Advanced Microeconomics
(Economics 104)
Spring 2011
Evolutionary stable strategy (ESS)

Consider a payoff symmetric game

$$G = \langle \{1,2\}, (A, A), (u_i) \rangle$$

where

$$u_1(a) = u_2(a')$$

when a' is obtained from a by exchanging a_1 and a_2.

- $a^* \in A$ is ESS iff for any $a \in A$, $a \neq a^*$ and $\varepsilon > 0$ sufficiently small

$$(1-\varepsilon)u(a^*, a^*) + \varepsilon u(a^*, a) > (1-\varepsilon)u(a, a^*) + \varepsilon u(a, a)$$

which is satisfied iff for any $a \neq a^*$ either

$$u(a^*, a^*) > u(a, a^*)$$

or

$$u(a^*, a^*) = u(a, a^*) \text{ and } u(a^*, a) > u(a, a)$$

If a^* is an ESS then (a^*, a^*) is a NE.

- Suppose not. Then there exists a strategy $a \in A$ such that

  $$u(a, a^*) > u(a^*, a^*)$$

  But for ε small enough

  $$(1-\varepsilon)u(a^*, a^*) + \varepsilon u(a^*, a) < (1-\varepsilon)u(a, a^*) + \varepsilon u(a, a)$$

- A strategy a^* is an ESS if (a^*, a^*) is a NE, and $\forall a \neq a^*$ if $u(a^*, a^*) = u(a, a^*)$ then $u(a^*, a) > u(a, a)$ (any *strict* NE strategy is ESS).

Existence of ESS in 2×2 game

A game $G = \langle \{1,2\}, (A,A), (u_i) \rangle$ where $u_i(a) \neq u_i(a')$ for any a, a' has a mixed strategy which is ESS.

	a	a'
a	w, w	x, y
a'	y, x	z, z

- If $w > y$ or $z > x$ then (a,a) or (a',a') are strict NE, and thus a or a' are ESS.
- If $w < y$ and $z < x$ then the game has a symmetric mixed strategy NE (α^*, α^*) in which

$$\alpha^*(a) = (z-x)/(w-y+z-x)$$

  To verify that α^* is ESS, we need to show that $u(\alpha^*, \alpha) > u(\alpha, \alpha)$ for any $\alpha \neq \alpha^*$.