Economics 209B
Behavioral / Experimental Game Theory
(Spring 2008)

Lecture 3: Equilibrium refinements and selection

- Theory cannot provide clear guesses about with equilibrium will occur in games with multiple equilibria.

- A major concern of game theorists is to understand which equilibria are selected.

- In order to restrict the set of equilibria, game theorists use a number of refinements.

- Learn about the empirical properties of the refinements that are widely used in game theory.

- The use of market-generated data for this purpose is problematic (many crucial parameters and variables are unobserved).

- In the laboratory, by contrast, we can control all the relevant parameters.

- Our objective is to confront the theory with some experimental data and visa versa.

- Any attempt to use theory to explain experimental data must answer a number of questions:

- Do we assume that all subjects are identical or do we allow for heterogeneity?

- Do we assume a single equilibrium is played in each repetition of a game?

- Do we allow for mistakes or behavioral biases from the outset or assume full rationality?

Provide a parsimonious account of the data (Occam's Razor), and maximize our chance of falsifying the theory (Popper's sense).

Monotone games

- A monotone game is an extensive-form game with simultaneous moves and an irreversibility structure on strategies.

- It captures a variety of situations in which players make partial commitments.

- We characterize conditions under which equilibria result in efficient outcomes.

- The game has many equilibrium outcomes so the theory lacks predictive power.

- To produce stronger predictions, we restrict attention to sequential, or Markov, or symmetric equilibria.
  - Whether any of these refinements is reasonable in practice is an empirical question.

- Multiple equilibria cannot be avoided in general and the theory cannot tell us which equilibrium is most likely to be played.
  - Identify the important factors in creating the “salience” of certain equilibria.

The game

- An indivisible public project with cost K and N players, each of whom has an endowment of E tokens.

- The players simultaneously make *irreversible* contributions to the project at a sequence of dates $t = 1, ..., T$.

- The project is carried out if and only if the sum of the contributions is large enough to meet its cost.

- If the project is completed, each player receives A tokens *plus* to the number of tokens retained from his endowment.

[1] The aggregate endowment is greater than the cost of the project (completion is feasible)

$$NE > K.$$

[2] The aggregate value of the project is greater than the cost (completion is efficient)

$$NA > K.$$

[3] The project is not completed by a single player (either it is not feasible or it is not rational)

$$\min\{A, E\} < K.$$

The one-shot game

In the static (one-shot) game, all players make simultaneous binding decisions.

Proposition (one-shot) *(i) There exists a pure-strategy Nash equilibrium with no completion. Conversely, there exists at least one pure-strategy equilibrium in which the project is completed with probability one. (ii) The game also possesses mixed-strategy equilibria in which the project is completed with positive probability.*

The indivisibility of the public project makes each contributing player "pivotal" (Bagnoli and Lipman (1992)).

The dynamic game

The sharpest result is obtained for the case of pure-strategy sequential equilibria.

Proposition (pure strategy) *Suppose that $A > E$ and $T \geq K$. Then, under the maintained assumptions, in any pure strategy sequential equilibrium of the game, the public project is completed with probability one.*

In any pure strategy equilibrium, the probability of completion is either zero or one, so it is enough to show that the no-completion equilibrium is not sequential.

Mixed strategies expand the set of parameters for which there exists a no-completion equilibrium.

Proposition (mixed strategy) *Suppose that $A > E$ and $T \geq K$. Then there exists a number $A^*(E, K, N, T)$ such that, for any $E < A < A^*$ there exists a mixed strategy equilibrium in which the project is completed with probability zero.*

The use of mixed strategies in the continuation game can discourage an initial contribution and support an equilibrium with no completion.

The games in which $K = NE$ provide a useful benchmark (no possibility of taking a free ride on the contributions of other players).

Proposition (no-free-riding) *Suppose that $K = NE$, $A > E$ and $T \geq K$. Then the project is completed with probability one in any sequential equilibrium of the game.*

The result does not rule out the use of mixed strategies, even along the equilibrium path.

Taking $K = NE$ as a benchmark for the absence of free riding, the free-rider problem must be worse when the total endowment exceeds this level.

