Economics 209B
Behavioral / Experimental Game Theory
(Spring 2008)

Lecture 5: Alternative Equilibria – Cursed Equilibrium

Introduction

- Game theory assumes rationality and focuses on Nash equilibrium and its refinements.

- Players are rational not only in selecting their actions but also in forming beliefs about the other players.

- In reality, agents have systematically biased beliefs and use decision rules that are inconsistent with rationality.

Rational behavior

- The assumption of rational choice in a model of human behavior is not as restrictive as it sounds:
  - consistent preferences over all possible alternatives, and choices that correspond to the most preferred alternative from the feasible set.

- While consistency is an empty box (no *a priori* reason to rule out status, power, envy, altruism), equilibrium is a more restrictive concept.

Consider a state space Ω, a (subjective) probability measure over Ω, a set of actions A, a C set of consequences, a consequence function

$$g : A \times \Omega \rightarrow C.$$

Given a preference relation $\succsim$ on the set C and any set $B \subseteq A$ of actions, a rational agent chooses an action $a^* \in B$ such that

$$g(a^*) \succsim g(a)$$

for all $a \in B$.

- Modify Nash equilibrium to incorporate realistic limitations to rational choice modeling of games.

- In QRE, players do not choose best response with probability one (better response instead of best responses).

- But players have rational expectations – players' beliefs are correct, on average.

Cursed equilibrium

- An epistemic approach to investigate the strategic implications of systematic biases in Bayesian games.

- Players best response but hold systematic biases about the other players' actions (not strategies).

- Players underestimate the correlation between the other players' actions and their private information.

- Closely related literature (solution concepts that are based on bounded rationality):
  - Psychological motivations – Rabin (1993)
  - Optimistic beliefs – Yildiz (2007)
  - Unawareness – Feinberg (2004, 2005)
  - Limited foresight – Jehiel (1995)
  - Quantal response equilibrium – Mckelvey and Palfrey (1995)
  - Procedural rationality – Osborne and Rubinstein (1998)

An example - Akerlof (1970)

A car is a lemon, worth \$0 to both seller (s) and buyer (b), or a peach, worth \$3,000 to b and \$2,000 to s. Suppose b believes each occurs with prob. 1/2.

A χ-cursed b believes that with prob. χ s sells with prob. 1/2 irrespective of the type of car, so that the car being sold is a peach with prob.

$$(1-\chi)\cdot 0+\chi\cdot 1/2=\chi/2$$

and therefore worth $1,500\cdot\chi$. Hence, a buyer cursed by $\chi>2/3$ will wish to buy the car.

- A standard Bayesian game where players' private information is represented by their (payoff) types.

- Each player believes that with prob. χ other players playing their *average* distribution of actions rather than their type-contingent strategy.

- The extent to which a player is "cursed" is given by $\chi \in [0, 1]$. Setting $\chi = 0$ corresponds to the fully rational Bayesian Nash equilibrium.

Main results

Consider a finite Bayesian game $G = (A, T, p, u)$ where

– A_k - a finite set of player k's actions and $A \equiv A_1 \times A_2 \times \cdots \times A_N$

– T_k - a finite set of player k's types and $T \equiv T_0 \times T_1 \times T_2 \times \cdots \times T_N$

– p - a common prior (puts positive weight on each $t_k \in T_k$)

– $u_k : A \times T \to \mathbb{R}$ - player k's payoff function

A (mixed) strategy σ_k for player k specifies a probability distribution over actions for each type $\sigma_k : T_k \rightarrow \Delta A_k$ so $\sigma_k(a_k \,|t_k)$ is the probability that type t_k plays action a_k.

