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ECON 201A Economic Theory
Choice Theory
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**Revealed preference
(Kreps Ch. 5)**

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In 'classic' consumer behavior we have taken \succsim as the primitive concept and derived the restrictions that the u -maximization model imposes on observed demand.

- These are the Slutsky restrictions — the substitution terms matrix is symmetric and negative semidefinite.
- These restrictions are (in principle) testable, but require assumptions on the parametric form/shape of the demand function.
- The standard approach is postulating some (semi)parametric family of functions and error structure.

The most basic question to ask about choice data $(\mathbf{p}^t, \mathbf{x}^t)$

prices \mathbf{p}^t and associated chosen bundles \mathbf{x}^t for $t = 1, \dots, T$

is whether it is consistent with utility maximization.

Classical revealed preference theory provides a direct test: $(\mathbf{p}^t, \mathbf{x}^t)$ are consistent with maximizing a (well-behaved) u -function *iff* they satisfy the Generalized Axiom of Revealed Preference (GARP).

We say that u -function rationalizes the observed behavior $(\mathbf{p}^t, \mathbf{x}^t)$ if for all for $t = 1, \dots, T$

$$u(\mathbf{x}^t) \geq u(\mathbf{x}) \text{ for all } \mathbf{x} \text{ such that } \mathbf{p}^t \mathbf{x}^t \geq \mathbf{p}^t \mathbf{x},$$

that is, $u(\cdot)$ achieves its maximum value on the budget set at the chosen bundles.

Question Suppose that the data $(\mathbf{p}^t, \mathbf{x}^t)$ were generated by such a maximization process. What observable restrictions must the observed choices satisfy?

Answer None! Without any assumptions about $u(\cdot)$ there are no restrictions: $u(\cdot)$ can be a constant function (so the consumer was indifferent to all observed bundles...).

- We rule out this trivial case: what are the observable restrictions imposed by the maximization of a locally non-satiated u -function?
- Now, when \mathbf{x}^t was chosen when \mathbf{x} could have been strictly chosen $\mathbf{p}^t \mathbf{x}^t > \mathbf{p}^t \mathbf{x}$, the utility of $u(\mathbf{x}^t)$ must be strictly larger than the utility of $u(\mathbf{x})$.

We will say that \mathbf{x}^t is

- directly revealed preferred to \mathbf{x} , $\mathbf{x}^t R^D \mathbf{x}$, if $\mathbf{p}^t \mathbf{x}^t \geq \mathbf{p}^t \mathbf{x}$.
- strictly directly revealed preferred to \mathbf{x} , $\mathbf{x}^t P^D \mathbf{x}$, if $\mathbf{p}^t \mathbf{x}^t > \mathbf{p}^t \mathbf{x}$.
- revealed preferred to \mathbf{x} , $\mathbf{x}^t R \mathbf{x}$, if there exists a sequence $\{\mathbf{x}\}_{k=1}^K$ with

$$\mathbf{x}^1 = \mathbf{x}^t \text{ and } \mathbf{x}^K = \mathbf{x} \text{ such that } \mathbf{x}^k R^D \mathbf{x}^{k+1}$$

for every $k = 1, \dots, K - 1$ (R is the transitive closure of R^D).

It is clear that if the data $(\mathbf{p}^t, \mathbf{x}^t)$ were generated by a non-satiated u -function then

$$\mathbf{x}^t R \mathbf{x} \implies u(\mathbf{x}^t) \geq u(\mathbf{x}).$$

Consider any two observations \mathbf{x}^t and \mathbf{x}^s : we now have a condition to determine whether $u(\mathbf{x}^t) \geq u(\mathbf{x}^s)$ and an (observable) condition to determine whether $u(\mathbf{x}^s) > u(\mathbf{x}^t)$.

Obviously, these two conditions should not both be satisfied. This condition (GARP) can be stated in the notation introduced above.

Generalized Axiom of Revealed Preference (GARP)

$$\mathbf{x}^t R \mathbf{x}^s \text{ implies not } \mathbf{x}^s P^D \mathbf{x}^t.$$

In words, if \mathbf{x}^t is indirectly revealed preferred to \mathbf{x}^s , then \mathbf{x}^s is not strictly directly revealed preferred to \mathbf{x}^t .