Proposition (free-riding) *Suppose that $E > A$ and $T \geq K$. Then under the maintained assumptions, there exists a pure strategy sequential equilibrium of the game in which the public project is completed with probability zero.*

The essential ingredient in the construction of this equilibrium is the self-punishing strategy employed by Gale (2001).

Symmetric Markov perfect equilibrium (SMPE)

The class of SMPE takes a relatively simple form. The main predictions from SMPE can be summarized by four facts:

- There are no pure strategy SMPE, although mixed strategies may only be used off the equilibrium path.

- There is no completion of the public project in early periods when A "high" and no completion at all when A "low."

- The contribution probability at each state when A is "high" is at least as high as when A is "low."

- A game with horizon $T < T'$ is isomorphic to a continuation game starting in period $T' - T$ of the game with horizon T'.

Example 1

$$A = 3, E = 1, K = 2, N = 3, T = 5$$

τ/n	0	1
4	0.00	$--$
3	0.00	0.00
2	0.00	0.00
1	0.56| 0.55| 0.00	0.00
0	0.00| 0.21| 0.79	0.67

where n is the total number of contributions and τ is the number of periods remaining after the current period.

Example 2

$$A = 1.5, E = 1, K = 2, N = 3, T = 5$$

τ/n	0	1
4	0.00	$--$
3	0.00	0.00
2	0.00	0.00
1	0.00	0.00
0	0.00	0.33

Example 3

$$A = 3, E = 2, K = 2, N = 3, T = 5$$

$\tau/(n, n_i)$	0	$(1,0)$	$(0,1)$
4	0.00	$--$	$--$
3	0.00	0.00	0.00
2	0.00	0.00	0.00
1	0.50| 0.48| 0.00	0.00	0.00
0	0.00| 0.21| 0.79	0.42	0.42

where n_i is the total number of contributions to date by player i.

Example 4

$$A = 1.5, E = 2, K = 2, N = 3, T = 5$$

$\tau/(n, n_i)$	0	$(1,0)$	$(0,1)$
4	0.00	$--$	$--$
3	0.00	0.00	0.00
2	0.00	0.00	0.00
1	0.00	0.00	0.00
0	0.00	0.21	0.21

The Markov property reduces the set of sequential equilibria, sometimes substantially.

Summary of the equilibrium properties in the complete network

E, *K*, *N*	*T*	*A*	Pure	Mixed	SMPE
1, 2, 3	2	1.5	N	Y	0
		3	N	N	.62, .62, .89
	5	1.5	N	Y	0
		3	N	N	.62, .62, .89
1, 3, 3	2	1.5	Y	Y	0, 1
		3	Y	Y	1
	5	1.5	N	N	0, 1
		3	N	N	1
2, 2, 3	2	1.5	Y	Y	0
		3	N	N	.63, .62, .89
	5	1.5	Y	Y	0
		3	N	N	.63, .62, .89
1, 2, 3	1	1.5	Y	N	0
		3	Y	N	0, .51

Equilibrium outcomes

- The SMPE explains the qualitative patterns of contributions in the complete network.

- The other results on provision rates are all consistent with the qualitative predictions of the SMPE.

- The deviations from the SMPE contribution probabilities at earlier and later periods go in opposite directions.

- QRE replicates the tendency of early contributions in games, which could not be captured by the SMPE.

Frequencies of contribution in the complete network

$A=3$, $E=1$, $K=2$, $N=3$

τ / n	0	1	2
4	0.09 (270)		
3	0.08 (207)	0.11 (38)	0 (2)
2	0.11 (165)	0.07 (54)	0.25 (8)
1	0.37 (117)	0.07 (76)	0.10 (10)
0	0.36 (36)	0.60 (94)	0.08 (24)

τ / n	0	1	2
1	0.18 (270)		
0	0.62 (159)	0.54 (54)	0 (9)

( ) - # of obs.

A=1.5, E=1, K=2, N=3

τ / n	0	1	2
4	0.09 (270)		
3	0.05 (207)	0.03 (36)	0 (3)
2	0.06 (177)	0.06 (54)	0.25 (4)
1	0.26 (144)	0.19 (70)	0.17 (6)
0	0.20 (57)	0.48 (88)	0.09 (23)

τ / n	0	1	2
1	0.18 (270)		
0	0.35 (150)	0.33 (64)	0 (7)

( ) - # of obs.

