Economics 209B
Behavioral / Experimental Game Theory
(Spring 2008)

Lecture 2: Games with Pure Information Externalities

Background

- Social learning describes any situation in which individuals learn by observing the behavior of others.

- Several economic theories explain the existence of uniform social behavior:
  - benefits from conformity
  - sanctions imposed on deviants
  - network / payoff externalities
  - social learning.

The canonical model of social learning

- A set of players N, a finite set of actions $\mathcal{A}$, a set of states of nature Ω, and a common payoff function

$$U(a, \omega)$$

where $a \in \mathcal{A}$ is the action chosen and $\omega \in \Omega$ is the state of nature.

- Player i receives a private signal $\sigma_i(\omega)$, a function of the state of nature ω, and uses this private information to identify a payoff-maximizing action.

The canonical assumptions

- Bayes-rational behavior
- Incomplete and asymmetric information
- Pure information externality
- Once-in-a-lifetime decisions
- Exogenous sequencing
- Perfect information

Direct methodological extensions

- Caplin and Leahy (*AER* 1994)
- Chamley and Gale (*ECM* 1994)
- Bala and Goyal (*RES* 1998)
- Avery and Zemsky (*AER* 1999)
- Çelen and Kariv (*GEB* 2004)
- Gale and Kariv (*GEB* 2004)

The model of BHW (JPE 1992)

- There are two decision-relevant events, say A and B, equally likely to occur *ex ante* and two corresponding signals a and b.

- Signals are informative in the sense that there is a probability higher than $1/2$ that a signal matches the label of the realized event.

- The decision to be made is a prediction of which of the events takes place, basing the forecast on a private signal and the history of past decisions.

- Whenever two consecutive decisions coincide, say both predict A, the subsequent player should also choose A even if his signal is different b.

- Despite the asymmetry of private information, eventually every player imitates her predecessor.

- Since actions aggregate information poorly, despite the available information, such herds / cascades often adopt a suboptimal action.

- Anderson and Holt (*AER* 1997) investigate the social learning model of BHW experimentally.

- They report that "rational" herds / cascades formed in most rounds and that about half of the cascades were incorrect.

- Extensions: Hung and Plott (*AER* 2001), Kübler and Weizsäcker (*RES* 2004), Goeree, Palfrey, Rogers and McKelvey (*RES* 2007).

The model of Smith and Sørensen (ECM 2000)

- Two phenomena that have elicited particular interest are *informational cascades* and *herd behavior*.
  - Cascade: players 'ignore' their private information when choosing an action.
  - Herd: players choose the same action, not necessarily ignoring their private information.
- Smith and Sørensen (2000) show that with a continuous signal space herd behavior arises, yet there need be no informational cascade.

The model of Çelen and Kariv (GEB 2004)

Signals

- Each player $n \in \{1, ..., N\}$ receives a signal θ_n that is private information.

- For simplicity, $\{\theta_n\}$ are independent and uniformly distributed on $[-1, 1]$.

Actions

- Sequentially, each player n has to make a binary irreversible decision $x_n \in \{0, 1\}$.

Payoffs

- $x = 1$ is profitable if and only if $\sum_{n \leq N} \theta_n \geq 0$, and $x = 0$ is profitable otherwise.

Information

- Perfect information

$$\mathcal{I}_n = \{\theta_n, (x_1, ..., x_{n-1})\}$$

- Imperfect information

$$\mathcal{I}_n = \{\theta_n, x_{n-1}\}$$

The decision problem

– The optimal decision rule is given by

$$x_n = 1 \text{ if and only if } \mathbb{E}\left[\sum_{i=1}^{N} \theta_i \mid \mathcal{I}_n\right] \geq 0.$$

Since $\mathcal{I}_n$ does not provide any information about the content of successors' signals, we obtain

$$x_n = 1 \text{ if and only if } \theta_n \geq -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n\right].$$

The cutoff process

– For any n, the optimal strategy is the *cutoff strategy*

$$x_n = \begin{cases} 1 & if \quad \theta_n \geq \hat{\theta}_n \\ 0 & if \quad \theta_n < \hat{\theta}_n \end{cases}$$

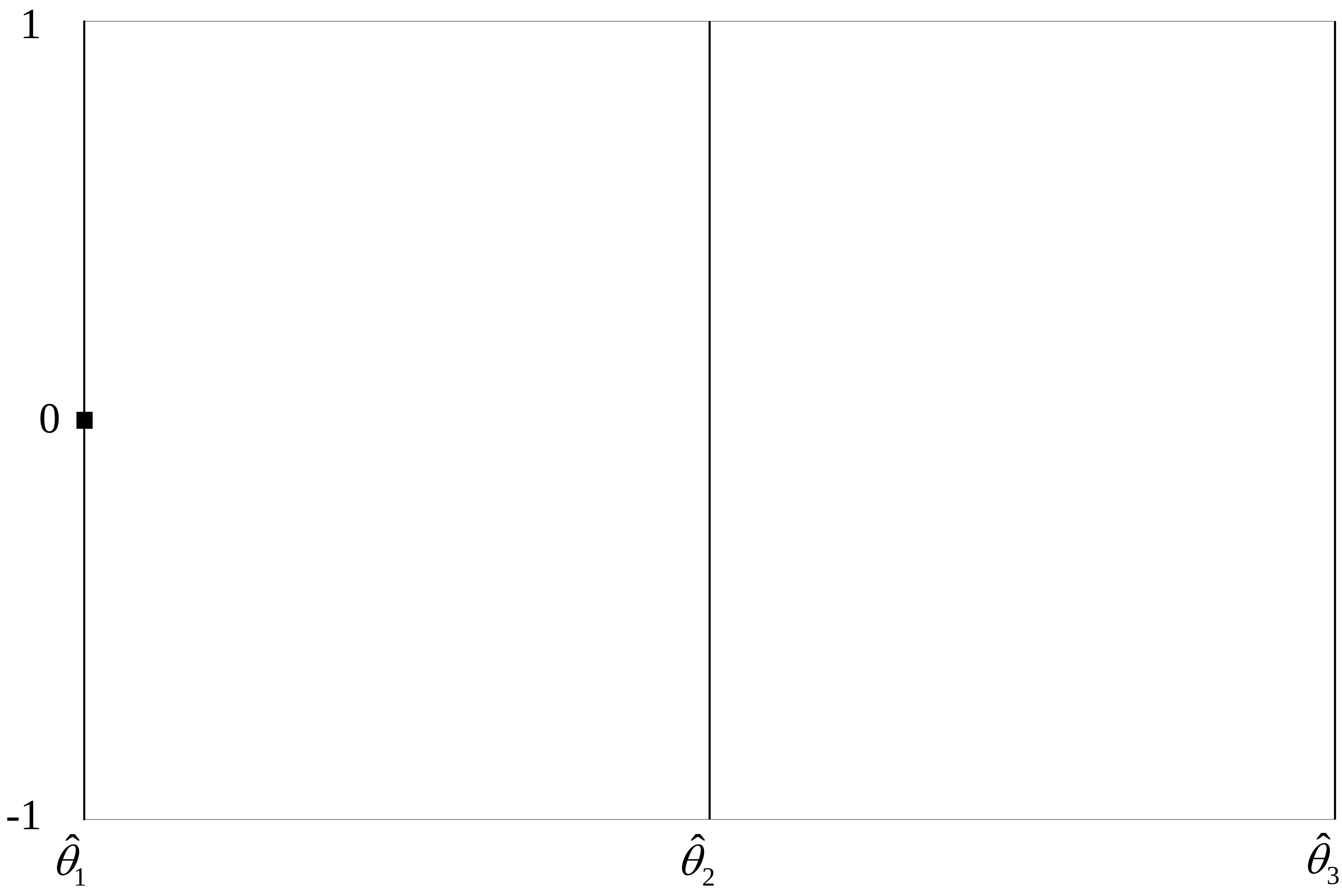
where

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n\right]$$

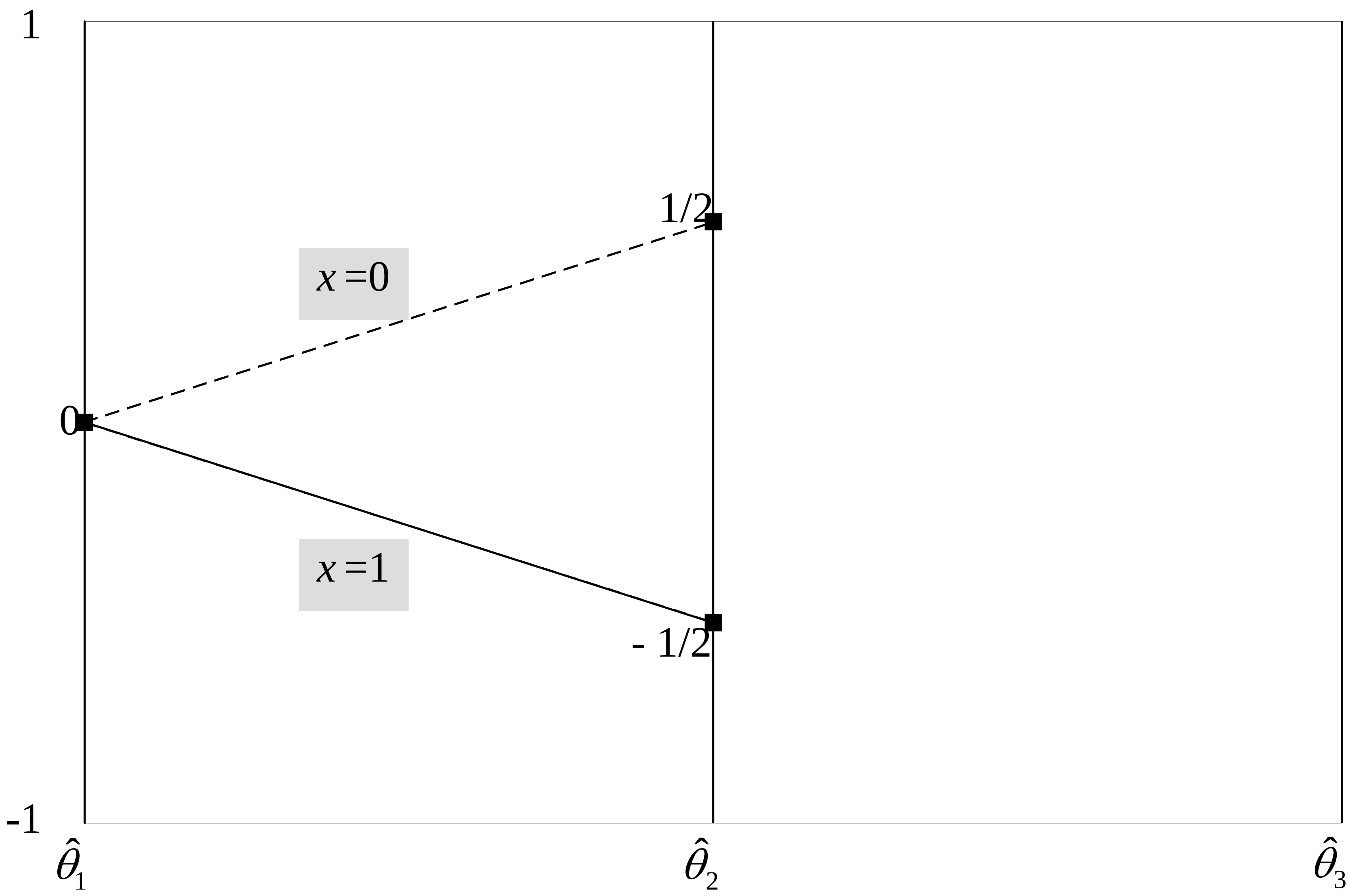
is the optimal history-contingent cutoff.

– $\hat{\theta}_n$ is sufficient to characterize the individual behavior, and $\{\hat{\theta}_n\}$ characterizes the social behavior of the economy.

