Advanced Microeconomics
(Economics 104)
Spring 2011
Introduction
Review Questions

Questions

- **Question 1**

  Consider a group of individuals A, B and C and the relation *at least as tall as* as in A is at least as tall as B. Does this relation satisfy the completeness and transitivity properties? Take the same group of individuals as above and consider the relation *strictly taller than*. Is it complete? Is this relation transitive?

- **Question 2**

  Determine if completeness and transitivity are satisfied for the following preferences defined on $x = (x_1, x_2)$ and $y = (y_1, y_2)$.

  - $x \succsim y \; iff$ (if and only if) $x_1 \geq y_1$ and $x_2 \geq y_2$ (solved as an example).
  - $x \succsim y \; iff \; \min\{x_1, x_2\} \geq \min\{y_1, y_2\}$, and
  - $x \succsim y \; iff \; x_1 > y_1$ or $x_1 = y_1$ and $x_2 > y_2$.

- **Question 3**

  Determine if completeness and transitivity are satisfied for the following preferences defined on $x = (x_1, x_2)$ and $y = (y_1, y_2)$

  $$x \succsim y \; iff \; \max\{x_1, x_2\} \geq \max\{y_1, y_2\}$$

  Illustrate a typical indifference curve graphically (Hint: pick a bundle $x = (x_1, x_2)$ and think what are the set of bundles that the consume indifferent between them and $x = (x_1, x_2)$). Accordingly, determine and explain graphically whether this preference relation satisfies convexity.

Answers

- **Question 1**

  The relation *at least as tall as* is complete and transitive.

  - To verify completeness, pick any two individuals A and B. Clearly, either individual A is at least as tall as individual B or individual B is at least as tall as individual A or both.
  - For transitivity, pick three individuals A, B and C and suppose that individual A is at least as tall as individual B and individual B is at least as tall as individual C. Obviously, individual A must be at least as tall as individual C. Thus, the relation at least as tall as satisfies the transitivity property.

  The relation *strictly taller than* does not satisfy completeness but is transitive.

  - In order to see that completeness fails, pick two individuals A and B with the same height. Clearly, it is not true that individual A is strictly taller than individual B and not true that individual B is strictly taller than individual A. Thus, the relation strictly taller than is not complete since two individuals of the same height can not be compared.
  - For transitivity, pick three individuals A, B and C and suppose that individual A is strictly taller than individual B and individual B is strictly taller than individual C. Obviously, individual A must be also strictly taller than individual C. Thus, the relation strictly taller than satisfies the transitivity property

- **Question 2**

  $x \succsim y \; iff$ (if and only if) $x_1 \geq y_1$ and $x_2 \geq y_2$.

  - Not complete: consider the following counter example: $x = (0, 1)$ and $y = (1, 0)$. Clearly, neither $x_i \geq y_i$ for all i nor $y_i \geq x_i$ for all i. So, neither $x \succsim y$ nor $y \succsim x$. Hence, the bundles $x = (0, 1)$ and $y = (1, 0)$ can not be compared.
  - Transitive: pick $x = (x_1, x_2)$, $y = (y_1, y_2)$ and $z = (z_1, z_2)$ and suppose that $x \succsim y$ and $y \succsim z$ towards showing that $x \succsim z$. By assumption, Since $x \succsim y$ then $x_i \geq y_i$ for all i and since $y \succsim z$ $y_i \geq z_i$ for all i. That is,

    $$x_1 \geq y_1 \text{and } x_2 \geq y_2$$

    and

    $$y_1 \geq z_1 \text{and } y_2 \geq z_2$$

Hence,

$$x_1 \geq z_1 \text{and } x_2 \geq z_2$$

Therefore, $x \succsim y$ and $y \succsim z$ imply that $x \succsim z$.