A=3, E=2, K=2, N=3

τ / n	(0,0)	(1,0)	(1,1)
4	0.14 (270)		
3	0.03 (165)	0.02 (52)	0.12 (26)
2	0.07 (153)	0.04 (50)	0.08 (25)
1	0.3 (126)	0.08 (60)	0 (30)
0	0.53 (45)	0.46 (84)	0.26 (42)

τ / n	0	1	2
1	0.34 (270)		
0	0.44 (75)	0.34 (70)	0.11 (35)

( ) - # of obs.

A =1.5, E =2, K =2, N =3

τ / n	(0,0)	(1,0)	(1,1)
4	0.06 (270)		
3	0.05 (228)	0.09 (22)	0.00 (11)
2	0.13 (195)	0.05 (40)	0.15 (20)
1	0.21 (126)	0.07 (70)	0.00 (35)
0	0.04 (63)	0.39 (92)	0.07 (46)

τ / n	(0,0)	(1,0)	(1,1)
1	0.26 (270)		
0	0.13 (111)	0.38 (70)	0.00 (35)

( ) - # of obs.

The relative frequencies of contributions from the different histories

$E=1$, $K=2$, $N=3$, $T=5$

A	(n,τ)	$h(1)$	$h(2)$	$h(3)$	$h(4)$	p-value
1.5	(1,2)	0.03 (34)	0.10 (20)	–	–	0.63
	(1,1)	0.06 (32)	0.25 (16)	0.32 (22)	–	0.05
	(1,0)	0.54 (28)	0.25 (8)	0.30 (10)	0.52 (42)	0.30
3	(1,2)	0.00 (30)	0.17 (24)	–	–	0.07
	(1,1)	0.00 (30)	0.06 (18)	0.14 (28)	–	0.21
	(1,0)	0.47 (30)	0.75 (18)	0.60 (20)	0.64 (28)	0.27

$E=2$, $K=2$, $N=3$, $T=5$

A	(n,τ)	$h(1)$	$h(2)$	$h(3)$	$h(4)$	p-value
1.5	(1,2)	0.56 (18)	0.45 (22)	–	–	0.25
	(1,1)	0.00 (10)	0.05 (20)	0.10 (40)	–	0.50
	(1,0)	0.50 (10)	0.33 (18)	0.47 (32)	0.31 (32)	0.12
3	(1,2)	0.05 (44)	0.00 (6)	–	–	0.10
	(1,1)	0.11 (38)	0.00 (6)	0.06 (16)	–	0.60
	(1,0)	0.43 (30)	0.67 (6)	0.57 (14)	0.41 (34)	0.53

Quantal Response Equilibrium (QRE)

For simplicity, suppose that each player has an endowment of one token ($E = 1$).

The contribution behavior of each uncommitted player at state (n, τ) follows a binomial logit distribution:

$$\lambda_{(n,\tau)} = \frac{1}{1 + \exp(-\beta_{(n,\tau)}\Delta_{(n,\tau)})},$$

where $\Delta_{(n,\tau)}$ is the difference between the expected payoffs from contributing and not contributing, and $\beta_{(n,\tau)}$ is a coefficient.

The calculation of QRE proceeds by backward induction, beginning with the final period.

QRE estimation results and the probability of contribution

A =3, E =1, K =2, N =3

β=10.05 (0.78), Log_lik =-472.52

τ / n	0	1	2
4	0.11		
3	0.14	0.07	0.00
2	0.18	0.10	0.00
1	0.20	0.17	0.00
0	0.75	0.65	0.00