A strategy profile σ is a Bayesian Nash equilibrium if for each player k, each type $t_k \in T_k$, and each a_k^* such that $\sigma_k(a_k^* \,|t_k) > 0$,

$$\begin{aligned} a_k^* \;\in\; & \operatorname*{arg\,max}_{a_k \in A_k} \sum_{t_{-k} \in T_{-k}} p_k(t_{-k} \,|t_k) \\ & \times \sum_{a_{-k} \in A_{-k}} \sigma_{-k}(a_{-k} \,|t_{-k}) u_k(a_k, a_{-k}; t_k, t_{-k}). \end{aligned}$$

For each type t_k of each player k, consider the average strategy of other players (averaged over the other players' types) by

$$\bar{\sigma}_{-k}(a_{-k}\,|t_k) \equiv \sum_{t_{-k}\in T_{-k}} p_k(t_{-k}\,|t_k)\sigma_{-k}(a_{-k}\,|t_{-k}).$$

A strategy profile σ is a <u>χ-cursed equilibrium</u> if for each player k, each type $t_k \in T_k$, and each a_k^* such that $\sigma_k(a_k^*\,|t_k) > 0$,

$$\begin{aligned} a_k^* \in\; & \arg\max_{a_k\in A_k} \sum_{t_{-k}\in T_{-k}} p_k(t_{-k}\,|t_k) \\ & \times \sum_{a_{-k}\in A_{-k}} [\chi\bar{\sigma}_{-k}(a_{-k}\,|t_k) + (1-\chi)\sigma_{-k}(a_{-k}\,|t_{-k})] \\ & \times u_k(a_k, a_{-k}; t_k, t_{-k}). \end{aligned}$$

Let $\hat{p}_{t_k}(t_{-k} \,|a_{-k}, \sigma_{-k})$ be type t_k of player k's beliefs about the prob. of facing type t_{-k} of players $j \neq k$ when they play action profile a_{-k} under strategy σ_{-k}.

In a χ-cursed equilibrium, for each player k,

$$\hat{p}_{t_k}(t_{-k} \,|a_{-k}, \sigma_{-k}) = [\chi + (1 - \chi)\frac{\sigma_{-k}(a_{-k} \,|t_{-k})}{\bar{\sigma}_{-k}(a_{-k} \,|t_k)}]p_k(t_{-k} \,|t_k).$$

Result I: *If* $G = (A, T, p, u)$ *is a finite Bayesian game, then for each* $\chi \in [0, 1]$, G *has a* χ*-cursed equilibrium.*

Proof (a separating pure-strategy equilibrium): Each type of each player plays a different pure strategy. When t_k observes the action a_{-k} played by types t_{-k}, he believes he is facing t_{-k} with prob.

$$1 - \chi + \chi p_k(t_{-k} | t_k)$$

and facing $t'_{-k} \neq t_{-k}$ with prob.

$$\chi p_k(t'_{-k} | t_k).$$

In a cursed equilibrium, each player k plays best replies to these beliefs.

Thus, he acts as if his payoff from playing action a_k when facing action a_{-k} and type profile t_{-k} is

$$\begin{aligned}\bar{u}_k^{\chi}(a_k, a_{-k}; t_k, t_{-k}) \ &\equiv \ (1-\chi)u_k(a_k, a_{-k}; t_k, t_{-k}) \\ &+\chi \sum_{\tau_{-k} \in T_{-k}} p_k(t_{-k} \,|t_k) u_k(a_k, a_{-k}; t_k, \tau_{-k}).\end{aligned}$$

A χ-cursed equilibrium of $G = (A, T, p, u)$ is a Bayesian Nash equilibrium of $\bar{G}^{\chi} = (A, T, p, \bar{u}^{\chi})$ (whenever G is finite, $\bar{G}^{\chi}$ is finite).

Result II: *If a pooling strategy profile* σ *is a* χ*-cursed equilibrium for some* $\chi \in [0, 1]$, *then* σ *is a* χ*-cursed equilibrium for each* $\chi' \in [0, 1]$.

Proof: In a pooling equilibrium, players' actions are independent of their types, so ignoring the relationship between others' actions and their information is not a mistake.

Applications

- Bilateral trade (no-trade theorems)
- Common-values auctions (winner's curse)
- Elections (swing-voter's curse)