GARP requires that if $\mathbf{x}^t R \mathbf{x}^s$ then $\mathbf{p}^s \mathbf{x}^s \leq \mathbf{p}^s \mathbf{x}^t$ (\mathbf{x}^t must cost at least as much as \mathbf{x}^s at the prices prevailing when \mathbf{x}^s is chosen).

As the name implies, GARP is a generalization of various other revealed preference tests...

Weak Axiom of Revealed Preference (WARP)

$$\mathbf{x}^t R^D \mathbf{x}^s \text{ and } \mathbf{x}^t \neq \mathbf{x}^s \text{ implies not } \mathbf{x}^s R^D \mathbf{x}^t.$$

Strong Axiom of Revealed Preference (SARP)

$$\mathbf{x}^t R \mathbf{x}^s \text{ and } \mathbf{x}^t \neq \mathbf{x}^s \text{ implies not } \mathbf{x}^s R \mathbf{x}^t.$$

! WARP and SARP require that there be a unique demand bundle at each budget, while GARP allows for multiple demanded bundles (flat spots in the indifference curves).

!! Afriat's (1967) Theorem tells us that if a (finite) data set generated by an individual's choices satisfies GARP, then the data can be rationalized by a well-behaved utility function.

Afriat's Theorem: The following conditions are equivalent:

- (i) The data satisfy GARP.
- (ii) There exists a non-satiated u -function that rationalizes the data.
- (iii) There exists a concave, monotonic, continuous, non-satiated u -function that rationalizes the data.

(iii) trivially implies (ii) and we have already seen that (ii) implies (i). All that is left is the proof that (i) implies (iii). Not quite so... Let's throw more on the mix...

(iv) There exist positive numbers (u^t, λ^t) for $t = 1, \dots, T$ that satisfy the so-called Afriat inequalities:

$$u^s \leq u^t + \lambda^t \mathbf{p}^t (\mathbf{x}^s - \mathbf{x}^t) \text{ for all } t, s.$$

We will show that (iv) implies (iii). The proof that (i) implies (iv) is omitted. See Varian (1982) for the argument, at your own risk...

Proof: consider the following u -function

$$u(\mathbf{x}) = \min_{t=1, \dots, T} \{u^t + \lambda^t \mathbf{p}^t (\mathbf{x} - \mathbf{x}^t)\}$$

(which is the lower envelope of a finite number of hyperplanes). This function is continuous, locally non-satiated and monotonic (as long as $\mathbf{p}^t > 0$), and concave (trust me on this).

We will show that this function rationalizes the data — achieves its constrained maximum at \mathbf{x}^t when prices are \mathbf{p}^t .

- We first show that $u(\mathbf{x}^t) = u^t$. If this is not the case, we have

$$u(\mathbf{x}^t) = u^m + \lambda^m \mathbf{p}^m (\mathbf{x} - \mathbf{x}^m) < u^t$$

which violates one of the Afriat inequalities and thus $u(\mathbf{x}^t) = u^t$.

- Now suppose that $\mathbf{p}^s \mathbf{x}^s \geq \mathbf{p}^s \mathbf{x}$. It follows that

$$\begin{aligned} u(\mathbf{x}) &= \min_t \{u^t + \lambda^t \mathbf{p}^t (\mathbf{x} - \mathbf{x}^t)\} \\ &\leq u^s + \lambda^s \mathbf{p}^s (\mathbf{x} - \mathbf{x}^s) \\ &\leq u^s \\ &= u(\mathbf{x}^s) \end{aligned}$$

Hence, $u(\mathbf{x}^s) \geq u(\mathbf{x})$ for any \mathbf{x} such that $\mathbf{p}^s \mathbf{x}^s \geq \mathbf{p}^s \mathbf{x}$. ■

A note on concave functions:

- A function $f : A \rightarrow \mathbb{R}$ defined over a convex set $A \subset \mathbb{R}^N$ is concave if for all $x, x' \in A$ and any $\alpha \in [0, 1]$

$$f(\alpha x + (1 - \alpha)x') \geq \alpha f(x) + (1 - \alpha)f(x').$$

- Think of $z = x' - x$, this condition can be rewritten as

$$f(x + z) \leq f(x) + \frac{f(x + \alpha z) - f(x)}{\alpha},$$

and for continuously differentiable function

$$f(x + z) \leq f(x) + \nabla f(x) \cdot z \text{ as } \alpha \rightarrow 0.$$

The u -function defined in the proof of Afriat's theorem

$$u(\mathbf{x}) = \min_t \{u^t + \lambda^t \mathbf{p}^t(\mathbf{x} - \mathbf{x}^t)\}$$

has a natural interpretation. If $u(\mathbf{x})$ is also differentiable then it must satisfy the T FOCs (the gradient vector):

$$\nabla u(\mathbf{x}^t) = \lambda^t \mathbf{p}^t. \quad (*)$$

And since it is also concave it must satisfy the standard concavity conditions

$$u(\mathbf{x}^t) \leq u(\mathbf{x}^s) + \nabla u(\mathbf{x}^s)(\mathbf{x}^t - \mathbf{x}^s) \quad (**)$$

Substituting from (*) into (**), we have

$$u(\mathbf{x}^t) \leq u(\mathbf{x}^s) + \lambda^t \mathbf{p}^t(\mathbf{x}^t - \mathbf{x}^s).$$

The Afriat numbers can thus be interpreted as utility levels (u^t) and marginal utilities (λ^t) that are consistent with the observed choices.

- Using similar methods there are (finite) tests for: homotheticity, weak and additive separability, expected utility, and more.
- They involve checking to see whether a solution exists to a particular set of linear Afriat inequalities.
- Well known graph theory algorithms can be used to verify whether or not these conditions are satisfied.

Afriat's theorem has (at least) two remarkable implications:

- If there is a locally non-satiated u that rationalizes the data then there must exist a continuous, monotonic, and concave u that rationalizes the data.
- If the underlying u had the “wrong” curvature at some points, we would never observe choices being made at such points (do not satisfy the right 2nd-order conditions).

(1) Market data do not allow us to reject the hypotheses of convexity and monotonicity of preferences.

- Since GARP is a necessary and sufficient condition for u -maximization, it must imply conditions analogous to comparative statics results of classic demand theory.
- These include the Slutsky decomposition of a price change into the income and the substitution effects (for finite changes in a price rather than just infinitesimal changes).

(2) Since revealed preferences provide a complete set of the restrictions imposed by u -maximization, they must contain all information available about preferences.

The critical cost efficiency index (CCEI)

An obvious difficulty: GARP provides an exact test of u -maximization – either the data satisfy GARP or they do not.

- But choices involve at least some errors: compute incorrectly, execute intended choices incorrectly, err in other less obvious ways...
- Afriat (1972) suggested the following approach: for any number $0 \leq e \leq 1$, define the direct revealed preference relation $R^D(e)$ as

$$\mathbf{x}^t R^D(e) \mathbf{x} \text{ if } e \mathbf{p}^t \mathbf{x}^t \geq \mathbf{p}^t \mathbf{x}$$

and define $P^D(e)$ and $R(e)$ accordingly.

- Let e^* be the largest value of e such that the data $(\mathbf{p}^t, \mathbf{x}^t)$ satisfies GARP. Afriat's CCEI is the value of e^* associated with the data.
- e^* can be interpreted as saying that the consumer is 'wasting' as much as $1 - e^*$ of his income by making inefficient choices.
- The closer e^* to one, the smaller the 'perturbation' required to remove all violations and thus the closer the data are to satisfying GARP.

Recovering preferences and forecasting behavior: a brief outline

The tightest possible bounds on indifference curves through an allocation \mathbf{x}^0 not observed in the data $(\mathbf{p}^t, \mathbf{x}^t)$ for $t = 1, \dots, T$.

- Consider the set of prices at which \mathbf{x}^0 could be chosen and be consistent
 - does not add violations of GARP – with the previously observed data.
- This set of prices is the solution to the system of linear inequalities constructed from the data and revealed preference relations. Call this set $S(\mathbf{x}^0)$.
- Use $S(\mathbf{x}^0)$ to generate set of observations, $RP(\mathbf{x}^0)$ and $RW(\mathbf{x}^0)$, revealed preferred/worse than \mathbf{x}^0 .