β = 10.51 (1.27), Log_lik = -278.55

τ / n	0	1	2
1	0.19		
0	0.76	0.65	0

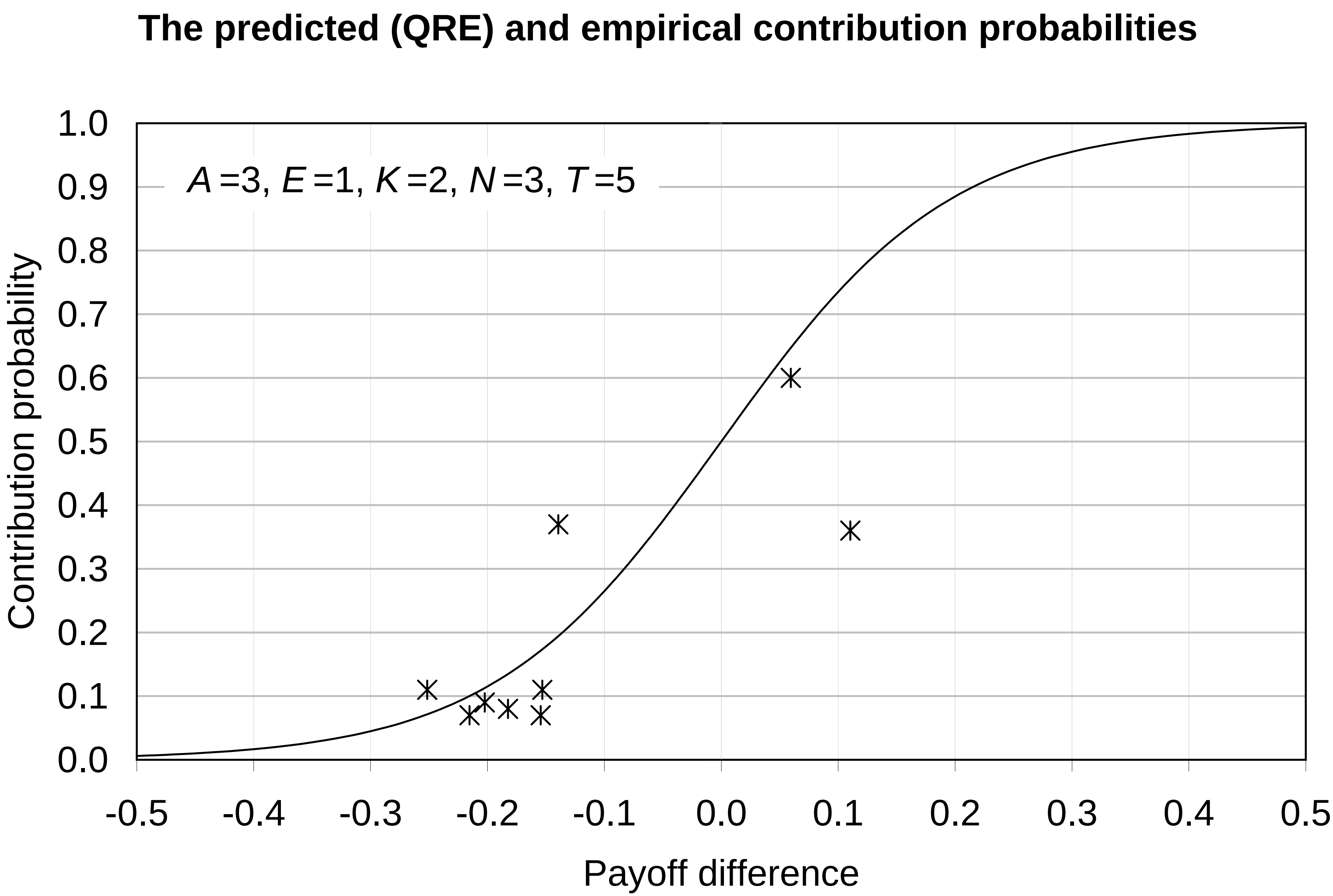
$A = 1.5$, $E = 1$, $K = 2$, $N = 3$

β=12.34 (0.83), Log_lik =-475.01

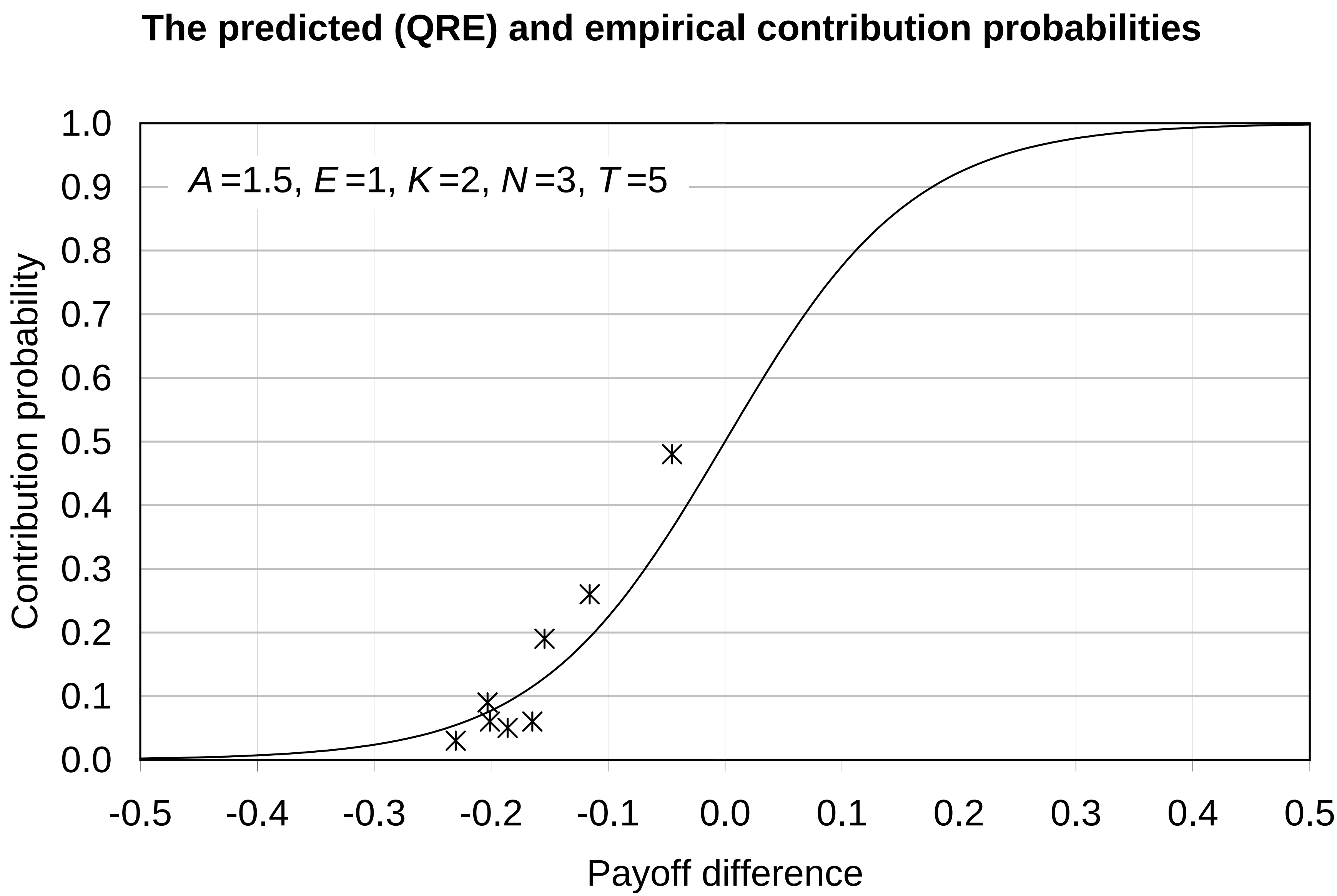
τ / n	0	1	2
4	0.08		
3	0.09	0.06	0.00
2	0.12	0.08	0.00
1	0.19	0.13	0.00
0	0.00	0.36	0.00

β = 2.26 (0.20), Log_lik = -296.41

τ / n	0	1	2
1	0.4		
0	0.3	0.42	0.09



The predicted (QRE) and empirical contribution probabilities
A =3, E =1, K =2, N =3, T =5
Contribution probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
Payoff difference



The predicted (QRE) and empirical contribution probabilities
A =1.5, E =1, K =2, N =3, T =5
Contribution probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
Payoff difference

Imperfect information

- Perfect information makes it easier for players to coordinate their actions, if they are so inclined.

- In the absence of perfect information, players beliefs play a larger role in supporting (possibly inefficient) equilibria.

- Asymmetry of the information structure may have an impact on the "selection" of equilibria.

- To complete the description of the game, we have to specify the information available to each player.

- The information structure is represented by a *directed* graph (or network).

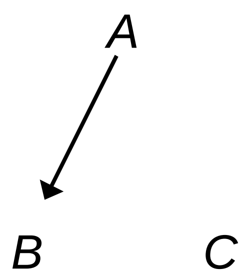
- Each player is located at a node of the graph and player i can observe player j if there is an edge leading from node i to node j.