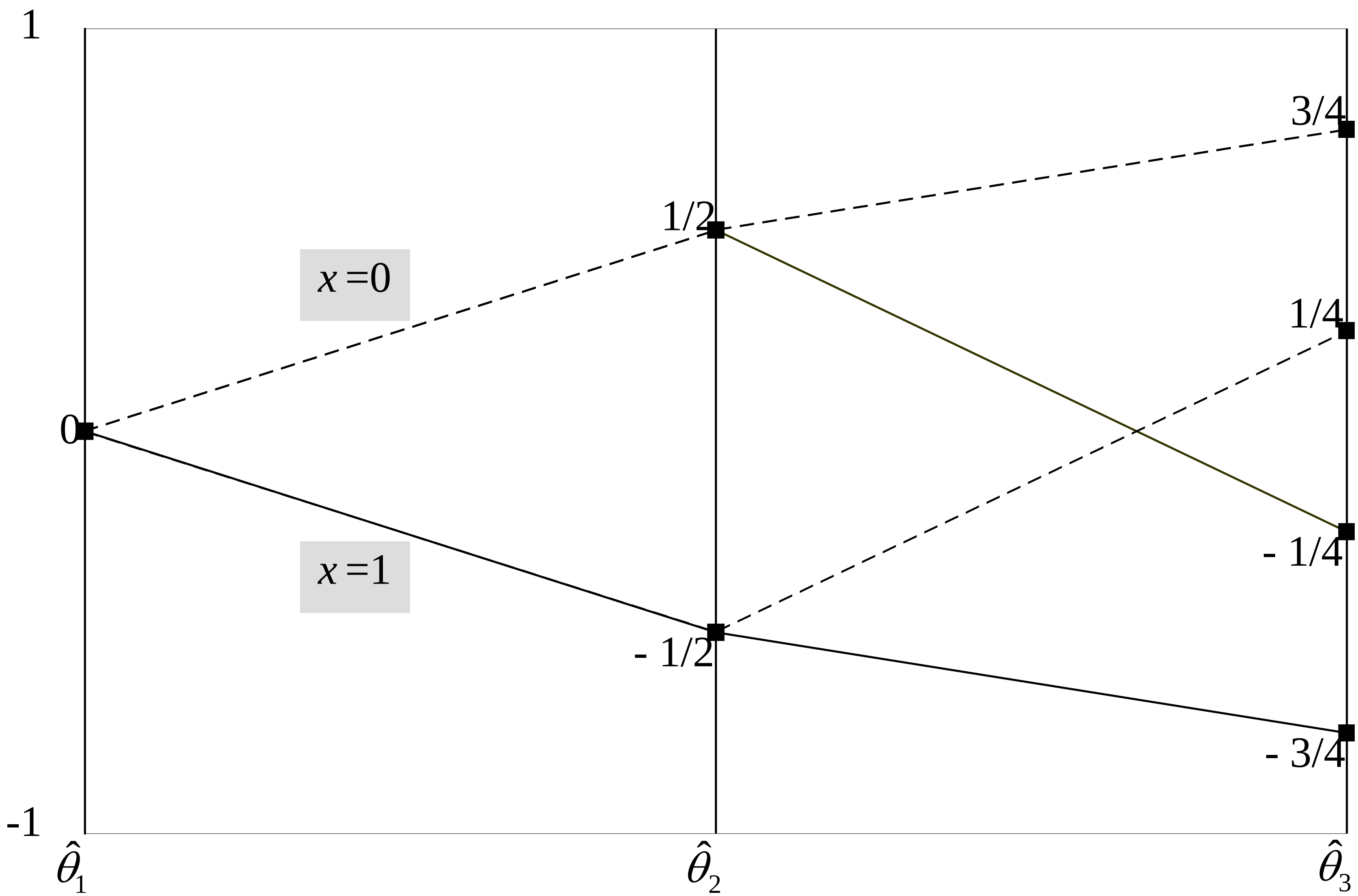
A three-agent example



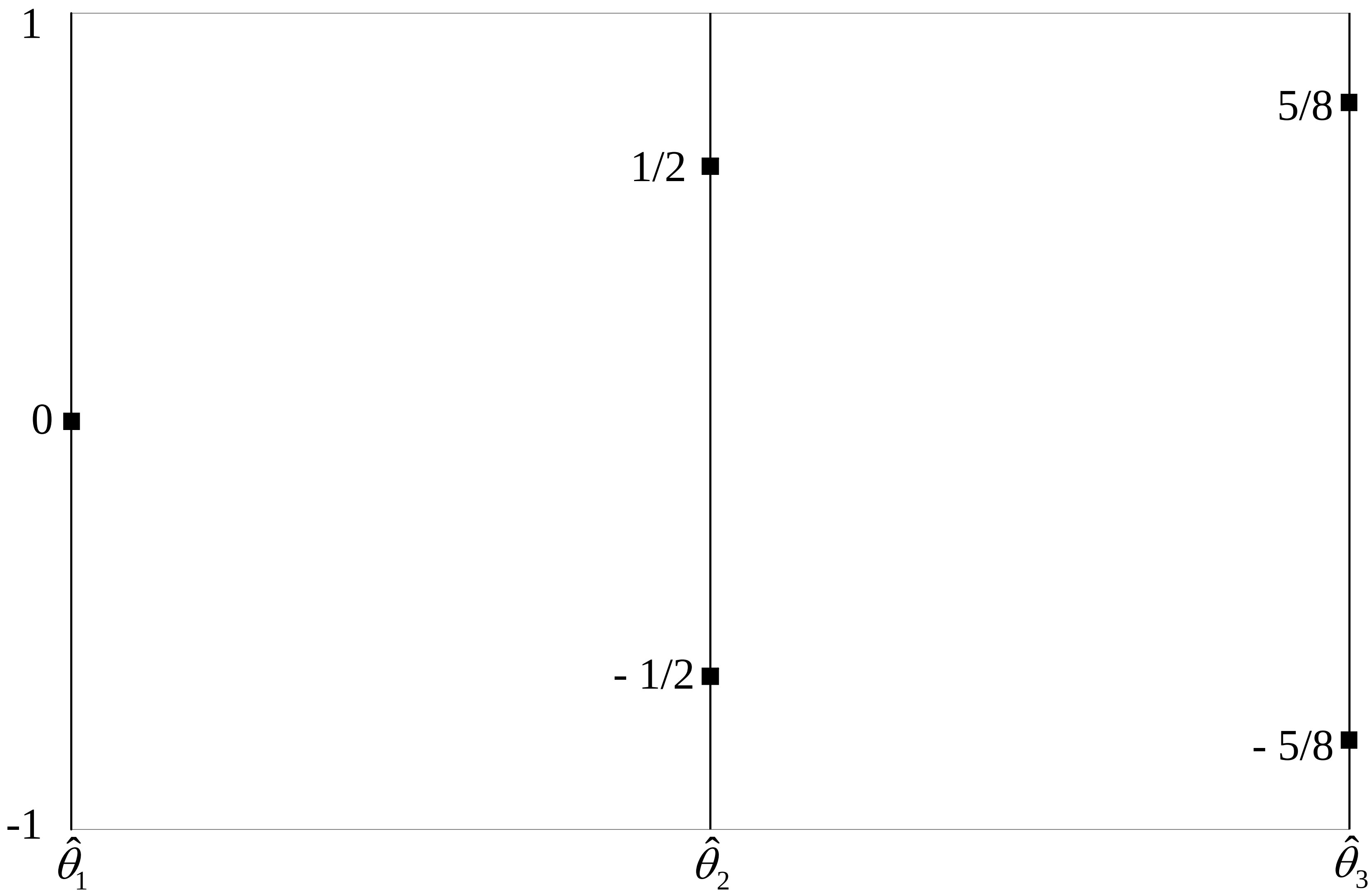
A three-agent example



A three-agent example under perfect information



A three-agent example under imperfect information

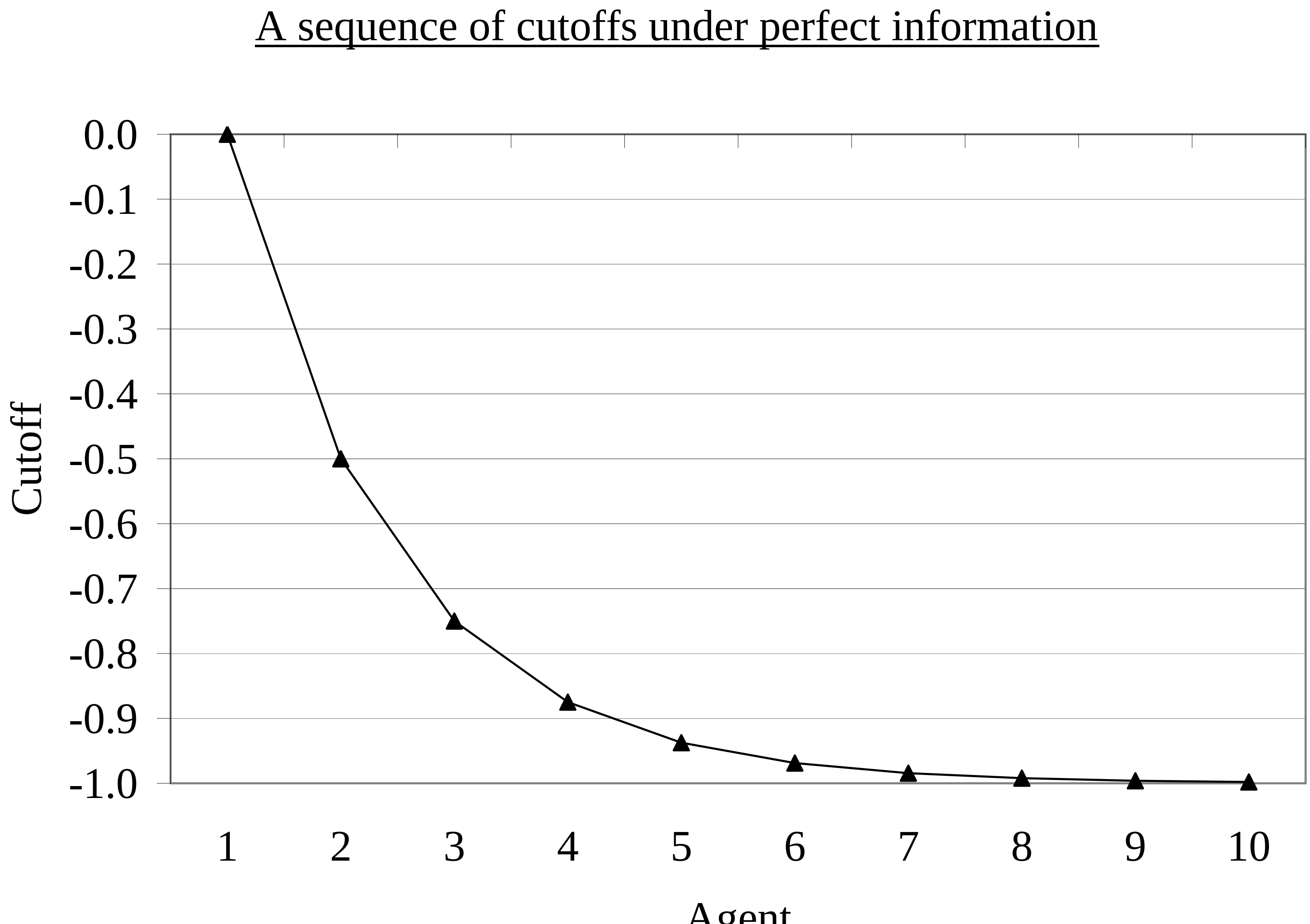


The case of perfect information

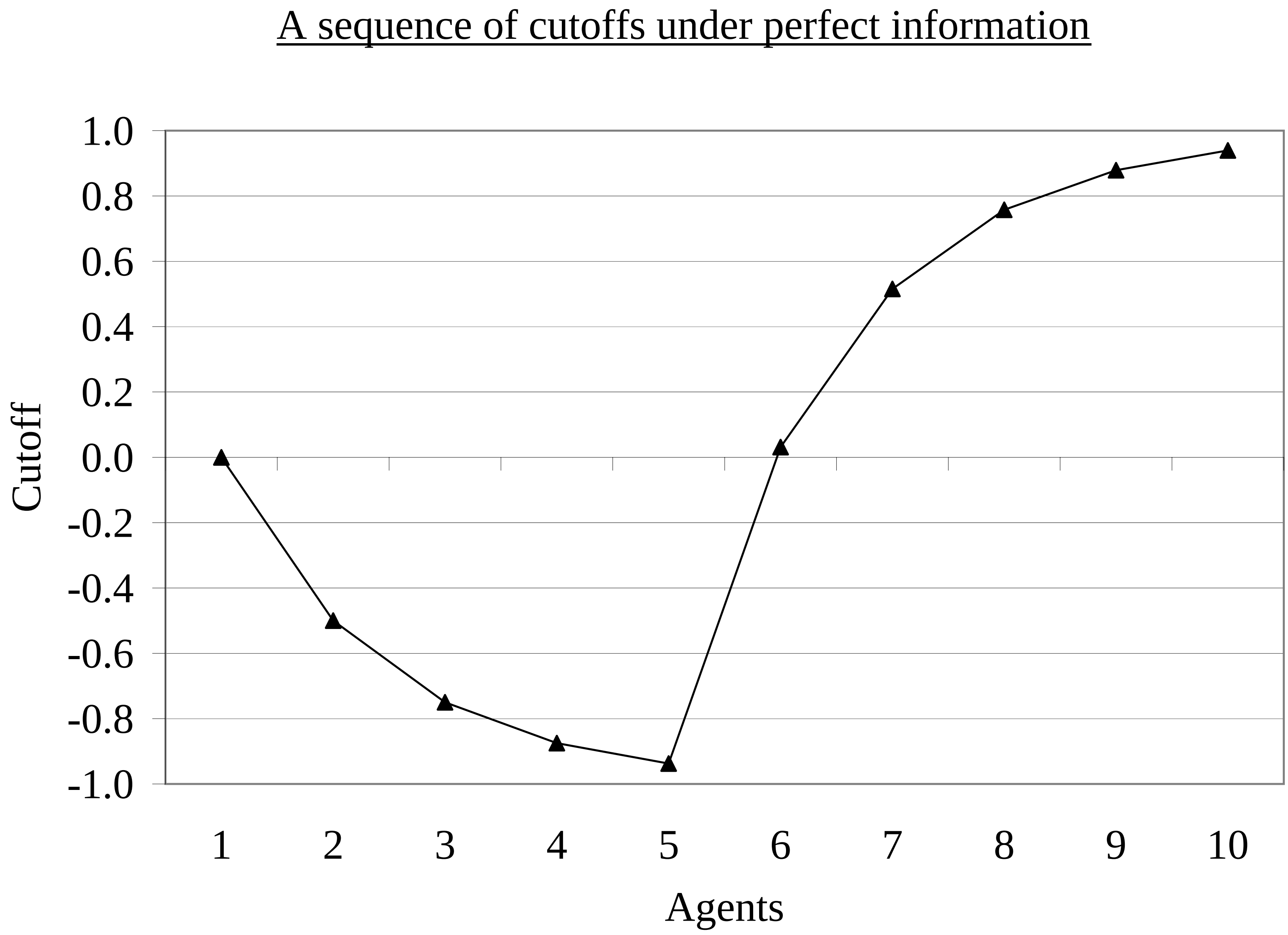
The cutoff dynamics follows the cutoff process

$$\hat{\theta}_n = \begin{cases} \frac{-1+\hat{\theta}_{n-1}}{2} & \text{if} \quad x_{n-1} = 1 \\ \frac{1+\hat{\theta}_{n-1}}{2} & \text{if} \quad x_{n-1} = 0 \end{cases}$$

where $\hat{\theta}_1 = 0$.



A sequence of cutoffs under perfect information
Cutoff
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
1
2
3
4
5
6
7
8
9
10
Agent


A sequence of cutoffs under perfect information
Cutoff
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
1
2
3
4
5
6
7
8
9
10
Agents

Informational cascades

- $-1 < \hat{\theta}_n < 1 \; \forall n$ so any player takes his private signal into account in a non-trivial way.