- $RP(\mathbf{x}^0)$ is simply the convex monotonic hull of all allocations revealed preferred to \mathbf{x}^0 and $RW(\mathbf{x}^0)$ is constructed as follows:

$$\mathbf{x}^0 R^D \mathbf{x}^t \text{ for all prices } \mathbf{p}^0 \in S(\mathbf{x}^0)$$

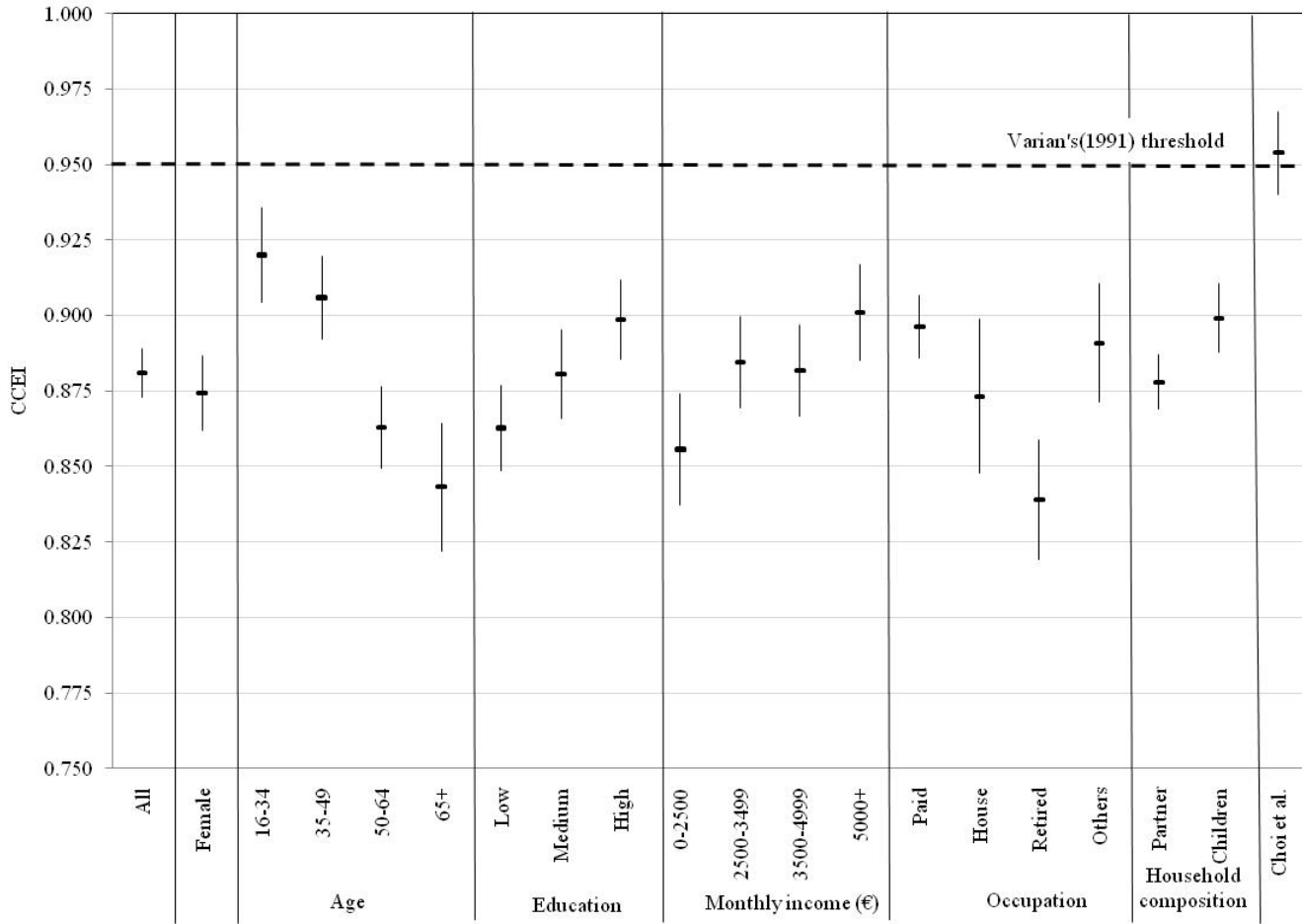
$$\Downarrow$$

$$\mathbf{x}^0 R \mathbf{x}^s \text{ for any } \mathbf{x}^s \text{ such that } \mathbf{x}^t R \mathbf{x}^s.$$

- $RP(\mathbf{x}^0)$ and the complement of $RW(\mathbf{x}^0)$ form the tightest inner and outer bounds on the set of allocations preferred to \mathbf{x}^0 .
- $RW(\mathbf{x}^0)$ and the complement of $RP(\mathbf{x}^0)$ form the tightest inner and outer bounds on the set of allocations worse than \mathbf{x}^0 .

