# Economics 202A
# Suggested Solutions to the Midterm

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## 1 Part I. True/False/Uncertain

Note: For clarity, these answers are longer than is needed.

### 1.1 Uncertain.

With high physical–capital externalities, high investment, all else equal, would lead to high income per person. And if either the countries started from a situation of low investment or if the externalities were large enough, there would be high growth. Thus the dismal economic performance of these countries is evidence against large physical–capital externalities. But it is possible that there are large externalities and that other factors were responsible for the poor performance.

### 1.2 True.

A jump in $K/L$ leads to a jump in $Y/L$ given standard (and reasonable) assumptions about the production function. With exogenous technological progress, $K/L$ would gradually return to its old path, so there would be no long–run effect on output per person. But since technological progress comes from people rather than exogenously, the decline in population acts to slow technological progress relative to what it otherwise would have been, and thus eventually causes output per person to fall below the path it otherwise would have taken.

### 1.3 True.

Hall and Jones's decomposition leaves out externalities from human capital and all sources of differences in human capital other than years of education. Since it is much more likely that there are positive than negative human–capital externalities, and since it is much more likely that rich countries have more rather than less human capital for a given amount of education than poor countries, these omissions almost surely cause Hall and Jones's procedure to underestimate the importance of human capital to cross-country income differences.

### 1.4 False!

The obvious problem is omitted variable bias: it is very likely that there are things other than human capital that affect countries' incomes that are correlated with average years of schooling. An additional problem is that years of schooling is a very incomplete measure of human capital.

### 1.5 Uncertain.

Hall and Jones's definition covers everything the government does that affects physical–capital accumulation, human–capital accumulation, and production vs. diversion (which in turn encompasses quite a bit); it also covers unspecified non–government institutions. Thus their definition is so broad and vague that it has limited (though not zero) empirical content. But although their general idea is vague, they operationalize it in a very concrete way, using measures of openness and government anti–diversion policies. Thus the statement is false for Hall and Jones's specific implementation of their idea.

## 2 Part II. Longer questions

### 2.1 Solow Model

In this problem economy is described by the Solow Model and it is initially "to the right" of the BGP. You were supposed to analyze the effect of an increase in the rate of depreciation $\delta$ in this situation.

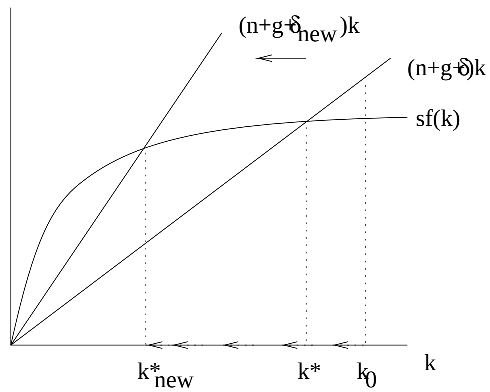


Figure 1: Question 2.1. Solow diagram

Consider first the Solow diagram (Figure 1). The break-even investment line is steeper after the change, so that the new $k^*$ is lower then the old one. This means that after the change the economy is even further from the BGP then it was before. Therefore the economy after the change will converge to a lower level of $k$ then it would otherwise.

What does it mean in terms of output per worker. We know that it is growing at the rate $g$ on the BGP. However, before the change, economy was not on the BGP. Before the change $k$ and therefore $y$ was falling and thus $Y/L$ was growing with the rate lower then $g$ (possibly negative). Then, when $\delta$ goes up, the growth rate of $k$ becomes "more negative":

$$\frac{\dot{k}}{k} = \frac{sf(k)}{k} - (n + g + \delta).$$

This implies that at the time of the change the growth rate of $Y/L$ falls further (can become "more negative" as well). Then, as $k$ is approaching new BGP, the rate of decrease of $k$ is falling and therefore the growth rate of $Y/L$ is increasing till it reaches the the value of $g$. Note that the growth rate of $Y/L$ is just the slope of the graph of $\ln Y/L$, so we can sketch it now (see Figure 2).