- The experiments involve all three-person networks ($N = 3$) with zero, one or two edges.

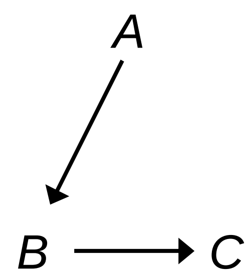
- Each network has a different architecture, a different set of equilibria, and different implications for the play of the game.

Asymmetric networks

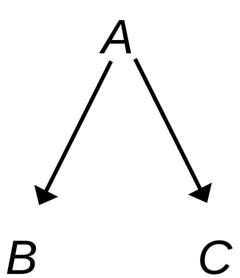
One-link



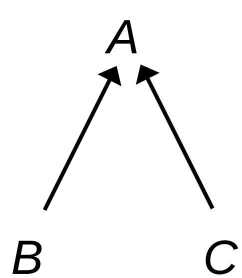
Line



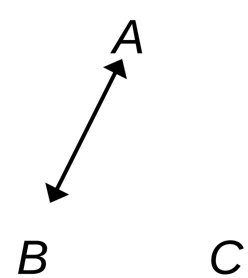
Star-out



Star-in



Pair



Salience

- The notion of salience was introduced into game theory by Schelling (1960), as part of his theory of focal equilibria.

- In Schelling's account, what makes an equilibrium focal is its psychological frame.

- Crawford et al. (*AER* 2008) provide a test of Schelling's notion of salience in the context of one-shot coordination games.

- Our concept of salience is different from "psychological" salience (based on structural properties of the game).

The one-link network

Adding one link to the empty network creates a simple asymmetry among the three players.

Proposition (one link) *Suppose that* $A > E = 1$ *and* $T \geq K = 2$*. Then, under the maintained assumptions, every pure-strategy sequential equilibrium completes the public project with probability one.*

Equilibria in which B contributes first and A contributes after observing B contribute seem "salient."

The line, star-out, star-in and pair networks

The remaining networks can each be obtained by adding a single link to the one-link network.

Proposition (networks) *Suppose that* $A > E = 1$ *and* $T \geq K = 2$*. Then, sequential rationality implies completion of the public project (with positive probability) in all of the networks except the empty network.*

Our focus in the sequel is to identify the impact of network architecture on efficiency and dynamics.

Equilibrium selection

- Uninformed players make a contribution early in the game to encourage other players to contribute.

- Informed players delay their contributions until they have observed another player contribute.

- Players who are symmetrically situated in a network have difficulty coordinating on an efficient outcome.

- Players who are otherwise similarly situated behave differently in different networks.

The frequencies of contributions in the one-link network

A=2, E=1, K=2, N=3, T=3

		A		*B*	*C*
1	n_i	--		--	--
	Freq.	0.104 (135)		0.570 (135)	0.163 (135)
2	n_i	0	1	--	--
	Freq.	0.039 (51)	0.500 (70)	0.345 (58)	0.035 (113)
3	n_i	0	1	--	--
	Freq.	0.222 (36)	0.583 (48)	0.158 (38)	0.046 (109)

( ) - # of obs.

The frequencies of contributions in the line network

$A=2$, $E=1$, $K=2$, $N=3$, $T=3$

		A		*B*		*C*
1	n_i	--		--		--
	Freq.	0.006 (180)		0.172 (180)		0.900 (180)
2	n_i	0	1	0	1	--
	Freq.	0.007 (148)	0.161 (31)	0.077 (13)	0.632 (136)	0.167 (18)
3	n_i	0	1	0	1	--
	Freq.	0.115 (61)	0.045 (112)	0.182 (11)	0.686 (51)	0.200 (15)

( ) - # of obs.

The frequencies of contributions in the star-out network

$A=2$, $E=1$, $K=2$, $N=3$, $T=3$

		A			*B,C*
1	n_i	--			--
	Freq.	0.006 (165)			0.318 (330)
2	n_i	0	1	2	--
	Freq.	0.027 (73)	0.195 (77)	0.071 (14)	0.187 (225)
3	n_i	0	1	2	--
	Freq.	0.089 (45)	0.922 (77)	0.000 (24)	0.044 (183)

( ) - # of obs.