Forecasting: \mathbf{x}^0 chosen from budget set \mathbf{p}^0 ? Use the same algorithm!!!

Mean CCEI scores



Wealth differentials

- ⇒ The heterogeneity in wealth is not well-explained either by standard observables (income, education, family structure) or by standard unobservables (intertemporal substitution, risk tolerance).
- ⇒ If consistency with utility maximization in the experiment were a good proxy for (financial) decision-making quality then the degree to which consistency differ across subjects should help explain wealth differentials.

	Ln(hhld wealth)			
CCEI	1.425** (0.565)	1.348* (0.714)	1.781** (0.746)	1.728** (0.750)
Combined CCEI		0.078 (0.381)	-0.091 (0.381)	-0.038 (0.384)
Risk attitude			-1.361 (0.838)	-1.366 (0.840)
Conscientiousness				0.103 (0.072)
Ln(hhld income '08)	0.601*** (0.127)	0.602*** (0.127)	0.520*** (0.121)	0.514*** (0.121)
Female	-0.228 (0.164)	-0.229 (0.164)	-0.299 (0.168)	-0.321 (0.169)
Age	-0.286 (0.316)	-0.284 (0.316)	-0.310 (0.319)	-0.282 (0.316)
Age ²	0.006 (0.005)	0.006 (0.005)	0.007 (0.005)	0.006 (0.005)
Age ³	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Partner	0.682*** (0.183)	0.682*** (0.183)	0.733*** (0.191)	0.714*** (0.191)
# of children	0.103 (0.092)	0.103 (0.093)	0.095 (0.095)	0.090 (0.095)
Education controls	Y	Y	Y	Y
Constant	5.932 (5.862)	5.888 (5.879)	7.797 (5.880)	7.371 (5.841)
R ²	0.1794	0.1778	0.1801	0.1819
# of obs.	517	517	494	494

	Ln(hhld wealth)		Fraction of hhld wealth			
			Checking	Saving	Stocks	House
CCEI	1.907** (0.751)	1.792** (0.705)	-0.100* (0.057)	-0.179* (0.096)	-0.003 (0.050)	0.336*** (0.129)
Ln(hhld income '08)	0.686*** (0.133)	0.25 (0.184)	-0.030** (0.013)	-0.062*** (0.021)	0.011 (0.010)	0.098*** (0.023)
Ln(hhld income '06)		0.467* (0.262)				
Ln(hhld income '04)		0.312* (0.176)				
Female	-0.144 (0.188)	-0.037 (0.184)	0.019 (0.018)	0.017 (0.029)	0.005 (0.012)	-0.039 (0.039)
partner	0.790*** (0.222)	0.740*** (0.222)	-0.033 (0.021)	-0.058* (0.033)	-0.008 (0.014)	0.136*** (0.044)
# of children	0.111 (0.107)	0.123 (0.101)	-0.004 (0.009)	-0.043*** (0.013)	0 (0.007)	0.069*** (0.019)
Constant	4.282 (5.947)	-0.415 (6.032)	0.045 (0.750)	1.42 (1.197)	-0.287 (0.388)	-0.777 (1.289)
R^2	0.236	0.269	0.051	0.104	0.029	0.146
# of obs.	377	377	512	502	514	479

Conclusion

Judgment about the quality of decision-making is generally made difficult by twin problems of *identification* and *measurement*:

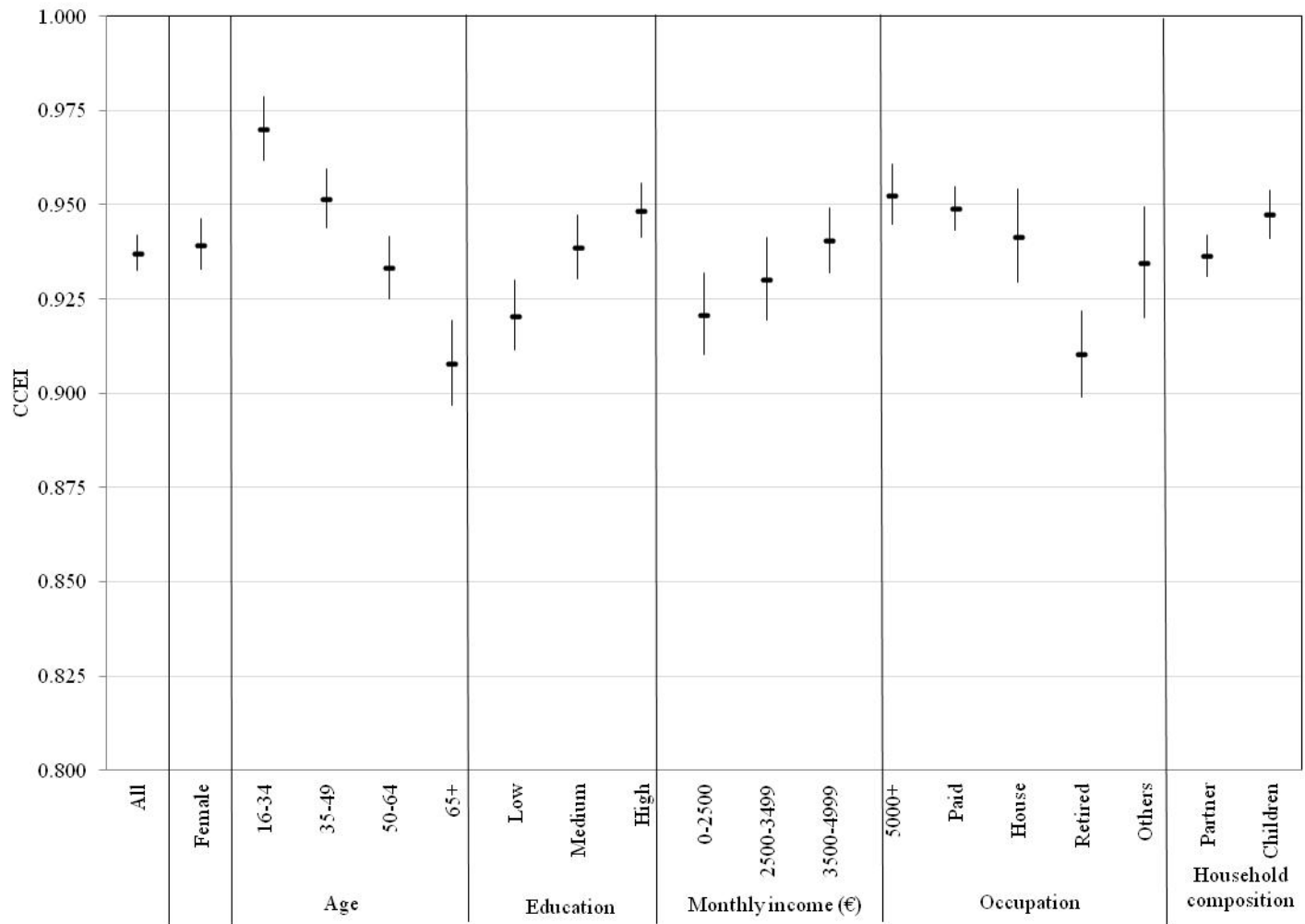
- **The identification problem** Distinguishing differences in decision-making quality from unobserved differences in preferences, information, beliefs or constraints.

Identification is important because welfare conclusions and thus (constrained) optimal policy will depend on the sources of any systematic differences in choices.