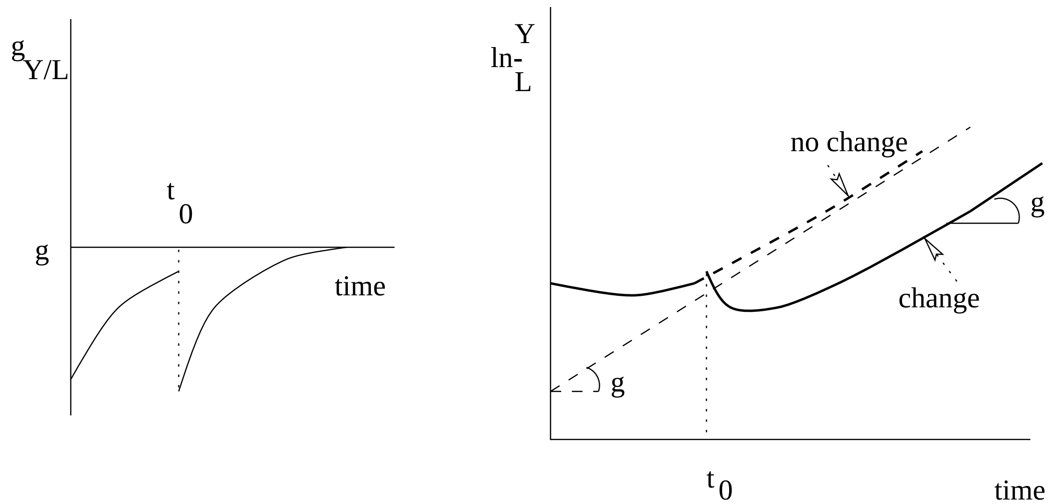


Figure 2: Answer to question 2.1

## 2.2 Ramsey model

Now we have a Ramsey model in which government is purchasing fraction $h$ of economies output and is financing the purchase through non-distortionary taxes. Since the taxes are not distortionary, Euler equation is not affected and therefore the $\dot{c} = 0$ locus is not affected.

We know that $\dot{k}$ is equal to actual investment minus break–even investment. Break-even investment did not change but actual investment per unit of effective labor is now $f(k) - c - G = (1-h)f(k) - c$, since $G = hf(k)$. Thus our equation of motion for capital now looks like

$$\dot{k} = (1-h)f(k) - c - (n+g)k.$$

Therefore the $\dot{k} = 0$ locus shifts down, but not parallely like we had in class — it has to start from the origin (see Figure 3).

At the time of the change there will be a discreet jump in $c$ to the new saddle path (since the change was unexpected), which in this case is just the new balanced growth path point. Recall that capital stock can not jump, because it is predetermined by the past investment history. Therefore after

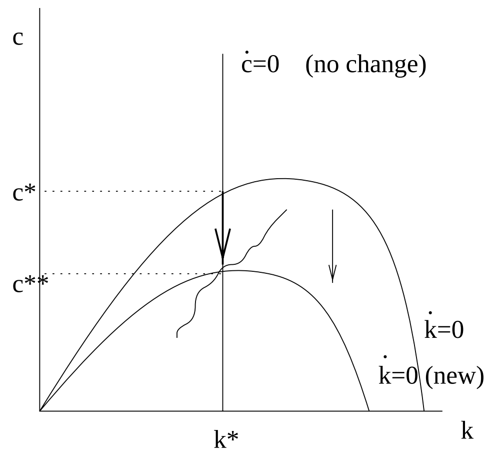


Figure 3: Question 2.2. Phase diagram

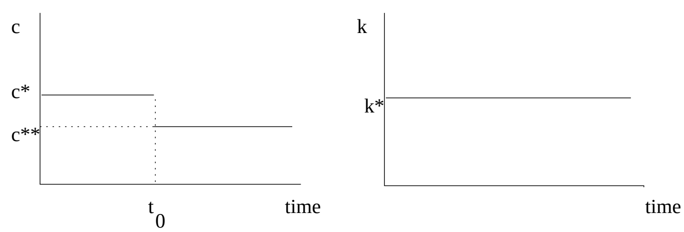


Figure 4: Answer to question 2.2

the initial change $k$ will stay the same and $c$ will be constant at a lower level, since the government is now taking a part of output from the consumers (see Figure 4).

Effectively, it does not matter that government is spending a fraction of output rather then a fixed amount $G$, since the capital stock and thus the output do not change.

