless of how long they have been unemployed. The escape rate from unemployment (or "exit hazard") is controlled by the choice of w^* and is just

$$exit \ hazard = P(w \ge w^*) = 1 - F(w^*).$$
 (2)

The content of this model is in the way that $\{F(w), b, c\}$ determine w^* . Unfortunately, we don't really see wage offer distributions (or even draws from this distribution in most data sets). So we have to focus on other implications. Notice that we can differentiate (1) w.r.t. b to get

$$\begin{array}{lcl} \displaystyle \frac{\partial w^*}{\partial b} & = & \displaystyle 1 - \left(\frac{1}{r}\int_{w^*}^{\infty}f(w)dw\right)\frac{\partial w^*}{\partial b} \\ \\ \quad \Rightarrow & \displaystyle \frac{\partial w^*}{\partial b} > 0 \end{array}$$

So there is a predicted positive effect of b on w^* . Using (2), then, the exit hazard rate from unemployment will be lower when b is raised (or ν is lowered).

Aside: the Pareto distribution

A trick that is sometimes used in the literature is to note that

$$\int_{w^*}^{\infty} (w - w^*) f(w) dw = \int_{w^*}^{\infty} (1 - F(w)) dw$$

This can be shown by applying integration by parts to the l.h.s., for an upper bound m:

$$\int_{w^*}^m (w - w^*) f(w) dw = (m - w^*) F(m) - \int_{w^*}^m F(w) dw = \int_{w^*}^m (F(m) - F(w)) dw$$

and now set $m = \infty$, so F(m) = 1. For a Pareto distribution $F(w) = 1 - (a/w)^{\gamma}$ for w > a. This means its pretty easy to compute the reservation wage assuming Pareto-distributed wage offers.

c) A Continuous Time Variant

A lot of search models are written in continuous time. We'll show how to do this taking a limit of a discrete time model as the length of each interval, htends to 0. The setup is the same as above except now the discount factor is $\beta(h) = e^{-rh}$, and w, b, ν are all instantaneous flow rates. We'll also assume that the number of offers received is Poisson distributed with arrival rate λ . This means that in an interval of length h the expected number of offers is λh and¹

$$P(0 \text{ offers}) = e^{-\lambda h}$$

$$P(1 \text{ offer}) = \lambda h e^{-\lambda h}$$

$$P(2 \text{ offers}) = (\lambda h)^2 e^{-\lambda h}/2...$$

¹Recall that when a random variable $x \in \{0, 1, ...\}$ is distributed as Poisson with parameter λ then $E[x] = \lambda$, and $P(x = \mathbf{x}) = \lambda^{\mathbf{x}} e^{-\lambda} / \mathbf{x}!$

Using these assumptions the Bellman equation can be written as

$$V = (b-\nu)h + e^{-rh} \left(e^{-\lambda h}V + \lambda h \ e^{-\lambda h} \int_0^\infty \max(V, \frac{w}{r}) f(w) dw \right)$$

+terms of order h^2 or higher

Ignoring the higher order terms (since we are going to let $h \to 0$) and following similar steps as above we get

$$V = (b-c)h + e^{-(r+\lambda)h}V + e^{-(r+\lambda)h}\lambda h V + e^{-(r+\lambda)h}\lambda h \int_0^\infty \max(0, \frac{w}{r} - V)f(w)dw \Rightarrow V\left(\frac{1 - e^{-(r+\lambda)h} - e^{-(r+\lambda)h}\lambda h}{h}\right) = b - \nu + e^{-(r+\lambda)h}\lambda \int_0^\infty \max(0, \frac{w}{r} - V)f(w)dw$$

Now we take the limit and use the facts that

$$\lim_{h \to 0} \frac{1 - e^{-(r+\lambda)h}}{h} = r + \lambda$$
$$\lim_{h \to 0} e^{-(r+\lambda)h}\lambda = \lambda$$

we get

$$rV = b - \nu + \lambda \int_0^\infty \max(0, \frac{w}{r} - V) f(w) dw$$

which is often the way people write down the Bellman equation to start. Finally, defining $w^* = rV$ as the reservation wage we can write this as

$$w^{*} = (b - \nu) + \frac{\lambda}{r} \int_{w^{*}}^{\infty} (w - w^{*}) f(w) dw$$
(3)

which is the same as (1) except that there is an arrival rate λ in front of the integral on the right. Another way of deriving this is to go back to a discrete time model and assume the person gets 1 offer with probability λ and no offers otherwise. Note that w^* is increasing in the arrival rate.