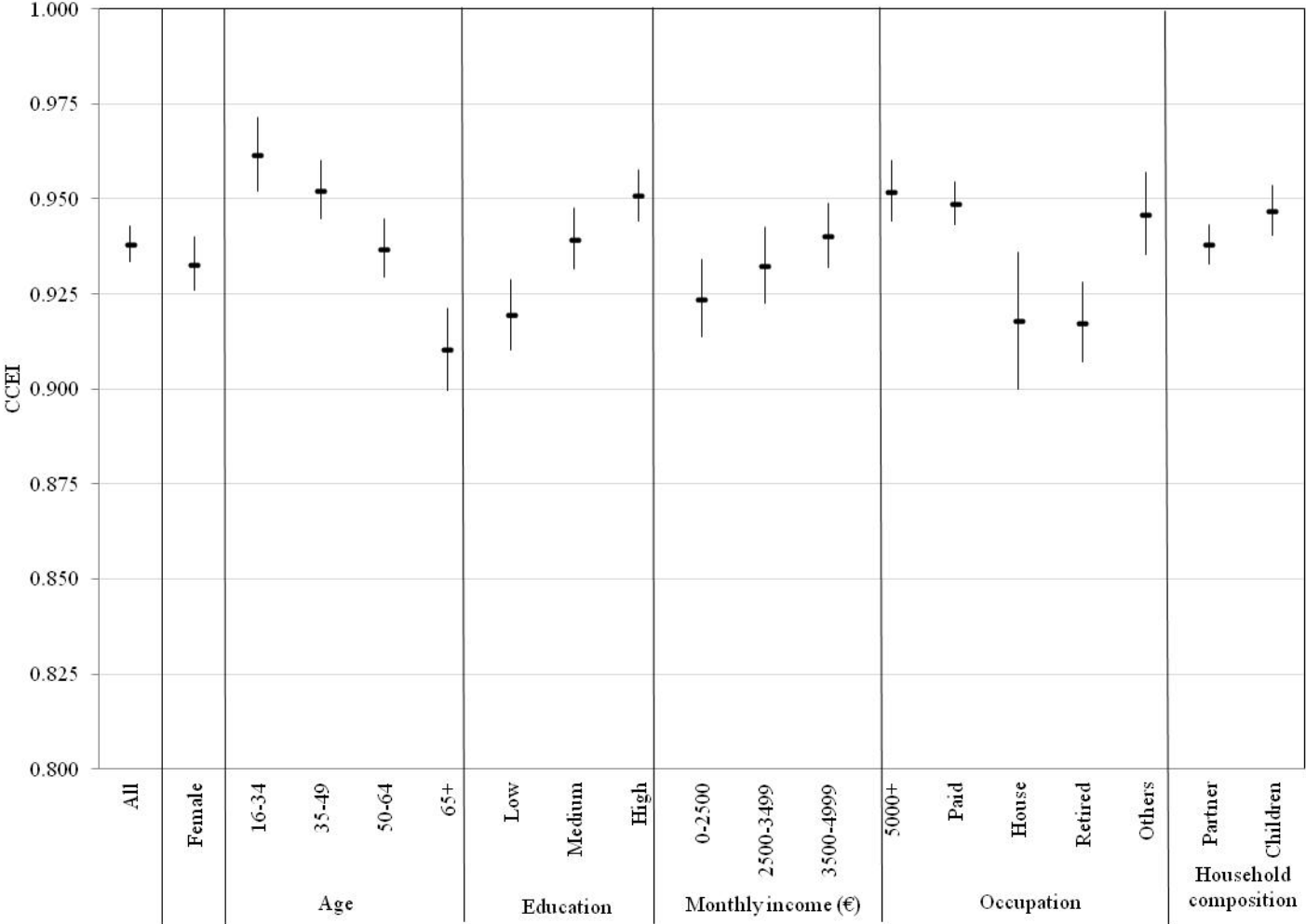
- **The measurement problem** Defining (and implementing) a *portable, practical, autonomous, quantifiable, and economically interpretable* measure of decision-making quality.

Ever since Laibson...

Mean CCEI scores: income in a few days and income 60 days after that



Mean CCEI scores: income in 60 days and income another 60 days after that



Stationarity, time invariance, and time consistency

- Time discount rates decline as tradeoffs are pushed into the temporal distance.
 - Subjects often choose the larger and later of two rewards when both are distant in time, but prefer the smaller and earlier one as both rewards draw nearer to the present.
- Interpreted as non-constant time discounting, these preference reversals have important implications.
 - Under standard assumptions, non-constant time discounting implies time-inconsistency – self-control problems and a demand for commitment thus emerge.