Herd behavior

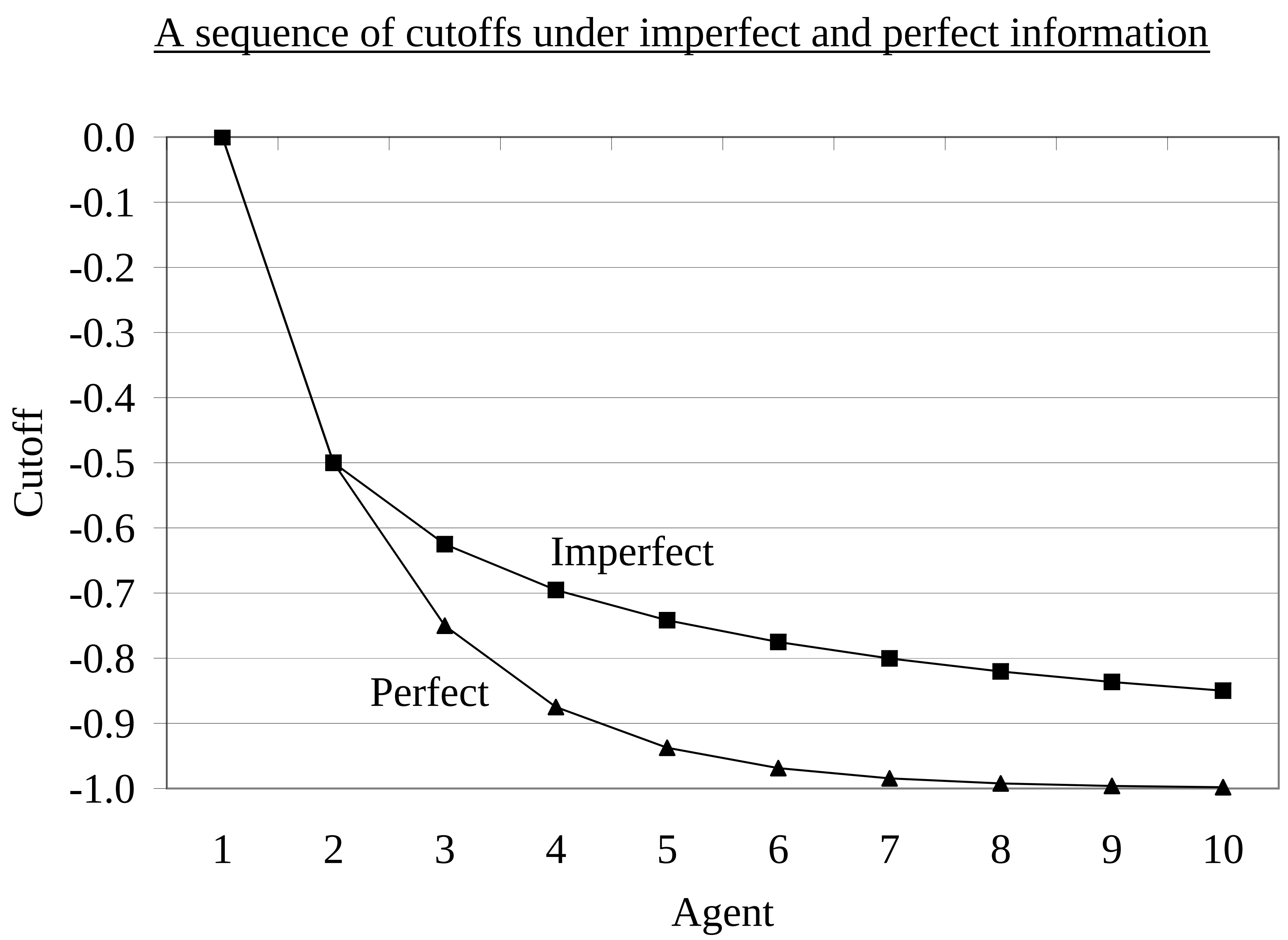
- $\{\hat{\theta}_n\}$ has the martingale property by the Martingale Convergence Theorem a limit-cascade implies a herd.

The case of imperfect information

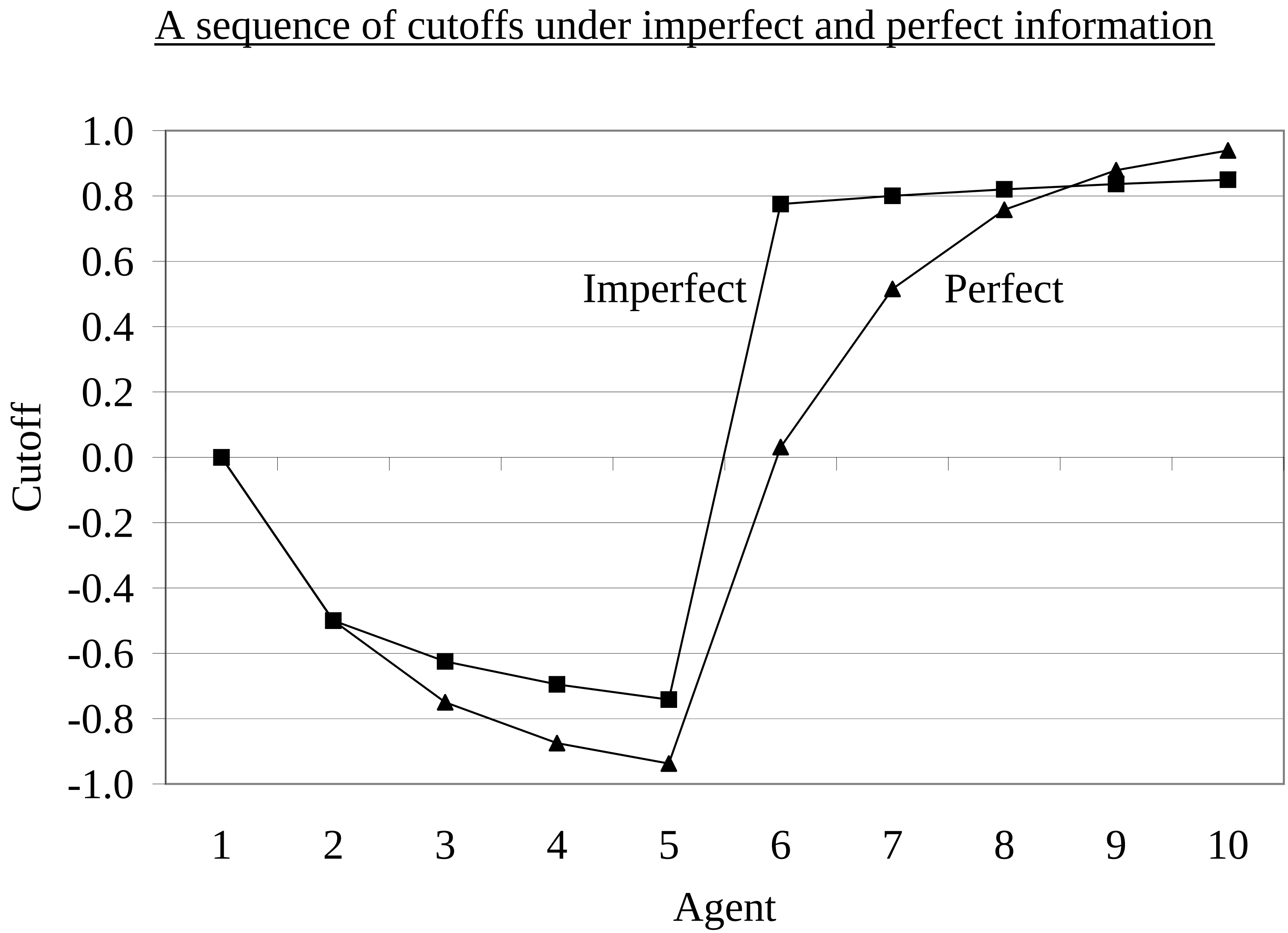
The cutoff dynamics follows the cutoff process

$$\hat{\theta}_n = \begin{cases} -\frac{1+\hat{\theta}_{n-1}^2}{2} & \text{if} \quad x_{n-1} = 1 \\ \frac{1+\hat{\theta}_{n-1}^2}{2} & \text{if} \quad x_{n-1} = 0 \end{cases}$$

where $\hat{\theta}_1 = 0$.



A sequence of cutoffs under imperfect and perfect information
Cutoff
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
Imperfect
Perfect
1
2
3
4
5
6
7
8
9
10
Agent


A sequence of cutoffs under imperfect and perfect information
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
Cutoff
Imperfect
Perfect
1
2
3
4
5
6
7
8
9
10
Agent

Informational cascades

– $-\mathbf{1} < \hat{\theta}_n < \mathbf{1}\ \forall n$ so any player takes his private signal into account in a non-trivial way.

Herd behavior

– $\{\hat{\theta}_n\}$ is not convergent and the divergence of cutoffs implies divergence of actions.

– Behavior exhibits periods of uniform behavior, punctuated by increasingly rare switches.

Takeaways

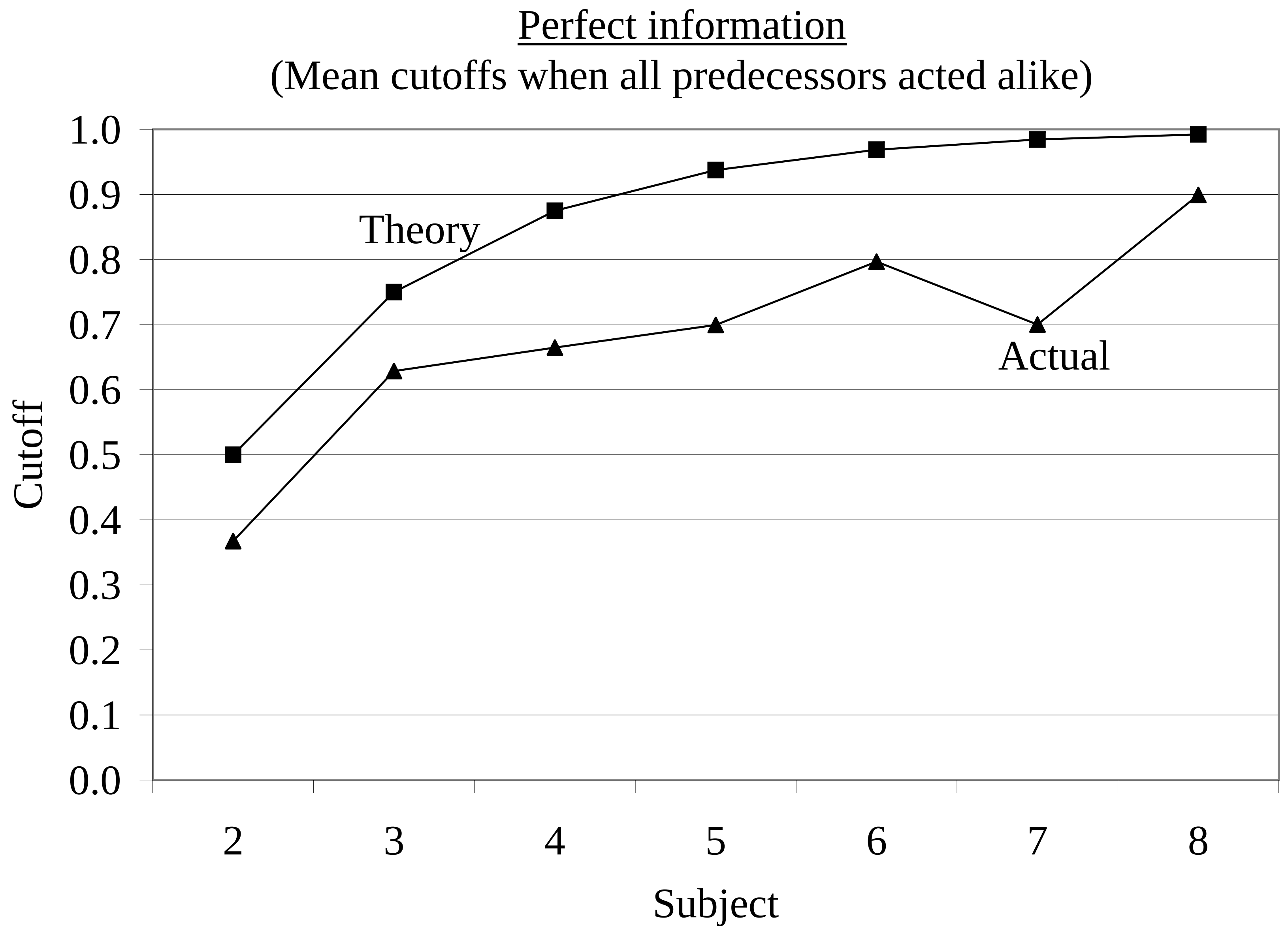
- The dynamics of social learning depend crucially on the extensive form of the game.
- Longer and longer periods of uniform behavior, punctuated by (increasingly rare) switches.
- A succession of fads: starting suddenly, expiring easily, each replaced by another fad.
- Why do markets move from ‘boom’ to ‘crash’ without settling down?

The experiments of Çelen and Kariv (AER 2004, ET 2005)

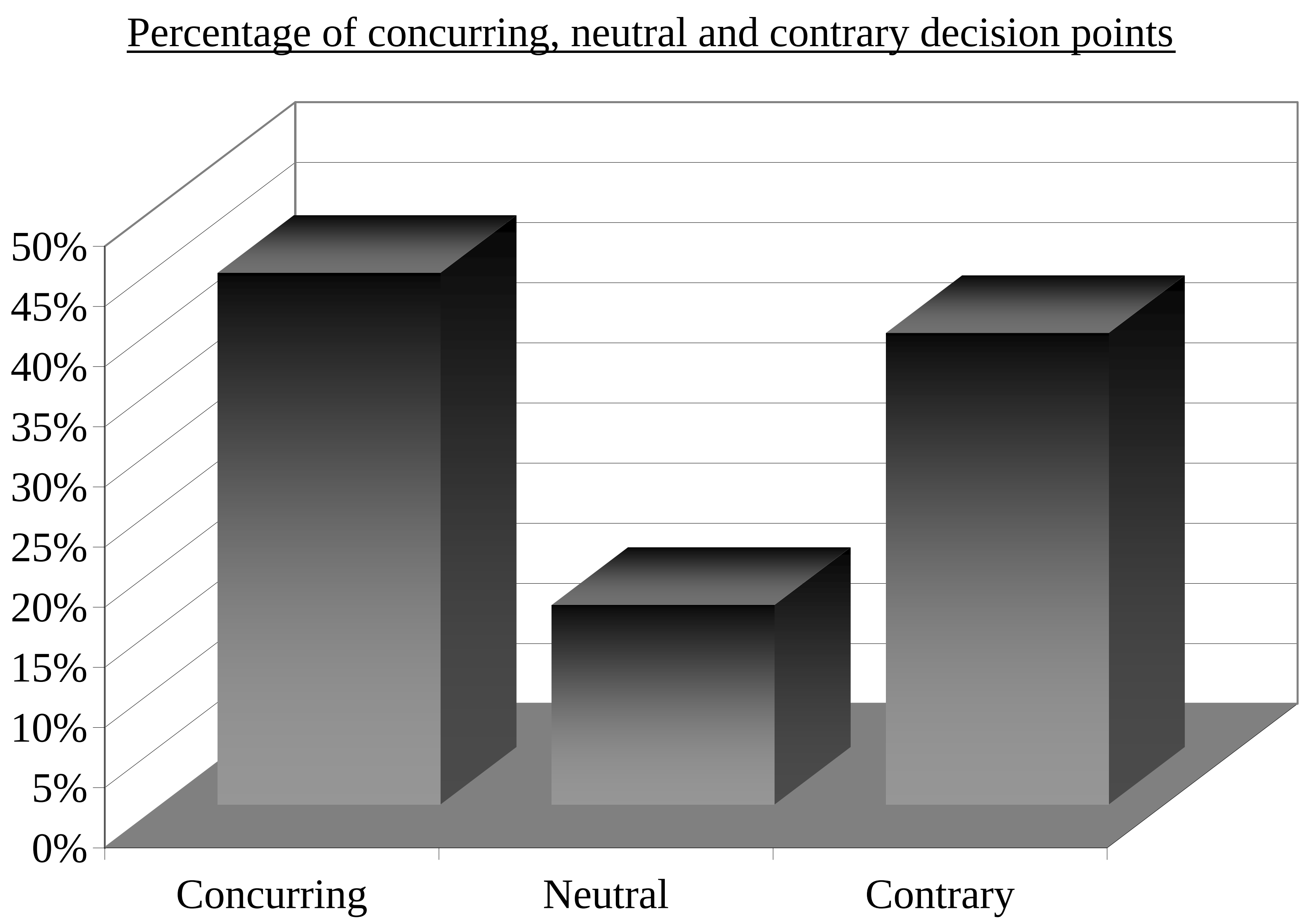
- In market settings, we observe behavior but not beliefs or private information.

