Advanced Microeconomics
(Economics 104)
Spring 2011
Introduction

Topics

– Terminology and notations:

  functions,
  preferences,
  utility representation, and
  profiles.

– Games and solutions:

  strategic vs. extensive games, and
  perfect vs. imperfect information.

– Rationality:

  a rational agent, and
  boundedly rational agent.

– Formalities:

  a strategic game of perfect information.

Terminology and notations (OR 1.7)

Sets

- $\mathbb{R}$ is the set of real numbers.
- $\mathbb{R}_+$ is the set of nonnegative real numbers.
- $\mathbb{R}^n$ is set of vectors on n real numbers.
- $\mathbb{R}^n_+$ is set of vectors of n nonnegative real numbers.

For $x, y \in \mathbb{R}^n$,

$$x \geq y \Longleftrightarrow x_i \geq y_i$$

for all i.

$$x > y \Longleftrightarrow x_i \geq y_i \text{ and } x_j > y_j$$

for all i and some j.

$$x >> y \Longleftrightarrow x_i > y_i$$

for all i.

Functions

A function $f : \mathbb{R} \to \mathbb{R}$ is

- increasing if $f(x) > f(y)$ whenever $x > y$,
- non decreasing if $f(x) \geq f(y)$ whenever $x > y$, and
- concave if

$$f(\alpha x + (1-\alpha)x') \geq \alpha f(x) + (1-\alpha) f(x')$$

  $\forall x, x' \in \mathbb{R}$ and $\forall \alpha \in [0,1]$.

Let X be a set. The set of maximizers of a function $f : X \to \mathbb{R}$ is given by $\arg\max_{x \in X} f(x)$.

Preferences

$\succsim$ - a binary relation on some set of alternatives $A \subseteq \mathbb{R}^n$. From $\succsim$ we derive two other relations on A:

- strict performance relation

$$a \succ b \iff a \succsim b \text{ and not } b \succsim a$$

- indifference relation $a\tilde{\ }b \iff a \succsim b$ and $b \succsim a$

$\succsim$ is said to be

- complete if
$$a \succsim b \text{ or } b \succsim a$$
$\forall a, b \in A$.
- transitive if
$$a \succsim b \text{ and } b \succsim c \text{ then } a \succsim c$$
$\forall a, b, c \in A$.

Utility representation

A function $u : A \to \mathbb{R}$ is a utility function representing $\succsim$ if for all $a, b \in A$

$$a \succsim b \iff u(a) \geq u(b)$$

$\succsim$ can be presented by a utility function only if it is complete and transitive (rational).

$\succsim$ is said to be

- continuous (preferences cannot jump...) if
  for any sequence of pairs $\{(a^k, b^k)\}_{k=1}^{\infty}$ with $a^k \succsim b^k$, and $a^k \to a$ and $b^k \to b$, $a \succsim b$.
- (strictly) quasi-concave if
  for any $b \in A$ the upper counter set $\{a \in A : a \succsim b\}$ is (strictly) convex.

These guarantee the existence of continuous well-behaved utility function representation.

Profiles

Let N be a the set of players.

$(x_i)_{i \in N}$ or simply (x_i)

- a profile, i.e., a collection of values of some variable, one for each player.

$(x_j)_{j \in N/\{i\}}$ or simply x_{-i}

- the list of elements of the profile $x = (x_j)_{j \in N}$ for all players except i.

(x_{-i}, x_i)

- a list x_{-i} and an element x_i, which is the profile $(x_i)_{i \in N}$.

Games and solutions (O 1.1; OR 1.1-1.3)

A game is a model of interactive (multi-person) decision-making. We distinguish between:

- noncooperative and cooperative games - the units of analysis are individuals or (sub) groups,
- strategic (normal) form games and extensive form games - players move simultaneously or precede one another, and
- Gams with perfect and imperfect information - players are perfectly or imperfectly informed about characteristics, events and actions.

A solution is a systematic description of outcomes in a family of games.

- Nash equilibrium.
- Subgame perfect equilibrium - extensive games with perfect information.
- Perfect Bayesian equilibrium - games with observable actions.
- Sequential equilibrium (and refinements) - extensive games with imperfect information.

The classic references are von Neumann and Morgenstern (1944), Luca and Raiffa (1957) and Schelling (1960) (see R and OR).

Rational behavior and bounded rationality (O 1.2; OR 1.4, 1.6)

Consider

- a A set of actions,
- a C set of consequences,
- a consequence function $g : A \to C$, and
- a preference relation $\succsim$ on the set C.

Given any set $B \subseteq A$ of actions, a *rational agent* chooses an action $a^* \in B$ such that

$$g(a^*) \succsim g(a)$$

for all $a \in B$.

And when $\succsim$ are specified by a utility function $U : C \to \mathbb{R}$

$$a^* \in \arg\max_{a \in B} U(g(a))$$

With uncertainty about

- the environment,
- events in the game, or
- actions of other players and their reasoning,

A rational agent is assumed to have in mind

- a state space Ω,
- a (subjective) probability measure over Ω, and
- a consequence function $g : A \times \Omega \to C$

A rational agent is an expected (vNM) utility $u(g(a, \omega))$ maximizer.

Formalities (O 2.1; OR 2.1)

A strategic game of perfect information:

a finite set N of players, and for each player $i \in N$

- a non-empty set A_i of actions
- a preference relation $\succsim_i$ on the set $A = \times_{j \in N} A_j$ of possible outcomes.

We will denote a strategic game by

$$\langle N, (A_i), (\succsim_i) \rangle$$

or by

$$\langle N, (A_i), (u_i) \rangle$$

when $\succsim_i$ can be represented by a utility function $u_i : A \to \mathbb{R}$.

A two-player finite strategic game can be described conveniently in a bi-matrix. For example, consider the 2×2 game

	L	R
T	A_1, A_2	B_1, B_2
B	C_1, C_2	D_1, D_2