The frequencies of contributions in the star-in network

A =2, E =1, K =2, N =3, T =3

		A	B,C	
1	n_i	--	--	
	Freq.	0.620 (150)	0.157 (300)	
2	n_i	--	0	1
	Freq.	0.439 (57)	0.080 (100)	0.229 (153)
3	n_i	--	0	1
	Freq.	0.094 (32)	0.173 (52)	0.215 (158)

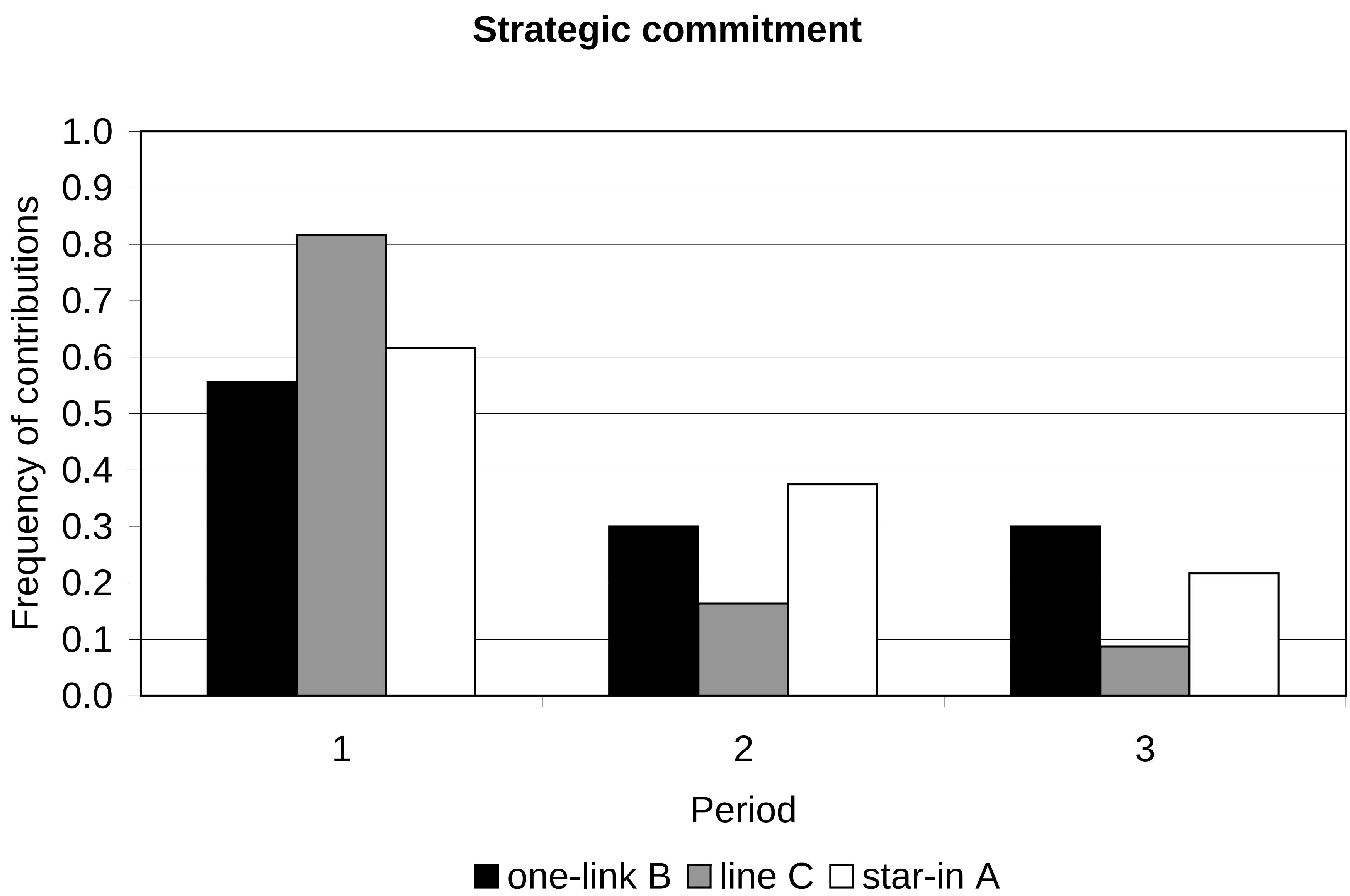
( ) - # of obs.