- In the laboratory, we can elicit subjects' beliefs and control their private information.

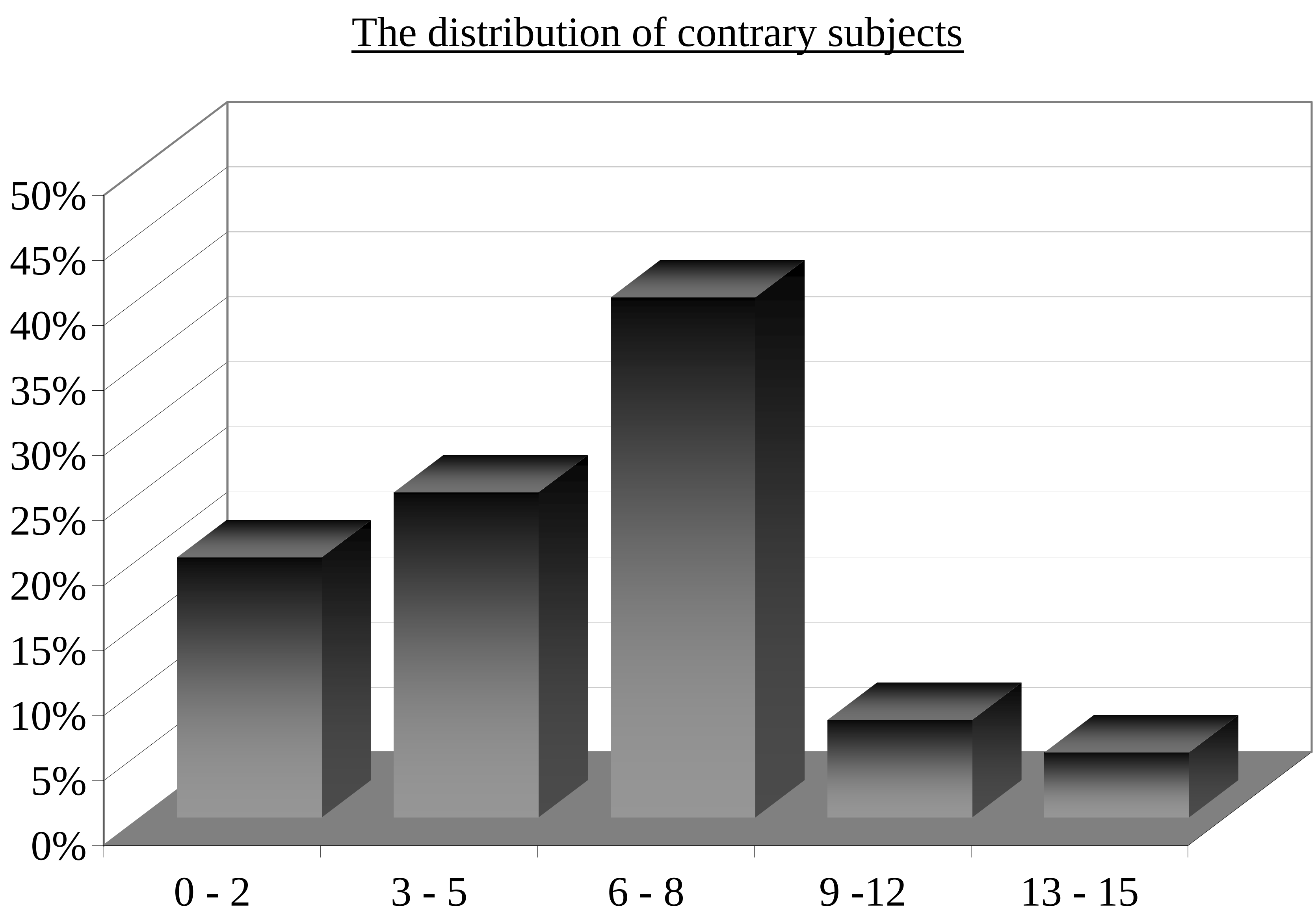
- Test the model's predictions and study the effects of variables about which our existing theory has little to say.



Perfect information
(Mean cutoffs when all predecessors acted alike)
Theory
Actual
Cutoff
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
2
3
4
5
6
7
8
Subject



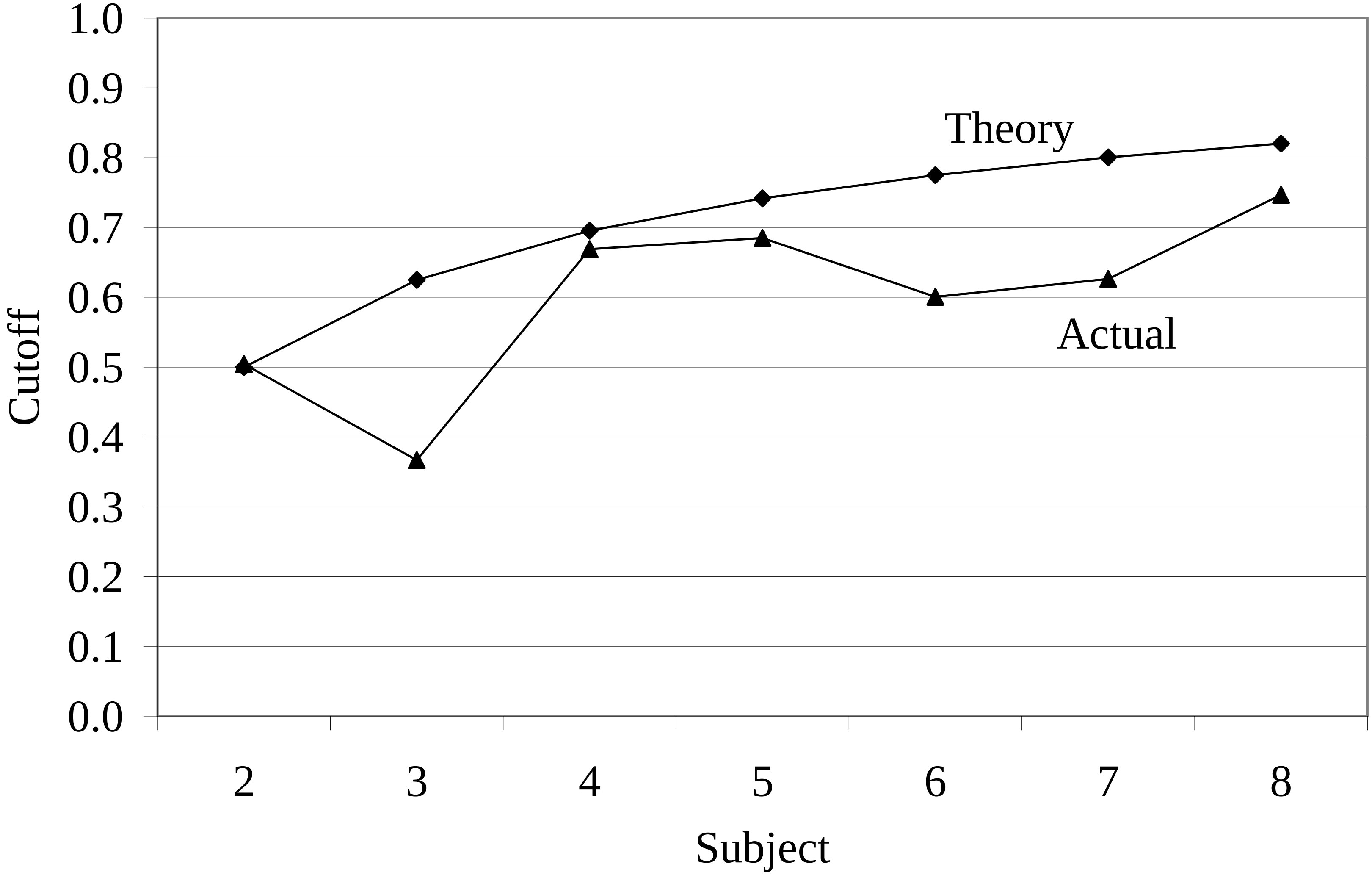
Percentage of concurring, neutral and contrary decision points
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Concurring
Neutral
Contrary

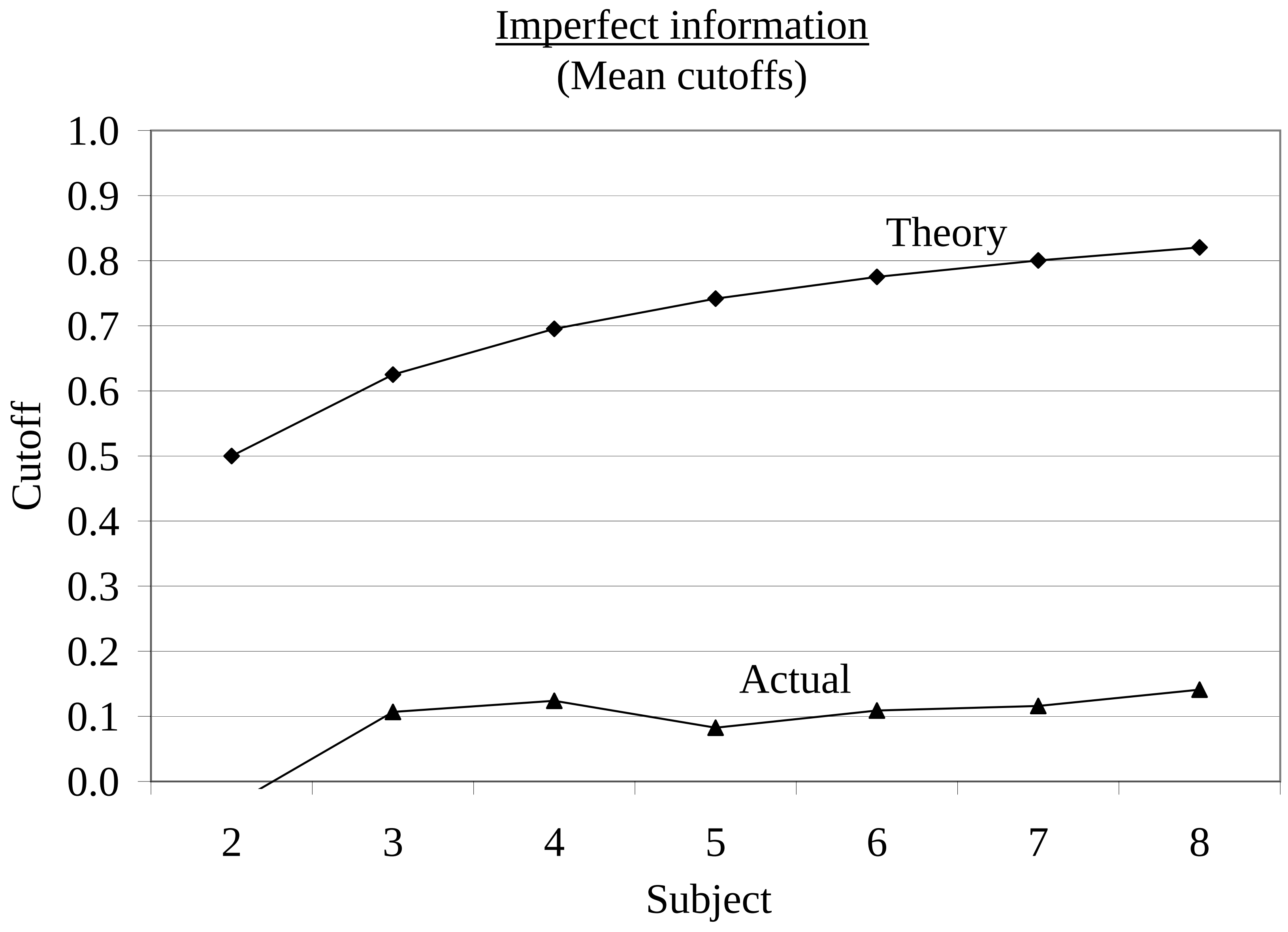


The distribution of contrary subjects
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
0 - 2
3 - 5
6 - 8
9 -12
13 - 15

Imperfect information

(Mean cutoffs in concurring decisions)




Imperfect information
(Mean cutoffs)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Cutoff
Theory
Actual
2
3
4
5
6
7
8
Subject

The econometric analysis

- At each decision turn n, with probability p_n a player is rationally, and with probability $1 - p_n$ he is noisy.

- The cutoff of a noisy player is a random draw from a distribution function G_n with support $[-1, 1]$ and mean $\tilde{\theta}_n$.

- Others cannot observe whether a player behavior is noisy, but the sequences $\{p_n\}$ and $\{G_n\}$ are common knowledge.

