Toward a Perception-Based Theory of Probabilistic Reasoning

Professor Lotfi A. Zadeh

Abstract

The past two decades have witnessed a dramatic growth in the use of probability-based methods in a wide variety of applications centering on automation of decision-making in an environment of uncertainty and incompleteness of information.

Successes of probability theory have high visibility. But what is not widely recognized is that successes of probability theory mask a fundamental limitation--the inability to operate on what may be called perception-based information. Such information is exemplified by the following. Assume that I look at a box containing balls of various sizes and form the perceptions: (a) there are about twenty balls; (b) most are large; and (c) a few are small. The question is: What is the probability that a ball drawn at random is neither large nor small? Probability theory cannot answer this question because there is no mechanism within the theory to represent the meaning of perceptions in a form that lends itself to computation. The same problem arises in the examples:

Usually Robert returns from work at about 6:00 p.m. What is the probability that Robert is home at 6:30 p.m.?

I do not know Michelle's age but my perceptions are: (a) it is very unlikely that Michelle is old; and (b) it is likely that Michelle is not young. What is the probability that Michelle is neither young nor old?

X is a normally distributed random variable with small mean and small variance. What is the probability that X is large?

Given the data in an insurance company database, what is the probability that my car may be stolen? In this case, the answer depends on perception-based information that is not in an insurance company database.

In these simple examples--examples drawn from everyday experiences--the general problem is that of estimation of probabilities of imprecisely defined events, given a mixture of measurement-based and perception-based information. The crux of the difficulty is that perception-based information is usually described in a natural language--a language that probability theory cannot understand and hence is not equipped to handle.

To endow probability theory with a capability to operate on perception-based information, it is necessary to generalize it in three ways. To this end, let PT denote standard probability theory of the kind taught in university-level courses. The three modes of generalization are labeled: (a) f-generalization; (b) f.g-generalization: and (c) nl-generalization. More specifically: (a) f-generalization involves fuzzification, that is, progression from crisp sets to fuzzy sets, leading to a generalization of PT that is denoted as PT+. In PT+, probabilities, functions, relations, measures, and everything else are allowed to have fuzzy denotations, that is, be a matter of degree. In particular,

probabilities described as low, high, not very high, etc. are interpreted as labels of fuzzy subsets of the unit interval or, equivalently, as possibility distributions of their numerical values; (b) f.g-generalization involves fuzzy granulation of variables, functions, relations, etc., leading to a generalization of PT that is denoted as PT++. By fuzzy granulation of a variable, X, what is meant is a partition of the range of X into fuzzy granules, with a granule being a clump of values of X that are drawn together by indistinguishability, similarity, proximity, or functionality. For example, fuzzy granulation of the variable *age* partitions its vales into fuzzy granules labeled very young, young, middle-aged, old, very old, etc. Membership functions of such granules are usually assumed to be triangular or trapezoidal. Basically, granulation reflects the bounded ability of the human mind to resolve detail and store information; and (c) Nl-generalization involves an addition to PT++ of a capability to represent the meaning of propositions expressed in a natural language, with the understanding that such propositions serve as descriptors of perceptions. Nl-generalization of PT leads to perception-based probability theory denoted as PTp.

An assumption that plays a key role in PTp is that the meaning of a proposition, *p,* drawn from a natural language may be represented as what is called a generalized constraint on a variable. More specifically, a generalized constraint is represented as X isr R, where X is the constrained variable; R is the constraining relation; and isr, pronounced ezar, is a copula in which *r* is an indexing variable whose value defines the way in which R constrains X. The principal types of constraints are: equality constraint, in which case isr is abbreviated to =; possibilistic constraint, with *r* abbreviated to blank; veristic constraint, with r = v; probabilistic constraint, in which case r = p, X is a random variable and R is its probability distribution; random-set constraint, r = rs, in which case X is set-valued random variable and R is its probability distribution; fuzzy-graph constraint, r = fg, in which case X is a function or a relation and R is its fuzzy graph; and usuality constraint, r = u, in which case X is a random variable and R is its usual--rather than expected--value.

The principal constraints are allowed to be modified, qualified, and combined, leading to composite generalized constraints. An example is: usually (X is small) and (X is large) is unlikely. Another example is: if (X is very small) then (Y is not very large) or if (X is large) then (Y is small).

The collection of composite generalized constraints forms what is referred to as the Generalized Constraint Language (GCL). Thus, in PTp, the Generalized Constraint Language serves to represent the meaning of perception-based information. Translation of descriptors of perceptions into GCL is accomplished through the use of what is called the constraint-centered semantics of natural languages (CSNL). Translating descriptors of perceptions into GCL is the first stage of perception-based probabilistic reasoning.

The second stage involves goal-directed propagation of generalized constraints from premises to conclusions. The rules governing generalized constraint propagation coincide with the rules of inference in fuzzy logic. The principal rule of inference is the generalized extension principle. In general, use of this principle reduces computation of desired probabilities to the solution of constrained problems in variational calculus or mathematical programming.

It should be noted that constraint-centered semantics of natural languages serves to translate propositions expressed in a natural language into GCL. What may be called the constraint-centered semantics of GCL, written as CSGCL, serves to represent the meaning of a composite constraint in GCL as a singular constraint X isr R. The reduction of a composite constraint to a singular constraint is accomplished through the use of rules that govern generalized constraint propagation.

Another point of importance is that the Generalized Constraint Language is maximally expressive, since it incorporates all conceivable constraints. A proposition in a natural language, NL, which is translatable into GCL, is said to be admissible. The richness of GCL justifies the default assumption that any given proposition in NL is admissible. The subset of admissible propositions in NL constitutes what is referred to as a precisiated natural language, PNL. The concept of PNL opens the door to a significant enlargement of the role of natural languages in information processing, decision, and control.

Perception-based theory of probabilistic reasoning suggests new problems and new directions in the development of probability theory. It is inevitable that in coming years there will be a progression from PT to PTp, since PTp enhances the ability of probability theory to deal with realistic problems in which decision-relevant information is a mixture of measurements and perceptions.

Lotfi A. Zadeh is Professor in the Graduate School and director, Berkeley initiative in Soft Computing (BISC), Computer Science Division and the Electronics Research Laboratory, Department of EECs, Univeristy of California, Berkeley, CA 94720-1776; Telephone: 510-642-4959; Fax: 510-642-1712;E-Mail: zadeh@cs.berkeley.edu. Research supported in part by ONR Contract N00014-99-C-0298, NASAContract NCC2-1006, NASA Grant NAC2-117, ONR Grant N00014-96-1-0556, ONR Grant FDN0014991035, ARO Grant DAAH 04-961-0341 and the BISC Program of UC Berkeley