University of California
Department of Economics

Economics 202A
Spring 1999
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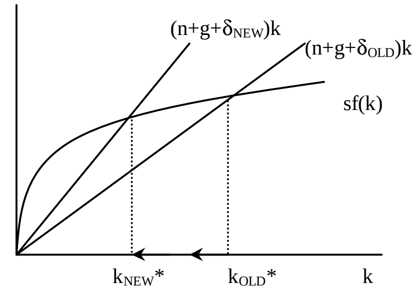
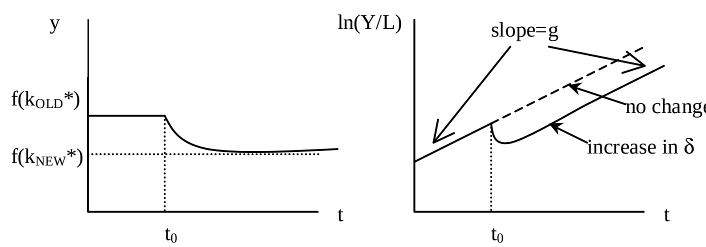
Suggested solutions to problem set 1.

1. Economy is initially on the BGP and δ falls permanently. We can use the Solow diagram to analyze this problem (*k=K/AL*, *y=Y/AL*). The break-even investment line becomes steeper and as a result the BGP level of *k* falls. At time of the change (t_0), *k* does not jump but instead starts to fall gradually (capital is a stock and thus never jumps). Output per unit of effective labor (*y*) follows the pattern of *k* (see picture below). Since output per worker *Y/L=yA*, on the BGP *Y/L* is growing with the rate *g* and is growing slower (or even falling) during the adjustment. This implies that the increase in the rate of depreciation has the level effect on the output per worker, but not the long-run growth effect. All this can be summarized on the graph. Without the change output per worker would continue growing with the rate *g*.



Intuitively, when the rate of depreciation increases the economy can not sustain in the long run (on the BGP) as high level of capital (and therefore output) as it could before, however the driving force of the growth of output per worker is the technological growth which remained unchanged.



2. Factor payments are: $w=\dfrac{\partial F(K,AL)}{\partial L}$, $r=\dfrac{\partial F(K,AL)}{\partial K}$.

a) $w=\dfrac{\partial F(K,AL)}{\partial L}=\dfrac{\partial ALf(\frac{K}{AL})}{\partial L}=A\left[f(k)+Lf'(k)\left(-\dfrac{K}{AL^2}\right)\right]=A\left[f(k)-kf'(k)\right].$

b) This is known as Euler theorem. Let's start from he CRS definition: *F(cK,cAL)=cF(K,AL)*. Now take the derivatives of both sides with respect to *c*:

$$\frac{\partial F(K,AL)}{\partial K}K+\frac{\partial F(K,AL)}{\partial AL}AL=F(K,AL).$$

Note also that $\frac{\partial F(K,AL)}{\partial AL} = \frac{\partial ALf(\frac{K}{AL})}{\partial AL} = f(k) - kf'(k) = \frac{w}{A}$ and thus we can rewrite the previous equation as $rK+wL=F(K,AL)$.

c) Share of output going to capital is rK/Y, share going to labor is wL/Y. Consider first the rate of growth of r. $r = \frac{\partial F(K,AL)}{\partial K} = \frac{\partial f(k)}{\partial k} = f'(k)$. Since the production function does not depend directly on time and k is constant on the BGP, r is constant on the BGP. This fits the first Kaldor's stylized fact. From the discussion in class we know that K and Y are growing at the same rate, thus K/Y is constant and the share of output going to capital rK/Y is thus constant. Since the sum of two shares is 1, this implies that the share of the output going to labor is also constant. This fits the second Kaldor's fact. See (d) for the rate of growth of w.

d) We will use 'take logs and differentiate' method here.
$w = A[f(k) - kf'(k)] \Rightarrow \ln w = \ln A + \ln[f(k) - kf'(k)]$, taking the time derivative, we get
$\frac{\dot{w}}{w} = \frac{\dot{A}}{A} + \frac{f'(k)\dot{k} - \dot{k}f'(k) - kf''(k)\dot{k}}{f(k) - kf'(k)} = g + \frac{-kf''(k)\dot{k}}{f(k) - kf'(k)} > g$, since $f''(k)<0$ and $f(k)-kf'(k)$ is positive for the concave increasing function starting at 0[1]. Note that on the balanced growth path when capital is constant the growth rate of the wage will be g. Thus as capital is growing to k^*, the wage rate is growing faster than on the balanced growth path. There are two reasons for the wage rate to increase as k is increasing: the increase in the effectiveness of labor (this is true for the BGP as well) and the increase in capital - effective labor ratio k, which makes labor more productive.
$r = f'(k) \Rightarrow \frac{\dot{r}}{r} = \frac{f''(k)\dot{k}}{f'(k)} < 0$, thus the return on capital is decreasing (growing at a lower rate then on the BGP). As the amount of capital per unit of effective labor increases, each unit of capital becomes less productive and thus the return on capital decreases.