Equation (3) has the form $w^* = T(w^*)$. It can be shown that T is a contraction, so there is a very simple algorithm for computing w^* : simply start with a guess w^1 and compute $w^2 = T(w^1)$. Then keep iterating. Or you can perform the integration on the r.h.s. for a grid of values of w^* and find the (approximate) value such that $w^* = T(w^*)$. In the first figure at the end of the lecture I show how this looks assuming a normal distribution for wages.

c) A discrete time model with on-the-job search

Now we move a little closer to reality by assuming that people can receive job offers even when working. (In fact an interesting question is whether having a job raises or lowers the arrival rate of offers, but we are going to ignore that). As before, let b represent the net benefit (in \$) received by an unemployed person. Let c represent the disutility cost of work (vs. unemployment/search), so a person with wage w receives a net income flow of w - c, whereas a person who is searching receives a flow utility of b. We'll assume that there is a probability λ of an a new offer in each period (whether working or not) and a probability δ that any job dissolves (so δ is the "rate of job destruction"). Now there will be 2 value functions: a value V if unemployed, and a value function U(w)associated with holding a job that pays w.

Consider someone who enters the period with a job paying w. (To simplify algebra we will assume the pay comes at the end of the period). With probability λ they get a new offer \tilde{w} , which they will accept at the end of the period if $\tilde{w} > w$, which has probability 1 - F(w). With probability δ the new job disappears before it even starts and the person ends up at the end of the period with V. With probability $(1 - \delta)$ the job actually opens up and the individual ends up at the end of the period with $U(\tilde{w})$. The expected value of an acceptable job is

$$\int_w^\infty U(\widetilde{w}) \frac{f(\widetilde{w})}{1 - F(\widetilde{w})} d\widetilde{w}$$

Alternatively with probability $(1-\lambda)$ they get no offer. Thus the net probability of either getting no offer or getting an unacceptable offer is $(1-\lambda) + \lambda F(w) =$ $1 - \lambda(1 - F(w))$. With probability δ the (old) job disappears and the person ends up and the end of the period at V. With probability $(1 - \delta)$ it persists and the individual ends up at U(w) at the end of the period.

Combining all these thoughts (and assuming a discount rate 1/(1+r)) we get:

$$U(w) = \frac{w-c}{1+r} + \frac{1}{1+r}\lambda(1-F(w))\left[\delta V + (1-\delta)\int_{w}^{\infty} U(\widetilde{w})\frac{f(\widetilde{w})}{1-F(\widetilde{w})}d\widetilde{w}\right] + \frac{1}{1+r}(1-\lambda(1-F(w))\left[\delta V + (1-\delta)U(w)\right].$$

If you stare at this you will see why it simplifies things to assume that some of the newly accepted jobs actually end before they start. Some simplification yields:

$$U(w) = \frac{w-c}{r+\delta} + \frac{\delta}{r+\delta}V + \frac{\lambda(1-\delta)}{r+\delta}\int_{w}^{\infty} (U(\widetilde{w}) - U(w))f(\widetilde{w})d\widetilde{w}$$
(4)

For an unemployed worker with a reservation wage w^* the same logic as above will yield:

$$V = \frac{b}{1+r} + \frac{1}{1+r}\lambda(1-F(w^*)) \left[\delta V + (1-\delta)\int_{w^*}^{\infty} U(\tilde{w})\frac{f(\tilde{w})}{1-F(\tilde{w})}d\tilde{w}\right] + \frac{1}{1+r}(1-\lambda(1-F(w^*))V.$$