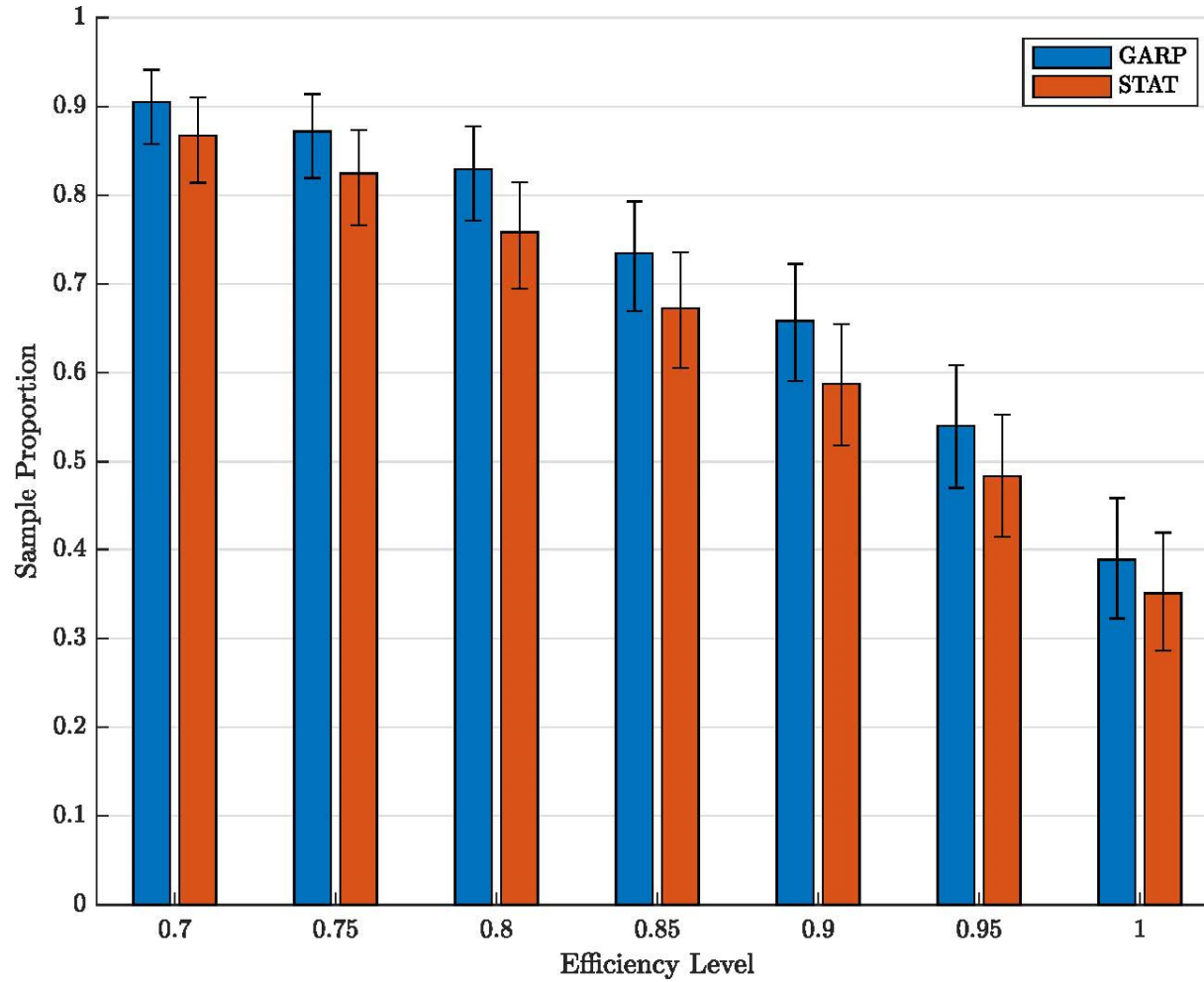
Stationarity

\succsim_t is stationary if for every $t, t' \geq 0$ and $\Delta_1, \Delta_2 \geq 0$

$$(x, t + \Delta_1) \sim_t (x', t + \Delta_2) \iff (x, t' + \Delta_1) \sim_t (x', t' + \Delta_2).$$

Ranking does not depend on the distance from t . Tested in the standard static experiment.

Stationarity



Exponential vs. quasi-hyperbolic