- Strongly monotonic: first, let's recall the definitions of monotonicity: we say that the preference relation $\succsim$ is monotonic if for any two bundles x any y such that $x >> y$, $x \succ y$ (by $x >> y$ we mean that each component of x is strictly larger than the corresponding component of y). And, we say that it is strongly monotonic if for any two bundles x any y such that $x \geq y$ and $x \neq y$, $x \succ y$ (by $x \geq y$ and $x \neq y$ we mean that x has at least as much of all components and strictly more of at least of one component). You should be able to show that if preference relation $\succsim$ is strongly monotonic, then it is monotonic. According to these definitions, the above preference relation is strongly monotonic: pick $x = (x_1, x_2)$ and $y = (y_1, y_2)$ such that

$$x_1 > y_1 \text{and } x_2 \geq y_2$$

then $x \succsim y$ but not $y \succsim x$. Hence, $x \succ y$. Since it is strongly monotonic it is also weakly monotonic.

$x \succsim y \; iff \; \min\{x_1, x_2\} \geq \min\{y_1, y_2\}$.

- Complete: pick any $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Clearly, either

$$\min\{x_1, x_2\} \geq \min\{y_1, y_2\}$$

holds or,

$$\min\{x_1, x_2\} \leq \min\{y_1, y_2\}$$

holds or both. Hence, either $x \succsim y$ or $y \succsim x$ or both.

- Transitive: pick any $x = (x_1, x_2)$, $y = (y_1, y_2)$ and $z = (z_1, z_2)$ and suppose that $x \succsim y$ and $y \succsim z$. To show that transitivity we need that $x \succsim y$. Since $x \succsim y$

$$\min\{x_1, x_2\} \geq \min\{y_1, y_2\}$$

and since $y \succsim z$

$$\min\{y_1, y_2\} \geq \min\{z_1, z_2\}$$

So, we conclude that

$$\min\{x_1, x_2\} \geq \min\{z_1, z_2\}$$

which implies that $x \succsim z$. Therefore, $x \succsim y$ and $y \succsim z$ imply that $x \succsim z$.

$x \succsim y \; iff \; x_1 > y_1$ or $x_1 = y_1$ and $x_2 > y_2$.

- Not complete: for a counter example pick two bundles x and y such that $x = y$. For example, $x = (1, 1)$ and $y = (1, 1)$. Clearly, since $x_1 = y_1$ and $x_2 = y_2$ neither $x \succsim y$ nor $y \succsim x$. Hence, the two bundles $x = (1, 1)$ and $y = (1, 1)$ can not be compared by this preference relation.
- Transitive: Pick $x = (x_1, x_2)$, $y = (y_1, y_2)$ and $z = (z_1, z_2)$ and suppose that $x \succsim y$ and $y \succsim z$ towards showing that $x \succsim z$. By assumption, Since $x \succsim y$ then

$$\text{either } x_1 > y_1 \text{or if } x_1 = y_1 \text{ then } x_2 > y_2$$

and since $y \succsim z$ then

$$\text{either } y_1 > z_1 \text{or if } y_1 = z_1 \text{ then } y_2 > z_2$$

Hence, it must hold that

$$\text{either } x_1 > z_1 \text{or if } x_1 = z_1 \text{ then } x_2 > z_2$$

which implies that $x \succsim z$.

- **Question 3**

Completeness and transitivity

- Complete: pick any $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Clearly, either

$$\max\{x_1, x_2\} \geq \max\{y_1, y_2\}$$

holds or,

$$\max\{x_1, x_2\} \leq \max\{y_1, y_2\}$$

holds or both. Hence, either $x \succsim y$ or $y \succsim x$ or both.
- Transitive: pick any $x = (x_1, x_2)$, $y = (y_1, y_2)$ and $z = (z_1, z_2)$ and suppose that $x \succsim y$ and $y \succsim z$. To show that transitivity we need that $x \succsim y$. Since $x \succsim y$

$$\max\{x_1, x_2\} \geq \max\{y_1, y_2\}$$

and since $y \succsim z$

$$\max\{y_1, y_2\} \geq \max\{z_1, z_2\}$$

So, we conclude that

$$\max\{x_1, x_2\} \geq \max\{z_1, z_2\}$$

which implies that $x \succsim z$. Therefore, $x \succsim y$ and $y \succsim z$ imply that $x \succsim z$.

A typical indifference curve is illustrated graphically in the figure attached from which it is obvious that this preference relation does not satisfy convexity.