Simplifying we get:

$$V = \frac{b}{r} + \frac{\lambda(1-\delta)}{r} \int_{w^*}^{\infty} (U(\widetilde{w}) - V) f(\widetilde{w}) d\widetilde{w}$$
(5)

Also, we must have $U(w^*) = V$, which is what it means for w^* to be the reservation wage. Look back at expression (4) for U(w), and evaluate at $w = w^*$. We get

$$V = U(w^*) = \frac{w^* - c}{r + \delta} + \frac{\delta}{r + \delta}V + \frac{\lambda(1 - \delta)}{r + \delta}\int_{w^*}^{\infty} (U(\widetilde{w}) - V)f(\widetilde{w})d\widetilde{w}$$

$$\Rightarrow V(1 - \frac{\delta}{r + \delta}) = \frac{w^* - c}{r + \delta} + \frac{\lambda(1 - \delta)}{r + \delta}\int_{w^*}^{\infty} (U(\widetilde{w}) - V)f(\widetilde{w})d\widetilde{w}$$

$$\Rightarrow V = \frac{w^* - c}{r} + \frac{\lambda(1 - \delta)}{r}\int_{w^*}^{\infty} (U(\widetilde{w}) - V)f(\widetilde{w})d\widetilde{w}$$

Comparing this to (5) we see that $w^* = b + c$. The reservation wage is just the benefit amount, plus the extra disutility of work versus unemployment. The reason is that there is no "opportunity cost" of taking a job: it does not slow down the arrival of offers so you might as well take any job with $w \ge b + c$ while you wait for something better.

What does U(w) look like? From equation (4), for higher values of w:

$$U(w) = \frac{w-c}{r+\delta} + \frac{\delta}{r+\delta}V + "little bit"$$
$$= \frac{w}{r+\delta} + \frac{\delta V - c}{r+\delta} + "little bit"$$

which is a linear term in w, plus a constant $\frac{\delta V-c}{r+\delta}$ plus the "*little bit*" which is the option value of a better job coming along. As w rises this option value is smaller and smaller. For low values of the wage the option value term can be sizeable (if there is a lot of variation in offers out there)

$$U(w) = \frac{w}{r+\delta} + \frac{\delta V - c}{r+\delta} + \frac{\lambda(1-\delta)}{r+\delta} \int_w^\infty (U(\widetilde{w}) - U(w)) f(\widetilde{w}) d\widetilde{w}$$

See the second figure at the end of the notes.

How do we solve for U(w)? Start with equation (4) and use the fact that V = U(b + c), yielding:

$$U(w) = \frac{w-c}{r+\delta} + \frac{\delta}{r+\delta}U(b+c) + \frac{\lambda(1-\delta)}{r+\delta}\int_w^\infty (U(\widetilde{w}) - U(w))f(\widetilde{w})d\widetilde{w}.$$

This is a *functional equation* of the form:

$$U(w) = T\{U(v)\}$$

i.e., T maps from the space of functions to the space of functions, and we are looking for a fixed point of T. Provided that T is a contraction mapping we can solve using a "successive approximation" approach. This is particularly easy when f(w) has a discrete support $\{w^1, w^2, ..., w^n\}$ and associated probabilities $\{\pi^1, \pi^2, \dots \pi^n\}$. Start at an initial guess for U(w), say $U^1(w)$ (which is just a list of utilities assigned to each point of support). Then working backward from the highest wage w^n , find the next set of guesses $U^2(w^k)$:

$$U^{2}(w^{n}) = \frac{w^{n} - c}{r + \delta} + \frac{\delta}{r + \delta}U^{1}(b + c),$$

$$U^{2}(w^{n-1}) = \frac{w^{n-1} - c}{r + \delta} + \frac{\delta}{r + \delta}U^{1}(b + c) + \frac{\lambda(1 - \delta)}{r + \delta}(U^{1}(w^{n}) - U^{1}(w^{n-1}))\pi^{n}$$

This will converge if the discount rate and rate of job destruction are large enough, and the arrival rate of offers is not too large.