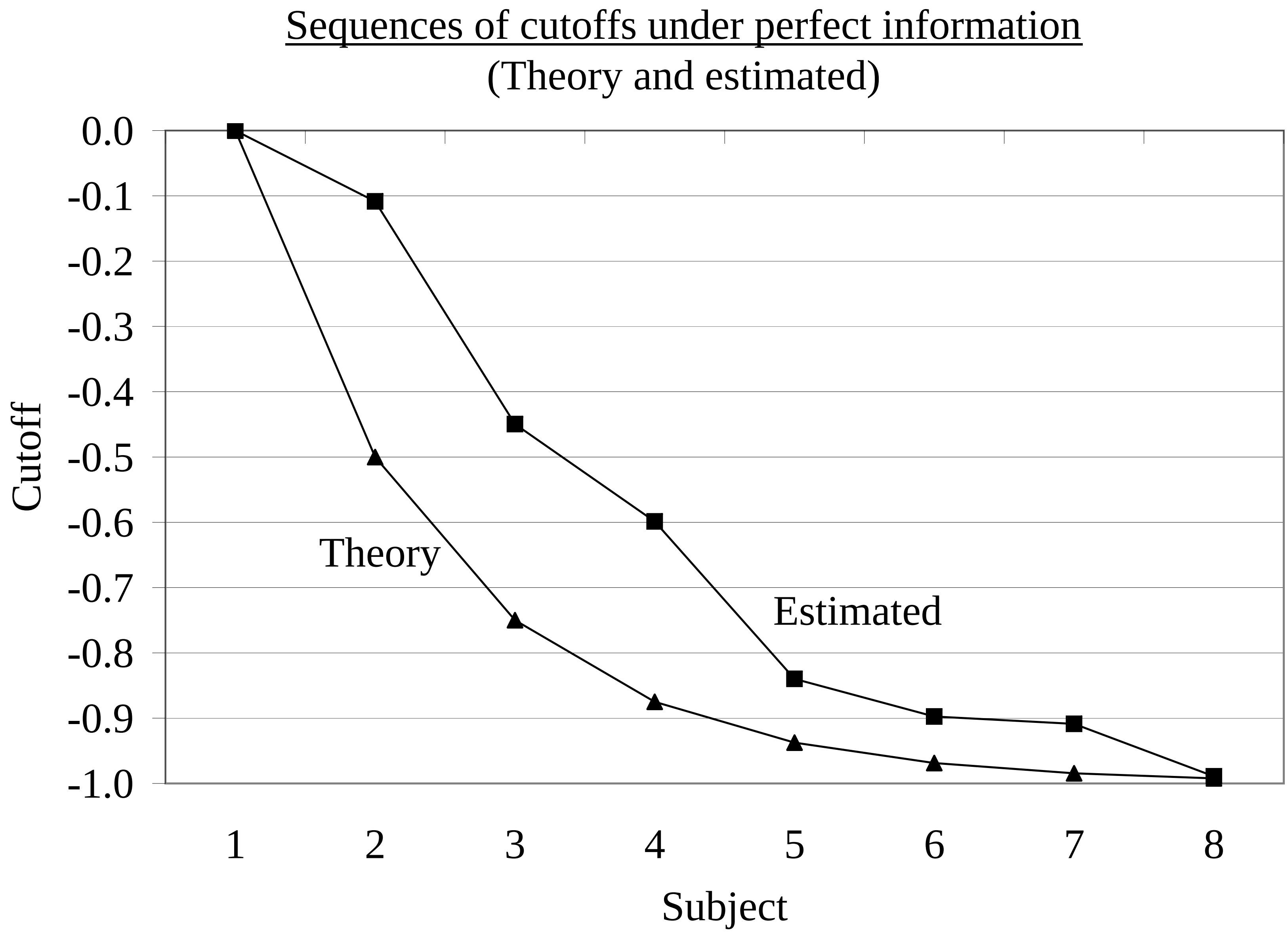
The estimated cutoff process

- The cutoff dynamics of rational players follow the process

$$\hat{\theta}_n = \hat{\theta}_{n-1} - \begin{cases} \frac{10+(1-p_{n-1})\tilde{\theta}_{n-1}+p_{n-1}\hat{\theta}_{n-1}}{2} & \text{if} \quad x_{n-1} = A, \\ \frac{-10+(1-p_{n-1})\tilde{\theta}_{n-1}+p_{n-1}\hat{\theta}_{n-1}}{2} & \text{if} \quad x_{n-1} = B, \end{cases}$$

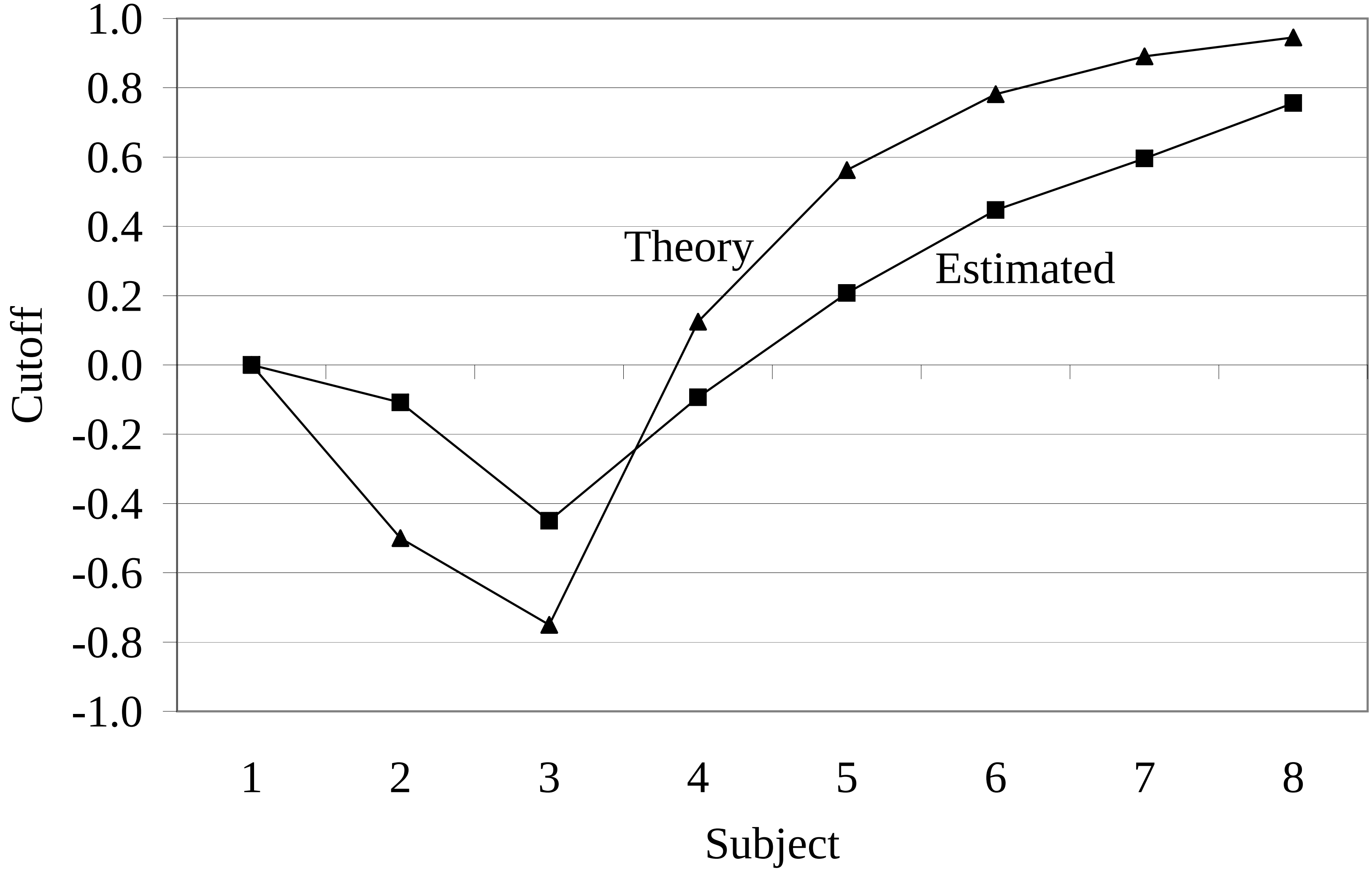
where $\hat{\theta}_1 = 0$.

- The estimated parameters for the first decision-turn are employed in estimating the parameters for the second turn, and so on.



Sequences of cutoffs under perfect information
(Theory and estimated)
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
Cutoff
Theory
Estimated
1
2
3
4
5
6
7
8
Subject

Sequences of cutoffs under perfect information
(Theory and estimated)



The model of Gale and Kariv (GEB 2004)

- Agents are bound together by a *social network*, a complex of relationships that brings them into contact with other agents.

- Markets are characterized by agents connected by complex, multilateral information networks.

- The network is represented by a family of sets $\{N_i\}$ where N_i denotes the set of agents $j \neq i$ who can be observed by agent i.

- Agents choose actions simultaneously and revise their decisions as new information is received.

Equilibrium

A weak perfect Bayesian equilibrium consists of a sequence of random variables $\{X_{it}\}$ and σ-fields $\{\mathcal{F}_{it}\}$ such that for each $i = 1, ..., n$ and $t = 1, 2, ...,$

(i) $X_{it} : \Omega \longrightarrow \mathcal{A}$ is $\mathcal{F}_{it}$-measurable,
(ii) $\mathcal{F}_{it} = \mathcal{F}\left(\sigma_i, \{X_{js} : j \in N_i\}_{s=1}^{t-1}\right)$, and
(iii) $E[U(x(\omega), \omega)] \leq E[U(X_{it}(\omega), \omega)]$, for any $\mathcal{F}_{it}$-measurable function $x : \Omega \rightarrow \mathcal{A}$.

Asymptotic properties

- The welfare-improvement principle
  - Agents have perfect recall, so expected utility is non-decreasing over time. This implies that equilibrium payoffs form a submartingale.

- The imitation principle
  - In a connected network, asymptotically, all agents must get the same average (unconditional) payoffs.

Convergence Let $\{X_{it}, \mathcal{F}_{it} : i = 1, ..., n, t = 1, 2, ...\}$ be an equilibrium. For each i, define $V_{it}^* : \Omega \to \mathbf{R}$ by

$$V_{it}^* = E[U(X_{it}, \cdot)|\mathcal{F}_{it}].$$

Then $\{V_{it}^*\}$ is a submartingale with respect to $\{\mathcal{F}_{it}\}$ and there exists a random variable $V_{i\infty}^*$ such that V_{it}^* converges to $V_{i\infty}^*$ almost surely.

Connectedness Let $\{X_{it}, \mathcal{F}_{it}\}$ be the equilibrium and let V_{it}^* be the equilibrium payoffs. If $j \in N_i$ and j is connected to i then $V_{i\infty}^* = E[V_{j\infty}^* | \mathcal{F}_{i\infty}]$.

Imitation Let i and j be two agents such that $j \in N_i$ and j is connected to i. Let E^{ab} denote the measurable set on which i chooses a infinitely often and j chooses b infinitely often. Then $V^a_{i\infty}(\omega) = V^b_{i\infty}(\omega)$ for almost every ω in E^{ab}.

- Apart from cases of disconnectedness and indifference, diversity of actions is eventually replaced by uniformity.

- This is the network-learning analogue of the herd behavior found in the standard social learning model.

- The convergence properties of the model are general but many important questions about learning in networks remain open.

- Identify the impact of network architecture on the efficiency and dynamics of social learning.

A three-person example

- The network consists of three agents indexed by $i = A, B, C$. The neighborhoods $\{N_A, N_B, N_C\}$ completely define the network.

- Uncertainty is represented by two equally likely events $\omega = -1, 1$ and two corresponding signals $\sigma = -1, 1$.

- Signals are informative in the sense that there is a probability $\frac{2}{3}$ that a signal matches the event.

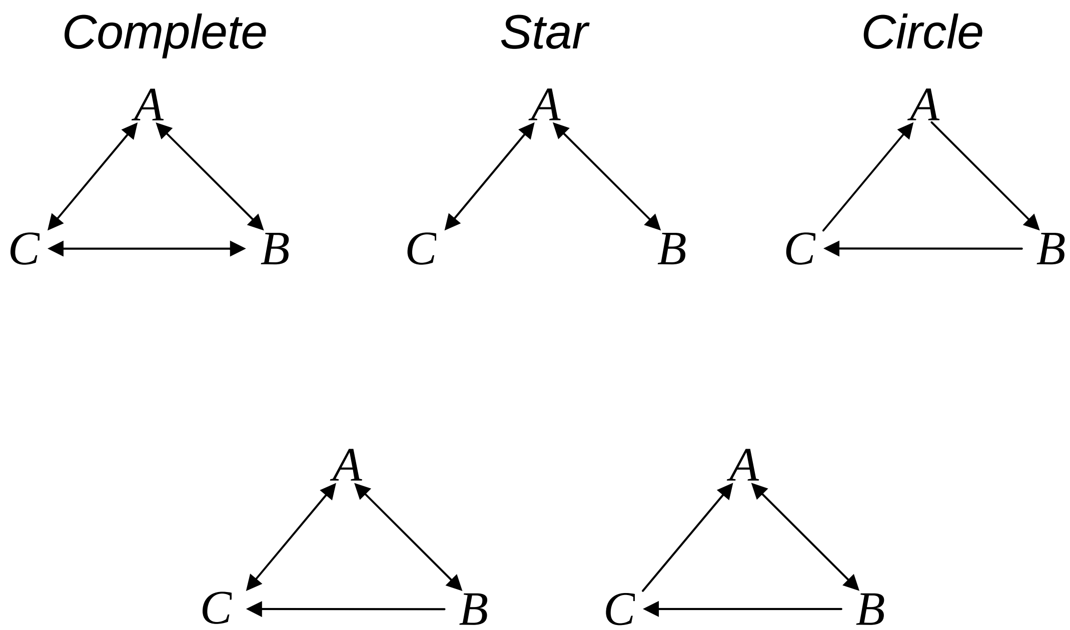
- With probability q an agent is informed and receives a private signal at the beginning of the game.

- At the beginning of each date t, agents simultaneously guess $a_{it} = -1, 1$ the true state.

- Agent i receives a positive payoff if his action $a_{it} = \omega$ and zero otherwise.

- Each agent i observes the actions a_{jt} chosen by the agents $j \in N_i$ and updates his beliefs accordingly.

- At date t, agent i's information set I_{it} consists of his private signal, if he observed one, and the history of neighbors' actions.



Complete
Star
Circle
A
B
C

Learning dynamics

- Learning is 'simply' a matter of Bayesian updating but agents must take account of the network architecture in order to update correctly.

- If all agents choose the same action at date 1, no further information is revealed at subsequent dates (an absorbing state).

- We can trace out possible evolutions of play when there is diversity of actions at date 1.

- The exact nature of the dynamics depends on the signals and the network architecture.

Complete network

$$N_A = \{B, C\}, N_B = \{A, C\}, N_C = \{A, B\}$$

	A	B	C
t/σ	1	0	0
1	1	-1	-1
2	-1	-1	-1
3	-1	-1	-1
4	-1	-1	-1
...	...	...	...