3. In this problem $\dot{K} = rK - \delta K$.

a) Consider the behavior of k. From the lecture we know that $\dot{k} = \frac{\dot{K}}{AL} - (n+g)k$. Substituting the condition for the growth of K, $\dot{k} = [r - (n+g+\delta)]k$. Consider a balanced growth path on which k is constant. Then all the variables are growing at constant rates as in standard Solow model. Recall that $r=f'(k)$. Then the condition for the BGP is $f'(k^*)=n+g+\delta$. If $k<k^*$, then $f'(k)>f'(k^*)= n+g+\delta$ and thus k is increasing, if $k>k^*$, then $f'(k)<f'(k^*)= n+g+\delta$ and k is decreasing. So the economy converges to its (non-trivial) balanced growth path.

b) The Golden Rule level of k is maximizing consumption. The first order condition is that the slope of the production function is equal to the slope of break-even investment: $f'(k_{GR})=n+g+\delta$. But this is exactly the same condition as the one determining the BGP in (a). Thus in this model the economy converges to the Golden Rule level of k.

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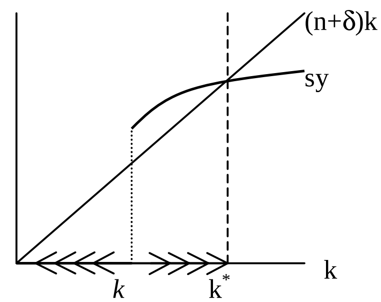
[1] Note that $\frac{f(k)}{kf'(k)} = \frac{1}{\alpha_k} > 1$, because share of capital is less then one given our assumptions about the production function, and thus $f(k)-kf'(k)>0$.

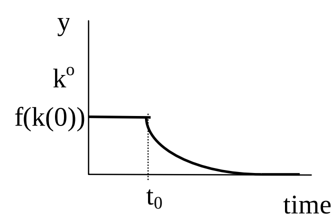
Intuition: the Golden Rule level of capital is characterized by the fact that the marginal return on capital is equal to the marginal cost of maintaining this capital. In the model of this problem, the BGP condition is that capital contribution to output (*rK*) is equal to break-even investment. Marginal cost of maintaining the capital is constant. If k falls below its Golden Rule level, marginal return on capital will be higher, but in our model it means that capital's contribution to output will exceed the break-even investment and *k* will increase. Basically, in this model capital is paying for itself and thus is choosing the 'optimal' level.



4. We consider the simpler version of the model with $Y=F(K,L)$, then $y=Y/L$ and $k=K/L$. Instead of having constant saving rate we now have $s=0$ if $y<f(\tilde{k})$ and $s>0$ if $y\geq f(\tilde{k})$. Note that this assumption can be equivalently stated as $s=0$ if $k<\tilde{k}$ and $s>0$ if $k\geq\tilde{k}$.

Final assumption that $sf(\tilde{k})>(n+d)\tilde{k}$ tells us that this critical level of *k* is to the left of the BGP level of capital k^*.

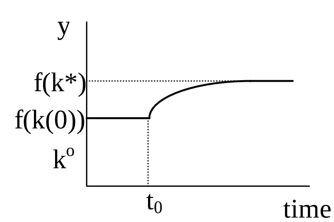
a) The break-even investment line will not be affected (except of the fact that $g=0$ now), however the actual investment will drop to *0* for all levels of $k<\tilde{k}$, then there will be a discontinuity because s becomes strictly positive and $f(\tilde{k})$ is strictly positive. As our final assumption says, actual investment at $\tilde{k}$ is above break-even (see the picture above).



(b) Since $A=1$, $y=f(k)$ is the output per worker. Qualitatively it behaves exactly like *k*.

The behavior of *k* is shown with arrows on the part (a) graph.

i. If initially $k(0)<\tilde{k}$, then savings will be equal to zero and therefore capital per worker will fall: $\dot{k}=-(n+\delta)k$ - until it reaches zero level. When $k=0$, no more changes occur. Therefore output per worker will fall over time until it reaches zero. Note that because of depreciation in the absence of investment, total level of capital *K* will decrease over time and so will the total output until they fall to zero.



ii. If initially $\tilde{k}<k(0)<k^*$ (*k* is <u>slightly</u> higher then $\tilde{k}$), then $\dot{k}=sf(k)-(n+d)k$ is positive and thus *k* is increasing until it reaches the BGP level of k^*. Therefore output per worker will increase and then settle down on the BGP level.