Nonstationarity

What happens if b is not a fixed number, but (for example) has a high value b_H for the first T periods of unemployment and a lower value b_L thereafter? From T onward the analysis above applies and the person has a reservation wage $w^* = b_L + c$. But before that, there is a value of unemployment V(d) that depends on duration (d). It is possible to work backward from period T to think about how V(d) and the associated reservation wage $w^*(d)$ evolve. See the third figure at the end of the notes.

d) Some Recent Empirical Studies

Card, Chetty and Weber use regression discontinuity designs to study the effects of lump sum severence payments (payable after 3 years on the job) and extended UI benefits (+10 extra weeks relative to base of 20 weeks, awarded if 36+ months of work in past 5 years) in Austria. They find that receipt of severance pay leads to longer time spent between jobs (not something that can be explained by the basic search model); while longer benefit entitlement leads to longer time out of work. See the graphs and figures at end of lecture.

Schmeider et al use RD designs to study the effects of extended UI benefits for German workers (eligibility based on age) in different years, and test whether the "entitlement effect" is larger or smaller when the labor market is stronger or weaker. See the graphs and tables at end.

These two papers show relatively small effects of benefit extensions on lost work time (e.g., $\frac{dTime-to-New-Job}{dDuration} = .1$). Schmeider et al show that the effect does not vary much with the state of the labor market.

Unemployment Rates Across States



Unemployment Rates of Older Workers (Age 45+) by Gender



Unemployment Rates by Gender (Ages 16-24)





Median Duration of In-Progress Unemployment Spells - Monthly CPS Data

Note: seasonally adjusted data

Reservation Wage Calculations



Value Function U(w) with on-the-job search





NOTE-These figures plot average nonemployment durations (time to next job) in each tenure-month category. They exclude observations with nonemployment durations of more than two years and ignore censoring. The vertical line denotes the cutoff for severance pay eligibility. Figure 5a uses the full sample. Figure 5b uses the "restricted sample" of individuals who have been employed at another firm (besides the one from which they were most recently laid off) for at least one month within the past five years.







TABLE 2

	(1) Restricted sample	(2) Restricted sample	(3) Full sample	(4) With controls
Severance pay	-0.127 (0.019)		-0.125 (0.017)	-0.115 (0.018)
Extended benefits		-0.084 (0.018)	-0.093 (0.016)	-0.064 (0.017)
Sample size	512,767	512,767	650,922	565,835

Effects of Severance Pay and EB on Durations: Hazard Model Estimates

NOTE--All specifications report estimates of Cox hazard models for nonemployment durations (time to next job) censored at twenty weeks; hence, coefficient estimates can be interpreted as percent change in average job finding hazard over first twenty weeks of the spell. Specifications 1 and 2 are estimated on the restricted sample of individuals who worked at another firm for at least one month within the past five years. Specification 1 includes an indicator for severance pay eligibility and a cubic polynomial for job tenure interacted with severance indicator. Specification 2 includes an indicator for extended-benefit eligibility and a cubic polynomial for months worked in past 5 years interacted with EB indicator. Specifications 3 and 4 report estimates of model specified in equation (15), with cubic polynomials for both job tenure and months worked interacted with severance pay and EB indicators. Specifications 3 and 4 are estimated on the full sample, defined in notes to Table 1. Specification 4 includes the following additional controls: gender, marital status, Austrian nationality, "blue collar" occupation indicator, age and its square, log previous wage and its square, and dummies for month and year of job termination. Standard errors shown in parentheses.



Figure 1: Potential Unemployment Insurance Durations by Period for Workers with High Prior Labor Force Attachment

Notes: The figure shows how potential unemployment insurance (UI) durations vary with age and over time for unemployed individuals workers who had worked for at least 52 months in the last 7 years without intermittent UI spell.

Figure 3: The Effect of Potential Duration in Unemployment Insurance (UI) Benefits on Months of Actual UI Benefit and Months of Nonemployment by Age - Period 1987 to 1999



(b) Nonemployment Duration

Notes: The top figure shows average durations of receiving UI benefits by age at the start of unemployment insurance receipt. The bottom figure shows average non-employment durations for these workers, where non-employment duration is measured as the time until return to a job and is capped at 36 months. Each dot corresponds to an average over 120 days. The continuous lines represent polynomials fitted separately within the respective age range. The vertical lines mark age cutoffs for increases in potential UI durations at age 42 (12 to 18 months), 44 (18 to 22 months) and 49 (22 to 26 months). The sample are unemployed worker claiming UI between July 1987 and March 1999 who had worked for at least 52 months in the last 7 years without intermittent UI spell.