Star network

$$N_A = \{B, C\}, N_B = \{A\}, N_C = \{A\}$$

	A	B	C
t/σ	1	0	0
1	1	-1	-1
2	-1	1	1
3	1	1	-1
4	1	1	1
...	...	...	...

Circle network

$$N_A = \{B\}, N_B = \{C\}, N_C = \{A\}$$

	A	B	C
t/σ	1	0	0
1	1	-1	-1
2	1	-1	1
3	1	1	1
4	1	1	1
...	...	...	...

Takeaways

- Convergence to a uniform action tends to be quite rapid, typically occurring within two to three periods.
- Significant differences can be identified in the equilibrium behavior of agents in different networks.
- Even in the three-person case the process of social learning in networks can be complicated.
- Because of the lack of common knowledge, inferences agents must draw in order to make rational decisions are quite subtle.

Experimental design

- Each experimental session consisted of 15 independent rounds and each round consisted of six decision-turns.
- The network structure and the information treatment ($q = \frac{1}{3}, \frac{2}{3}, 1$) were held constant throughout a given session.
- The ball-and-urn social learning experiments paradigm of Anderson and Holt (1997).
- A serious test of the ability of a structural econometric model based on the theory to interpret the data.

Selected data
(star network under high-information)

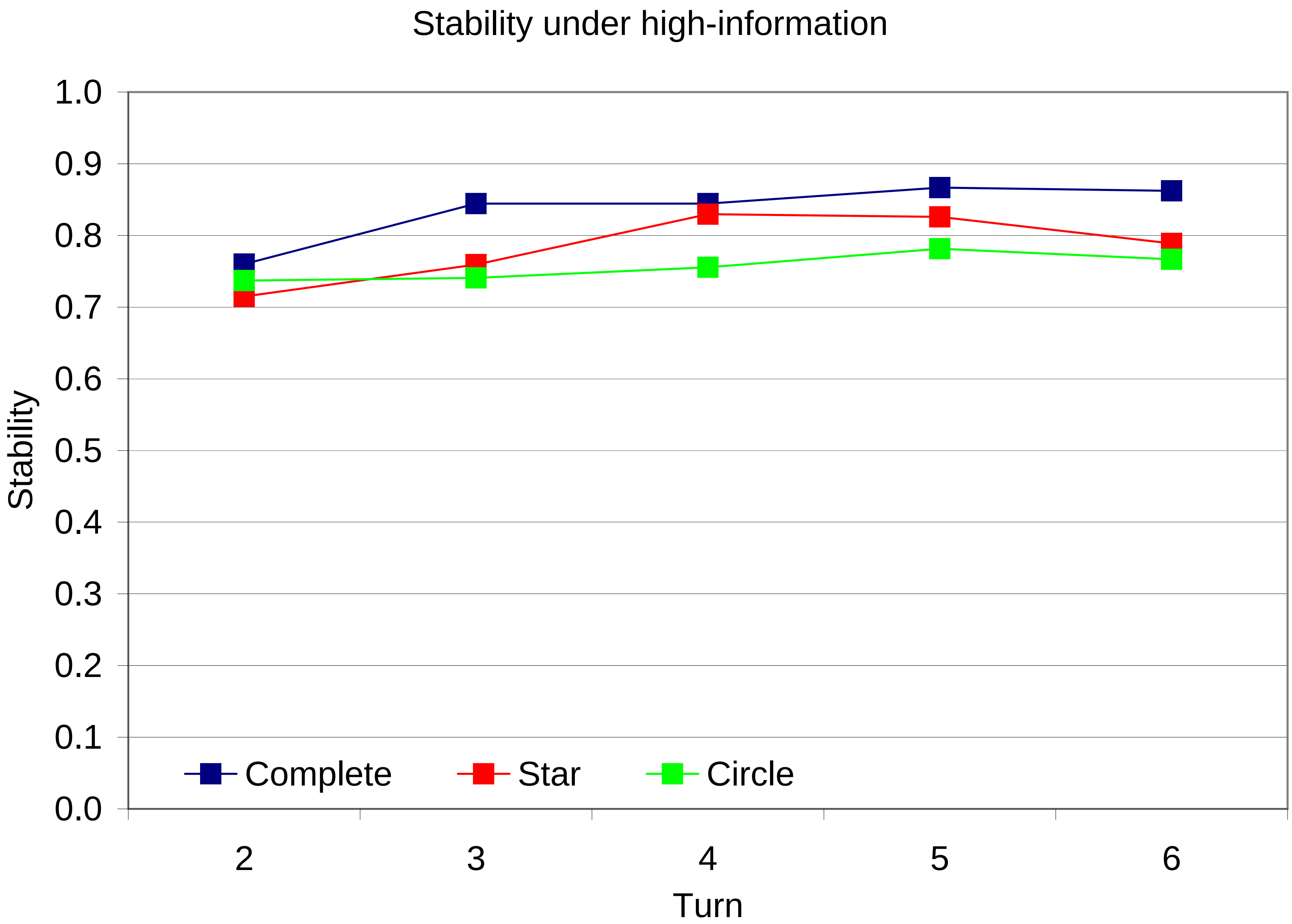
	A	B	C
t/σ	1	0	0
1	1	-1	-1
2	-1	1	1
3	-1	-1	-1
4	-1	-1	-1
5	1	1	-1
6	1	1	1

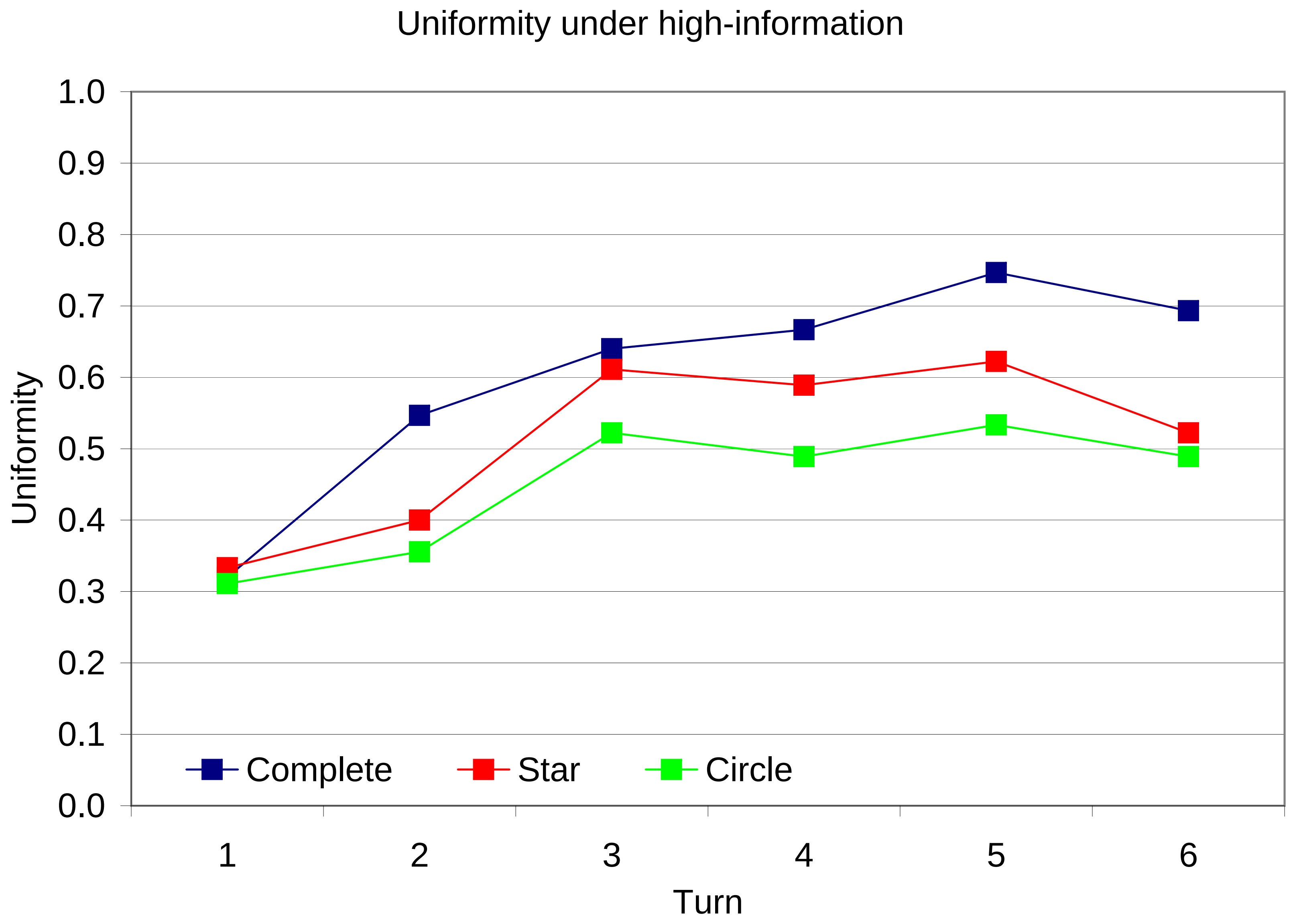
Herd behavior

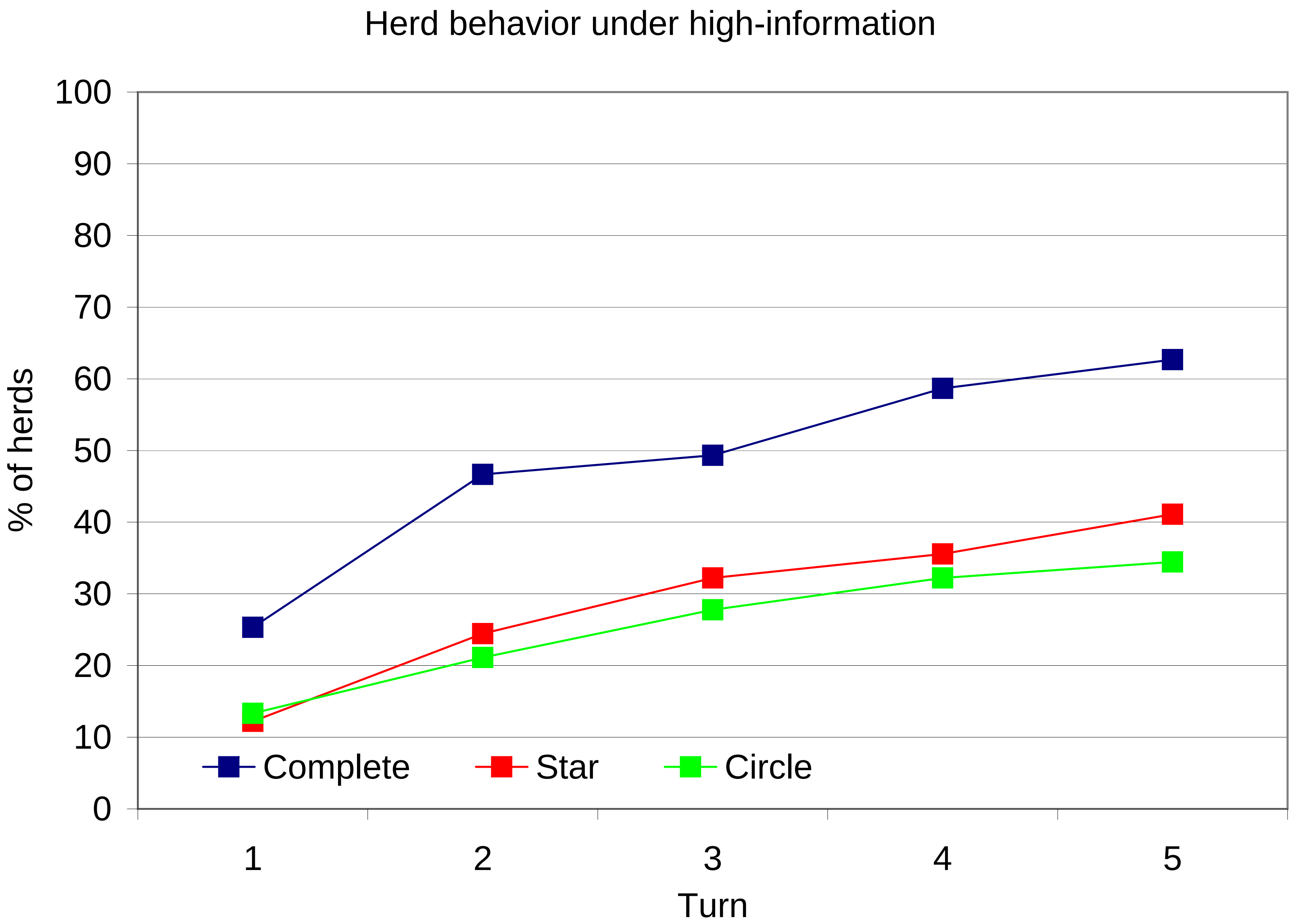
Herd behavior is characterized by two related phenomena:

- *Stability*: the proportion of subjects who continue to choose the same action.
- *Uniformity*: a score function that takes the value 1 if all subjects act alike and takes the value 0 otherwise.

Uniformity will persist and lead to herd behavior if stability takes the value 1 at all subsequent turns.