## 2.3 The Diamond model with labor supply in both periods of life

We modify Diamond model so that individuals now receive labor income in both periods of life. There is no technological progress and no population growth. The total amount of labor supplied each period is thus $2L$. The production function is $Y_t = BK_t^{\alpha}[2L]^{1-\alpha}$ and capital is depreciating fully every period: $\delta = 1$. Individuals are not discounting future. They maximize a utility function $U = \ln C_{1,t} + \ln C_{2,t+1}$.

(a) First we solve the consumer problem. Each individual is facing life-time budget constraint

$$C_{1,t} + \frac{C_{2,t+1}}{1 + r_{t+1}} = w_t + \frac{w_{t+1}}{1 + r_{t+1}}.$$

Since the pattern of income does not affect the intertemporal choice, the Euler equation will be the same as in standard Diamond model: $u'(C_{1,t}) = (1 + r_{t+1})u'(C_{2,t+1})$, or in our special case with logarithmic utility,

$$C_{2,t+1} = (1 + r_{t+1})C_{1,t}.$$

It follows immediately that

$$C_{1,t} = \frac{1}{2}\left(w_t + \frac{w_{t+1}}{1 + r_{t+1}}\right).$$

By definition, saving is the part of income that is not spent for consumption. Therefore the saving made when young is $S_{t+1} = w_t - C_{1,t}$,

$$S_{t+1} = \frac{1}{2}\left(w_t - \frac{w_{t+1}}{1 + r_{t+1}}\right).$$

Since individuals do not discount the future, they consume a half of the present value of their life-time income each period and thus save only if present value of the first-period income is higher then the present value of the second-period income.

(b) We know that factors are paid their marginal products. We should be careful though about two things. First is that labor supply is $2L$ rather then $L$ as we are used to think, thus

$$w_t = (1-\alpha)BK_t^\alpha[2L]^{-\alpha}.$$

Second thing to keep in mind is that $r = MPK - \delta$ and $\delta = 1$ in this case. Therefore

$$r_t = \alpha BK_t^{\alpha-1}[2L]^{1-\alpha} - 1.$$

(c) As we just mentioned in (a), $w_t - C_{1,t} = S_{t+1}$, and the standard intuition of the Diamond model applies. Since young generation of period $t$ is the only ones that will be able to transfer capital from period $t$ to period $t+1$, they are the only ones to own $K_{t+1}$. Also, to buy capital is the only way they can save. Therefore total savings of young people of period $t$ must be equal to $K_{t+1}$. $S_{t+1}L = K_{t+1}$, $(w_t - C_{1,t})L = K_{t+1}$.

(d) We will use our results from (c) to solve for $K_{t+1}$ as a function of $K_t$.

$$\begin{aligned} K_{t+1} &= (w_t - C_{1,t})L = S_{t+1}L = \\ &= \frac{1}{2}\left(w_t - \frac{w_{t+1}}{1+r_{t+1}}\right)L. \end{aligned}$$

We can rewrite everything in per-worker form for simplicity: $k = K/2L$,

$$\begin{aligned} w_t &= (1-\alpha)BK_t^\alpha[2L]^{-\alpha} = (1-\alpha)Bk_t^\alpha, \\ r_t &= \alpha BK_t^{\alpha-1}[2L]^{1-\alpha} - 1 = \alpha Bk_t^{\alpha-1} - 1. \end{aligned}$$

$$\begin{aligned} k_{t+1} &= \frac{1}{4}\left(w_t - \frac{w_{t+1}}{1+r_{t+1}}\right) = \\ &= \frac{1}{4}\left((1-\alpha)Bk_t^\alpha - \frac{(1-\alpha)Bk_{t+1}^\alpha}{\alpha Bk_{t+1}^{\alpha-1}}\right) = \\ &= \frac{1}{4}(1-\alpha)Bk_t^\alpha - \frac{1-\alpha}{4\alpha}k_{t+1}, \\ k_{t+1} &= \frac{\alpha(1-\alpha)}{1+3\alpha}Bk_t^\alpha. \end{aligned}$$

Turning back to total $K$, we get

$$K_{t+1} = \frac{\alpha(1-\alpha)}{1+3\alpha} B K_t^{\alpha},$$

since $2L$ cancel out on both sides.