Table 2: Regression Discontinuity Estimates of Potential Unemployment Insurance
(UI) Benefit Duration (P) on Months of Actual UI Benefit Receipt and Months of
Nonemployment

	(1)	(2)	(3)	(4)			
	Age bandwidth around age dis 2 years 1 year 0.5 year		nd age discol 0.5 years	0.2 years			
Panel A: Dependent Variable: Duration of UI Benefit receipt (B)							
D(age>=42)	1.78 [0.036]**	1.82 [0.052]**	1.73 [0.072]**	1.65 [0.11]**			
Effect of 1 add. Month of Benefits $\frac{dB}{dP}$	0.30	0.30	0.29	0.28			
Observations	452749	225774	112436	45301			
D(age>=44)	1.04 [0.047]**	1.16 [0.065]**	1.13 [0.092]**	1.24 [0.15]**			
Effect of 1 add. Month of Benefits $\frac{dB}{dP}$	0.26	0.29	0.28	0.31			
Observations	450280	225134	112597	45258			
D(age>=49)	1.40 [0.074]**	1.44 [0.084]**	1.44 [0.12]**	1.72 [0.18]**			
Effect of 1 add. Month of Benefits $\frac{dB}{dP}$	0.35	0.36	0.36	0.43			
Observations	329680	217942	109238	43812			
Panel B: Dependent Variable: Nonemployment Duration (D)							
D(age>=42)	0.78 [0.086]**	0.92 [0.12]**	1.04 [0.17]**	0.79 [0.27]**			
Effect of 1 add. Month of Benefits $\frac{dD}{dP}$	0.13	0.15	0.17	0.13			
Observations	452749	225774	112436	45301			
D(age>=44)	0.41 [0.089]**	0.63 [0.13]**	0.62 [0.18]**	0.78 [0.30]*			
Effect of 1 add. Month of Benefits $\frac{dD}{dP}$	0.10	0.16	0.15	0.20			
Observations	450280	225134	112597	45258			
D(age>=49)	0.43 [0.11]**	0.52 [0.13]**	0.56 [0.19]**	0.79 [0.29]**			
Effect of 1 add. Month of Benefits $\frac{dD}{dP}$	0.11	0.13	0.14	0.20			
Observations	329680	217942	109238	43812			

Notes: The coefficients estimate the magnitude of the change in benefit or Nonemployment duration at the age threshold. Each coefficient is estimated in a separate RD regression that controls linearly for age with different slopes on each side of cutoff. Standard errors (in parentheses) are clustered at the day level (* P < .05, ** P < .01).

At the age 42 discontinuity potential UI benefit durations (P) increase from 12 to 18 months, at the age 44 discontinuity from 18 to 22 months and at the age 49 discontinuity from 22 to 26 months. The sample consists of individuals starting unemployment insurance spells between July 1987 and March 1999, who had worked for at least 52 months in the last 7 years without intermittent UI spell. For the age 49 cutoff and bandwidth 2 years column, the regression only includes individuals 47 and older and younger than 50, due to the early retirement discontinuity at age 50 (see text).





(a) Effect of Pot. UI Durations on Nonemployment Durations $\frac{dD}{dP}$ vs. Change in Unemployment Rate



(b) Effect of Pot. UI Durations on Actual UI Durations $\frac{dB}{dP}$ vs. Change in Unemployment Rate

Notes: Each dot in the bottom figure corresponds to a rescaled marginal effect of one month additional potential UI duration estimated at an age cutoff in one year between 1987 and 2004 at any of the available cutoffs (42, 44, 45, 47, and 49). The horizontal lines are the regression lines from a regression of the estimated marginal effects on the change in the unemployment rate from year t-1 to t. The samples are described in Figures 2 and 4.