Stability under high-information
Stability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Complete
Star
Circle
2
3
4
5
6
Turn


Uniformity under high-information
Uniformity
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
4
5
6
Turn
Complete
Star
Circle



Herd behavior under high-information
100
90
80
70
60
50
40
30
20
10
0
% of herds
Complete
Star
Circle
1
2
3
4
5
Turn

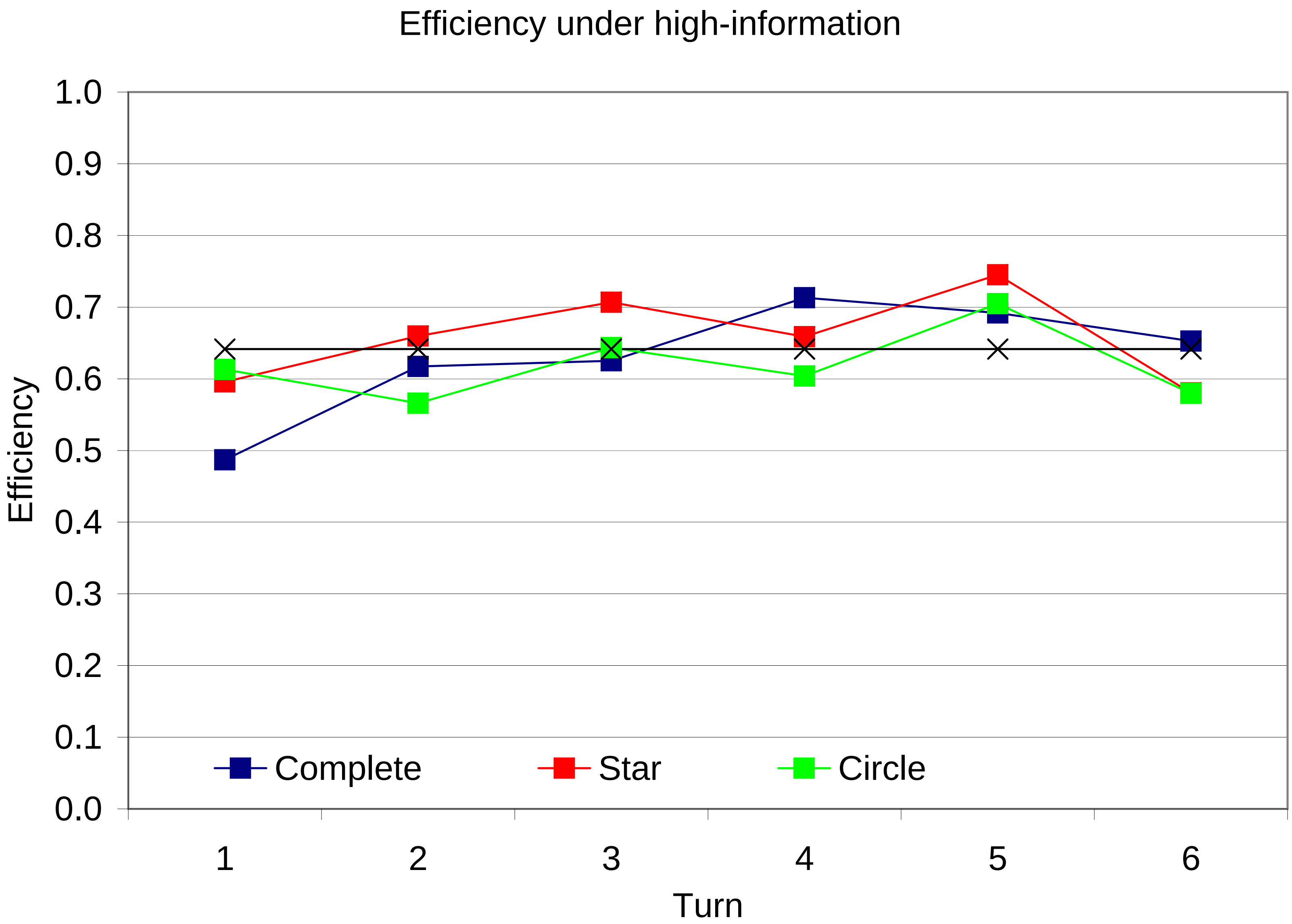
Informational efficiency

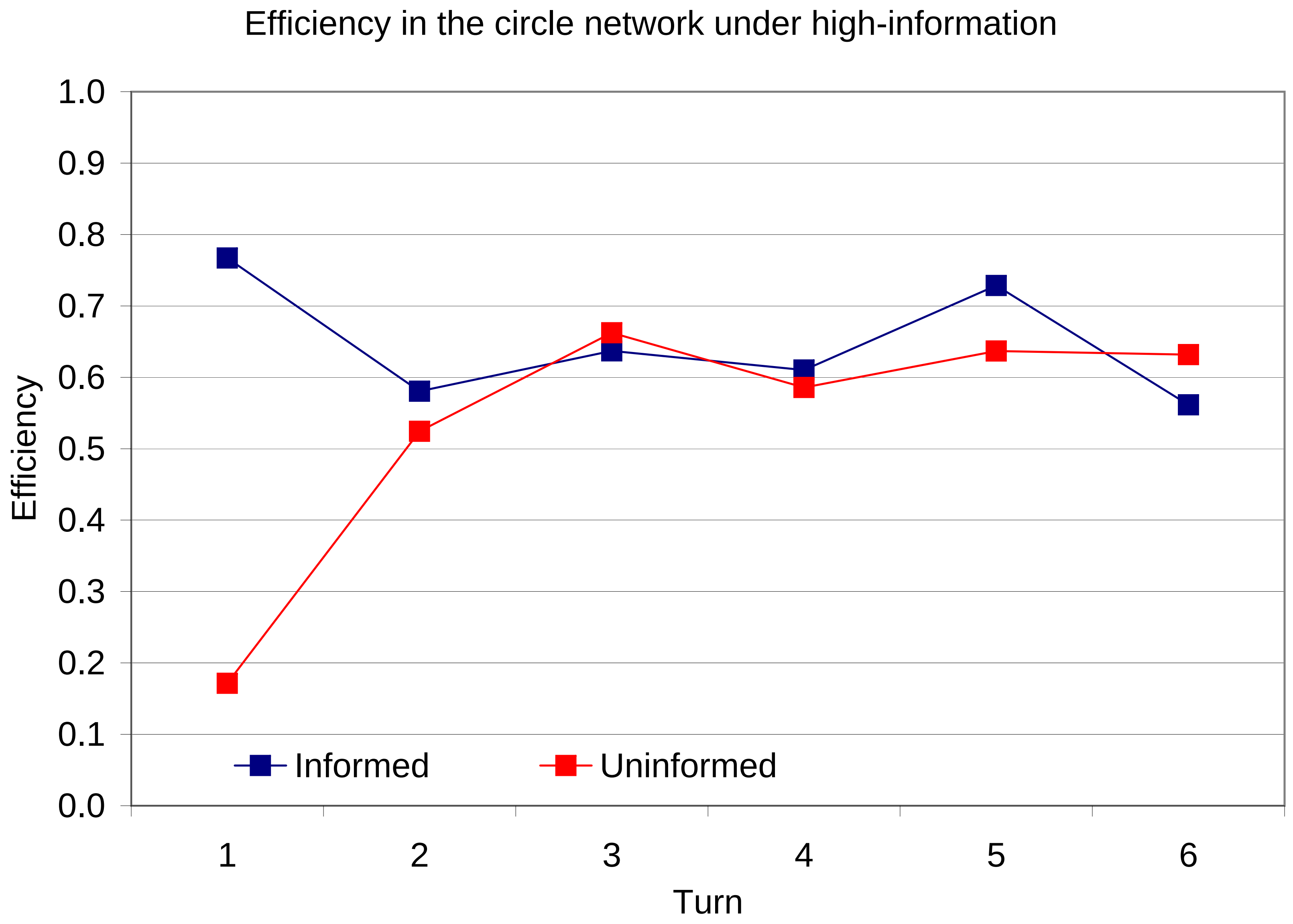
The efficiency of decisions is measured in two ways:

- *actual efficiency* $= \dfrac{\pi_a - \pi_r}{\pi_e - \pi_r}$

- *private-information efficiency* $= \dfrac{\pi_p - \pi_r}{\pi_e - \pi_r}$

The net actual (private) efficiency $\pi_a - \pi_r$ ($\pi_p - \pi_r$) as a fraction of the net pooled efficiency $\pi_e - \pi_r$.


Efficiency under high-information
Efficiency
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
4
5
6
Turn
Complete
Star
Circle


Efficiency in the circle network under high-information
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Efficiency
Informed
Uninformed
1
2
3
4
5
6
Turn

Summary of experimental data

- A strong tendency toward herd behavior and a marked efficiency of information aggregation.

- There are significant differences between the behavior of different networks and information treatments.

- Differences might be explained by the symmetry or asymmetry of the network or the information treatment.

- There is some variation across networks and treatments but the error rates are uniformly fairly low.

Quantal response equilibrium (QRE)

- Mistakes are made and this should be taken into account in any theory of rational behavior.

- The payoff from a given action is assumed to be a weighted average of the theoretical payoff and a logistic disturbance.

- The "weight" placed on the theoretical payoff is determined by a regression coefficient.

- The recursive structure of the model enables to estimate the coefficients of the QRE model for each decision-turn sequentially.

The logit equilibrium can be summarized by a choice probability function following a binomial logit distribution:

$$\Pr\left(a_{it} = 1 | I_{it}\right) = \frac{1}{1 + \exp\left(-\beta_{it} x_{it}\right)}$$

where β_{it} is a coefficient and x_{it} is the difference between the expected payoffs from actions 1 and -1.

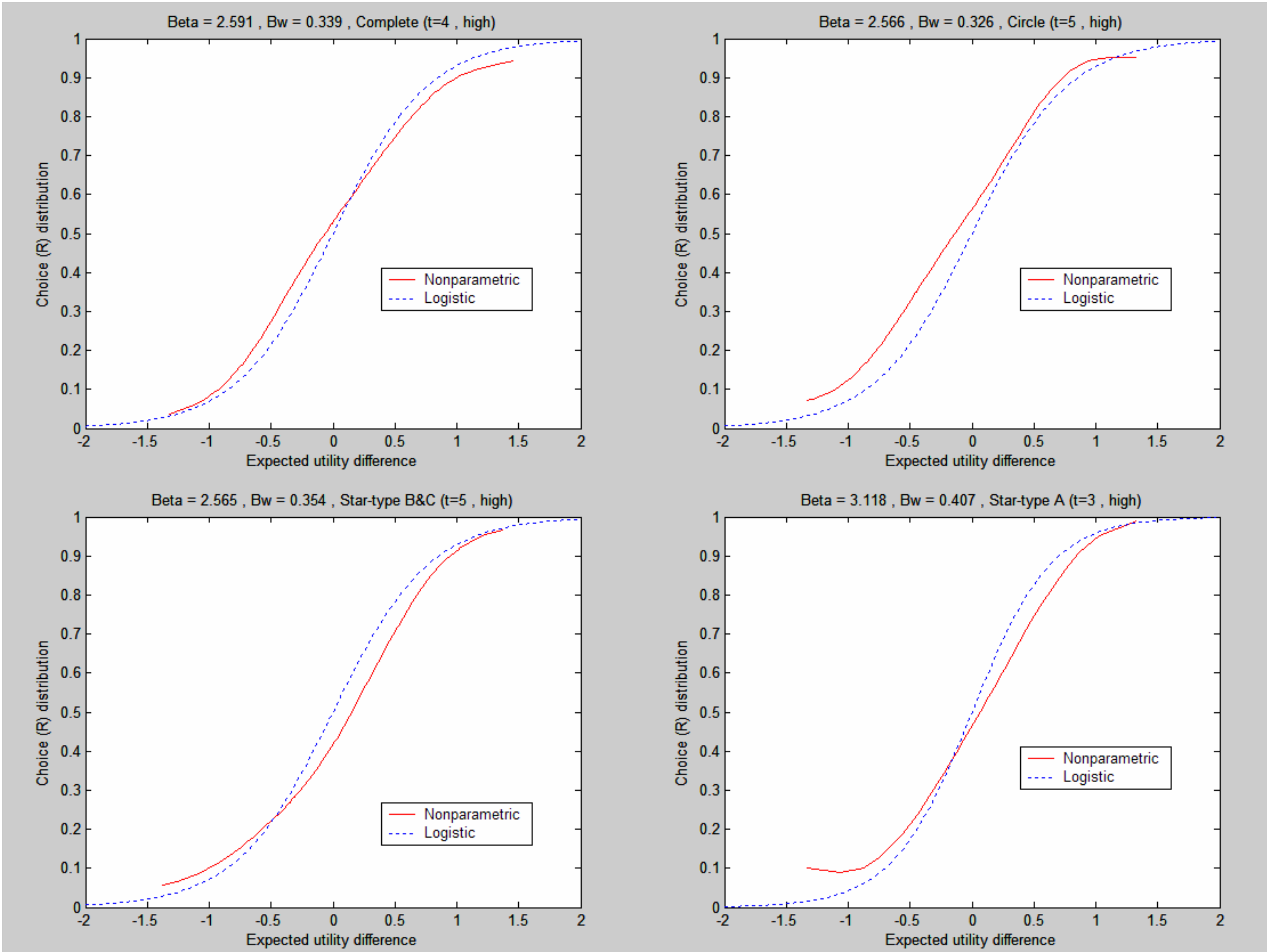
The regression coefficient β will be positive if the theory has any predictive power.

- Use the estimated coefficient from turn t to calculate the theoretical payoffs from the actions at turn $t + 1$.

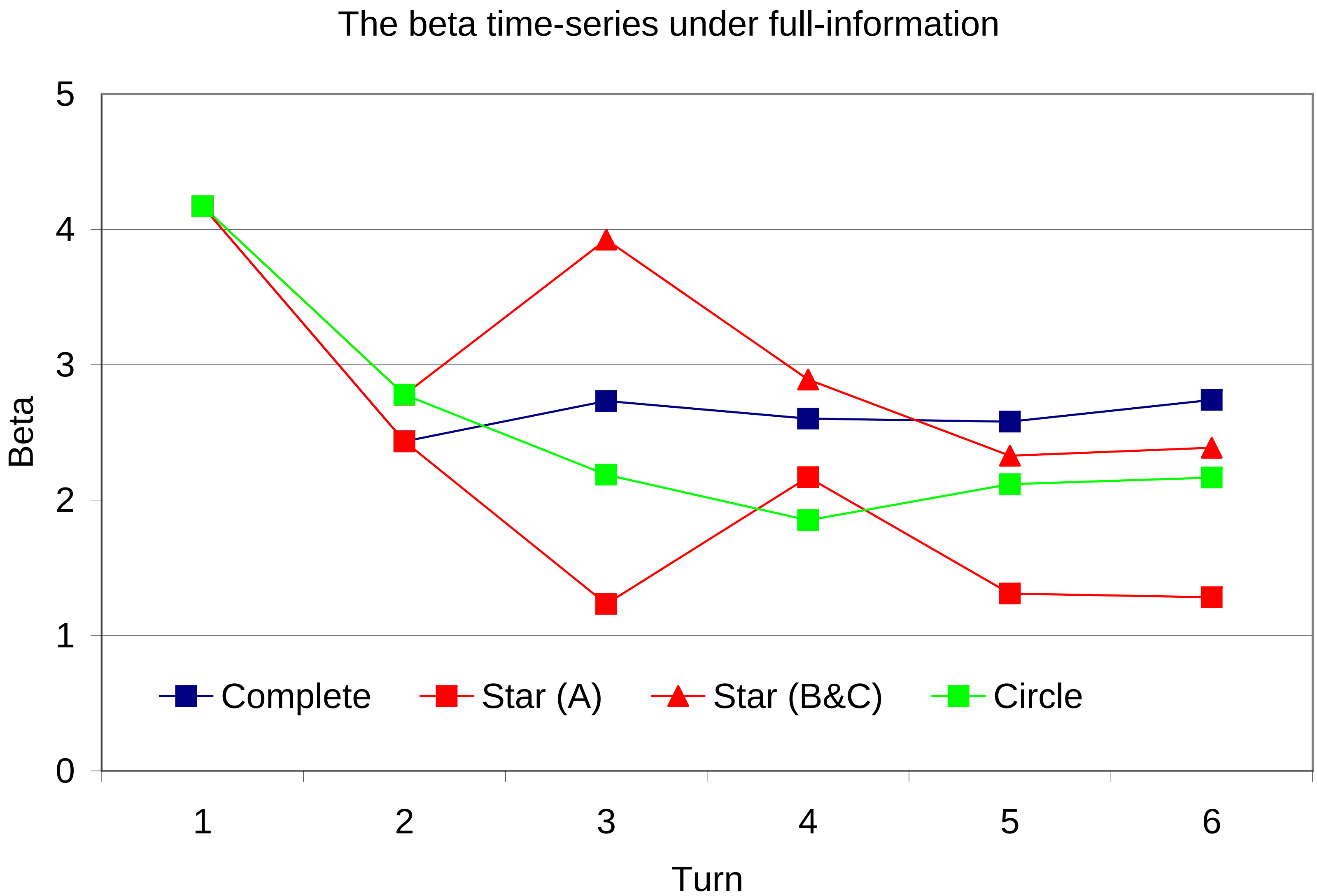
- The behavioral interpretation is that subjects have rational expectations and use the true mean error rate.

- The parameter estimates are highly significant and positive, showing that the theory does help predict the subjects' behavior.

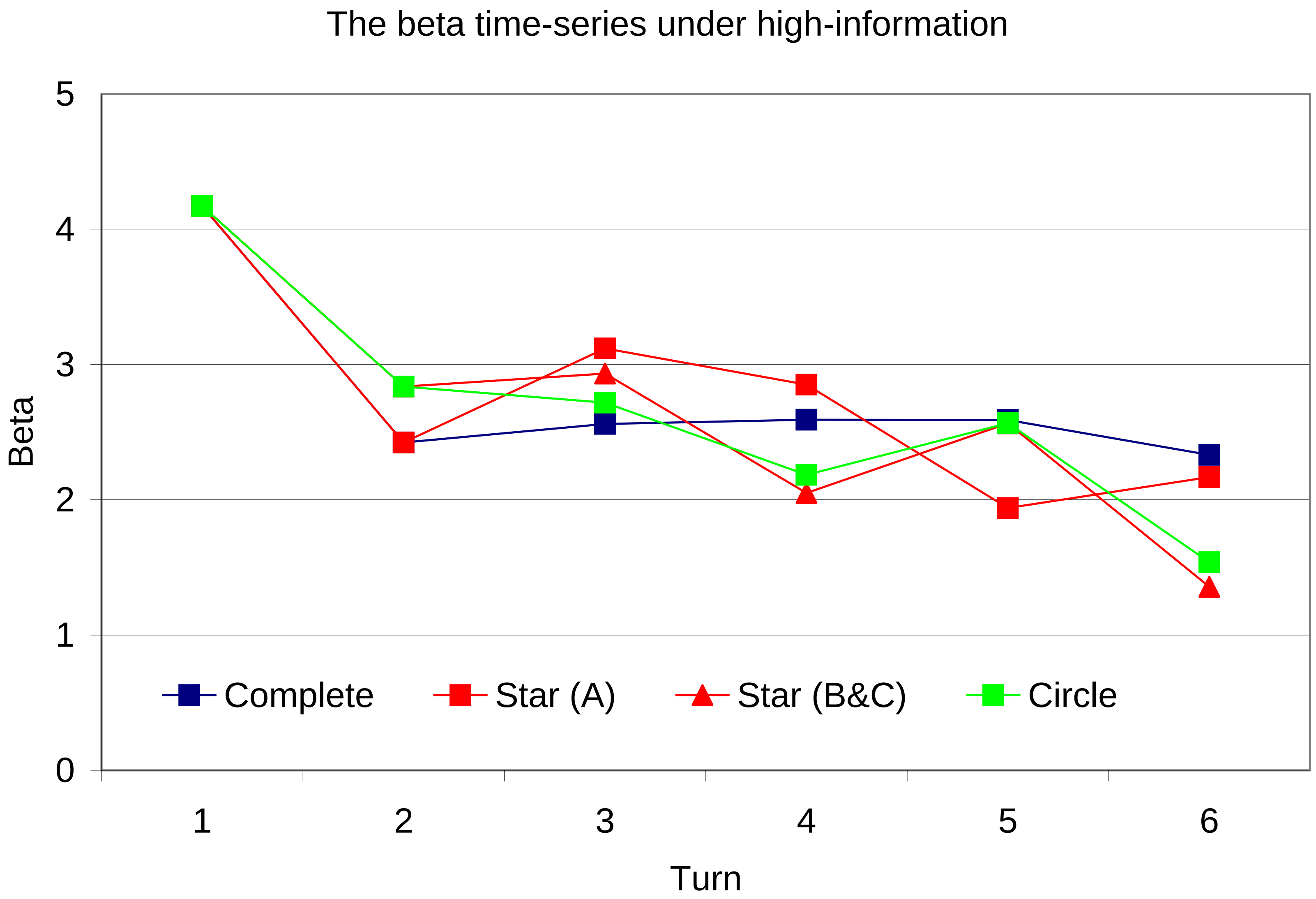
- A series of specification tests shows that the restrictions of the QRE model are confirmed by the data.



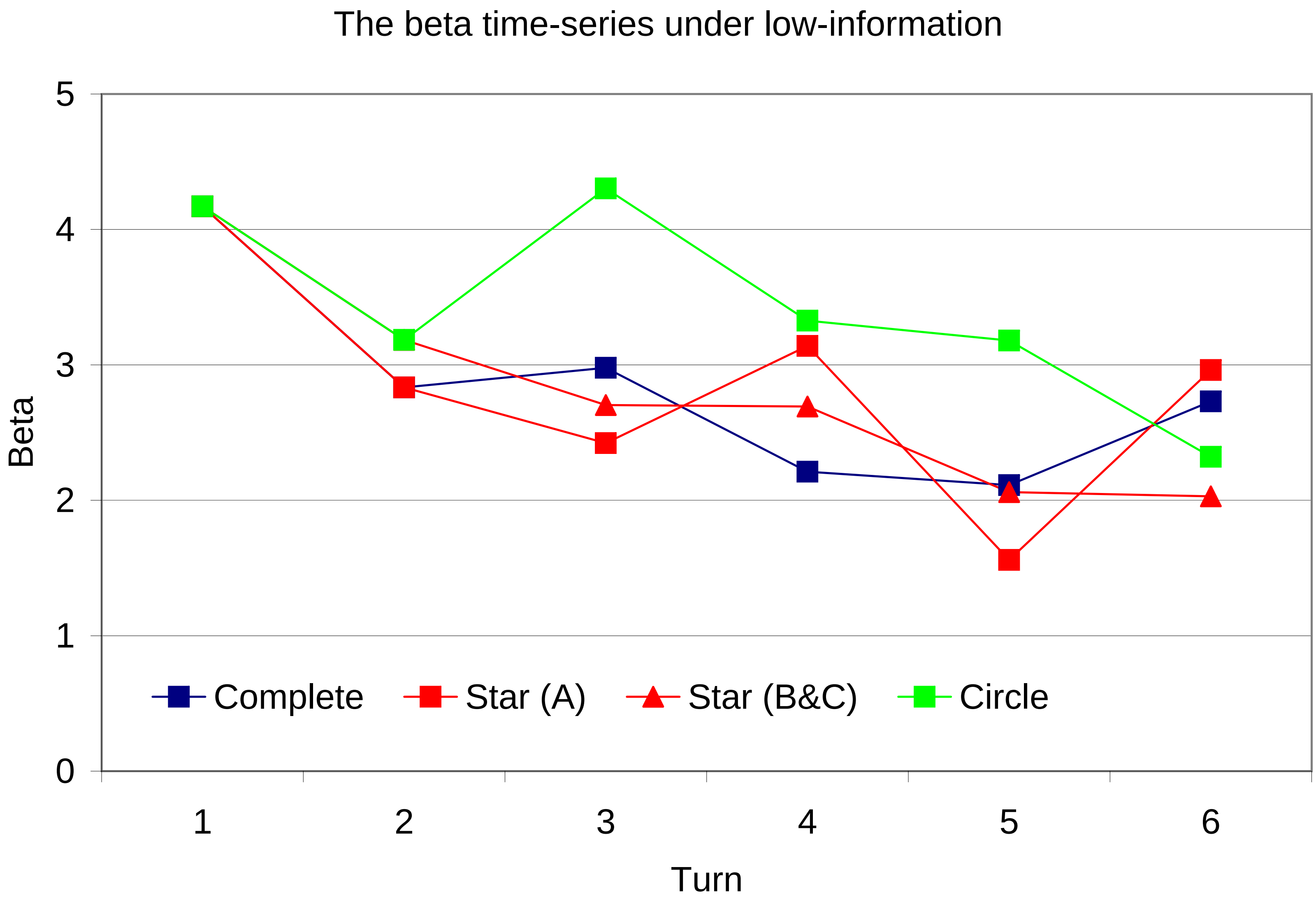
Beta = 2.591 , Bw = 0.339 , Complete (t=4 , high)
Beta = 2.566 , Bw = 0.326 , Circle (t=5 , high)
Beta = 2.565 , Bw = 0.354 , Star-type B&C (t=5 , high)
Beta = 3.118 , Bw = 0.407 , Star-type A (t=3 , high)
Choice (R) distribution
Expected utility difference
Nonparametric
Logistic

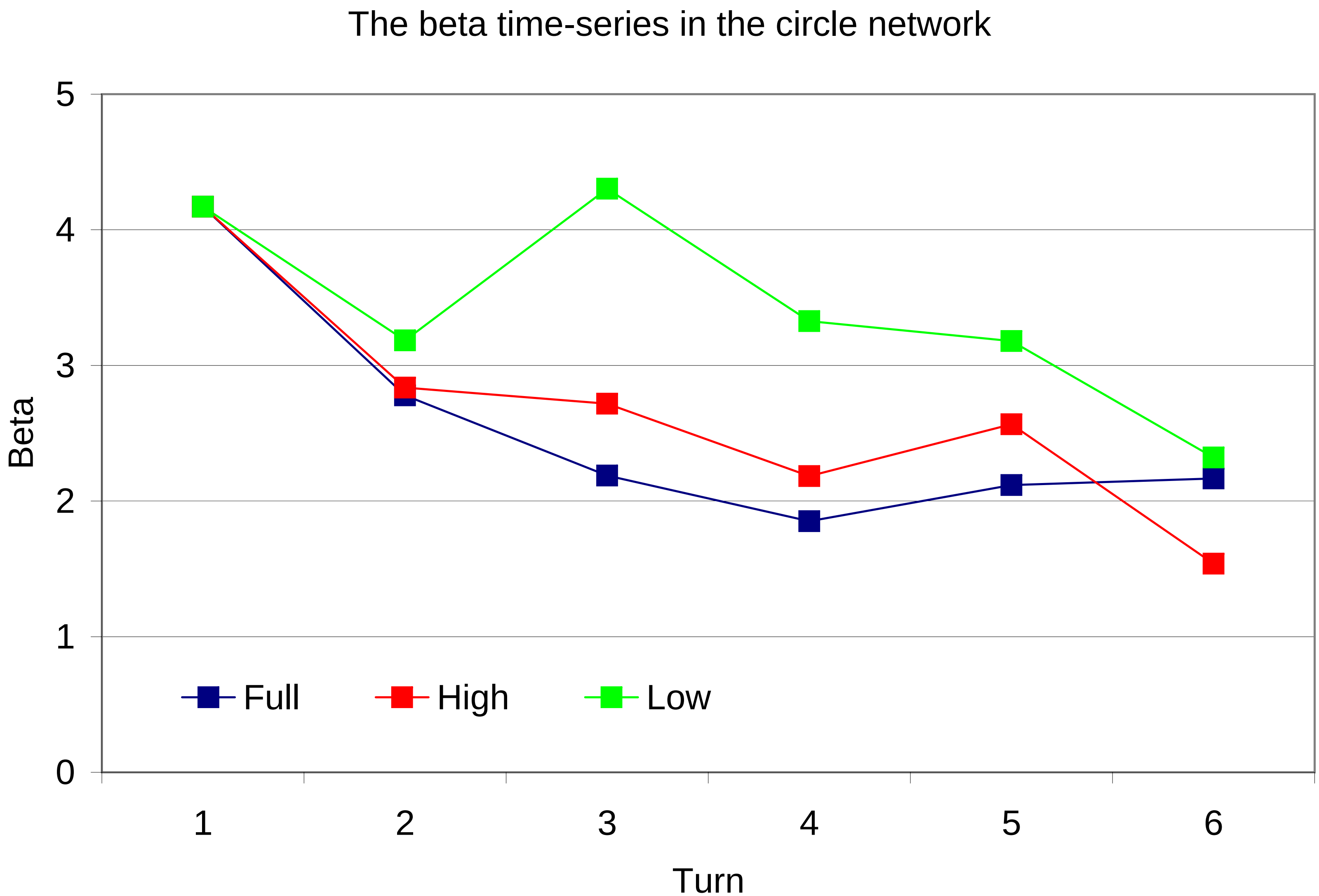


The beta time-series under full-information
Beta
5
4
3
2
1
0
1
2
3
4
5
6
Turn
Complete
Star (A)
Star (B&C)
Circle



The beta time-series under high-information
Beta
Turn
0
1
2
3
4
5
1
2
3
4
5
6
Complete
Star (A)
Star (B&C)
Circle


The beta time-series under low-information
Beta
5
4
3
2
1
0
1
2
3
4
5
6
Turn
Complete
Star (A)
Star (B&C)
Circle



The beta time-series in the circle network
5
4
3
2
1
0
Beta
1
2
3
4
5
6
Turn
Full
High
Low

Concluding remarks

- Use the theory to interpret data generated by experiments of social learning in three-person networks.

- The family of three-person networks includes several architectures, each of which gives rise to its own distinctive learning patterns.

- The theory, modified to include the possibility of errors, adequately accounts for large-scale features of the data.

- A strong support for the use of models as the basis for structural estimation and the use of QRE to interpret experimental data.