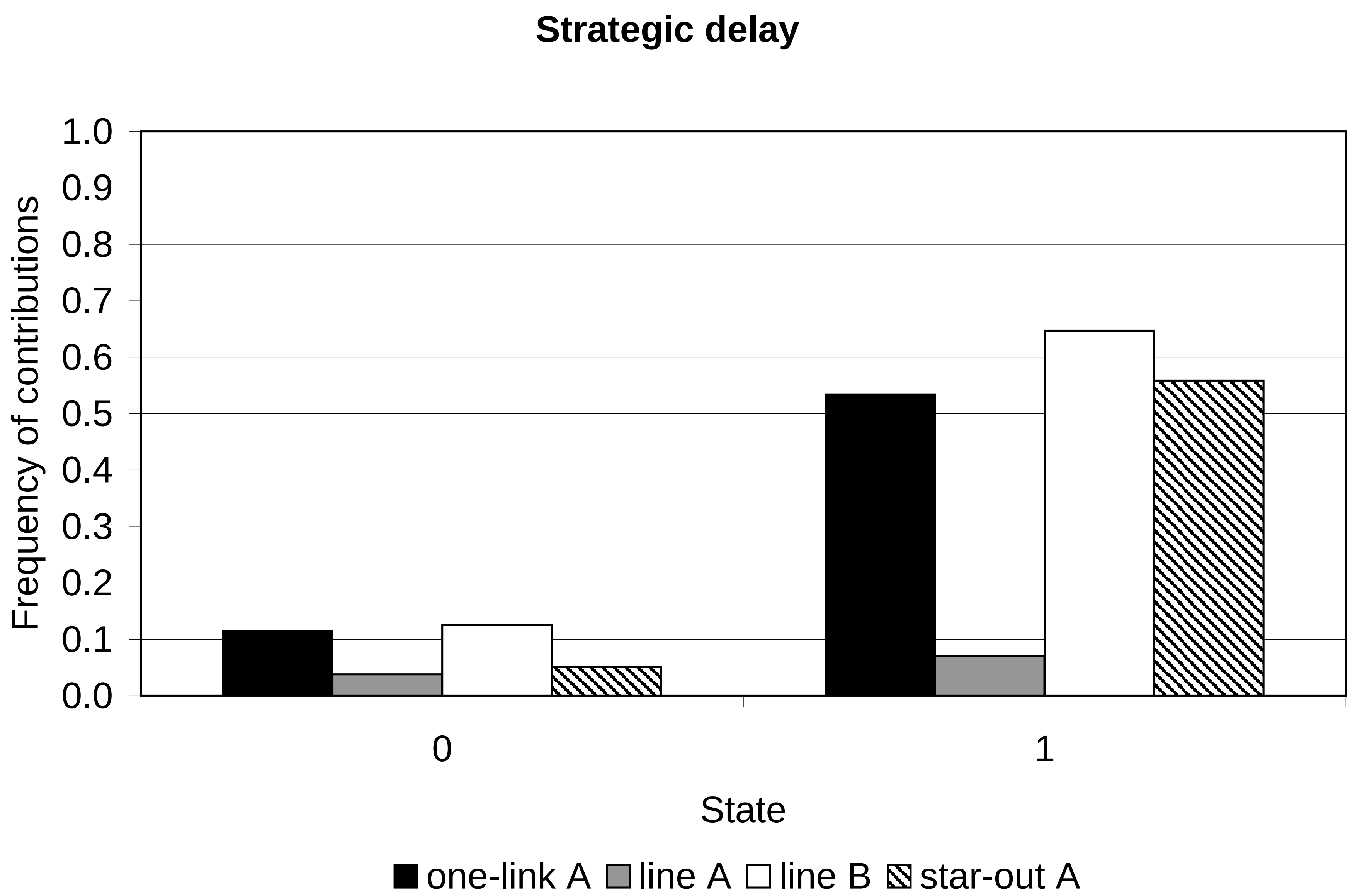
The frequencies of contributions in the pair network

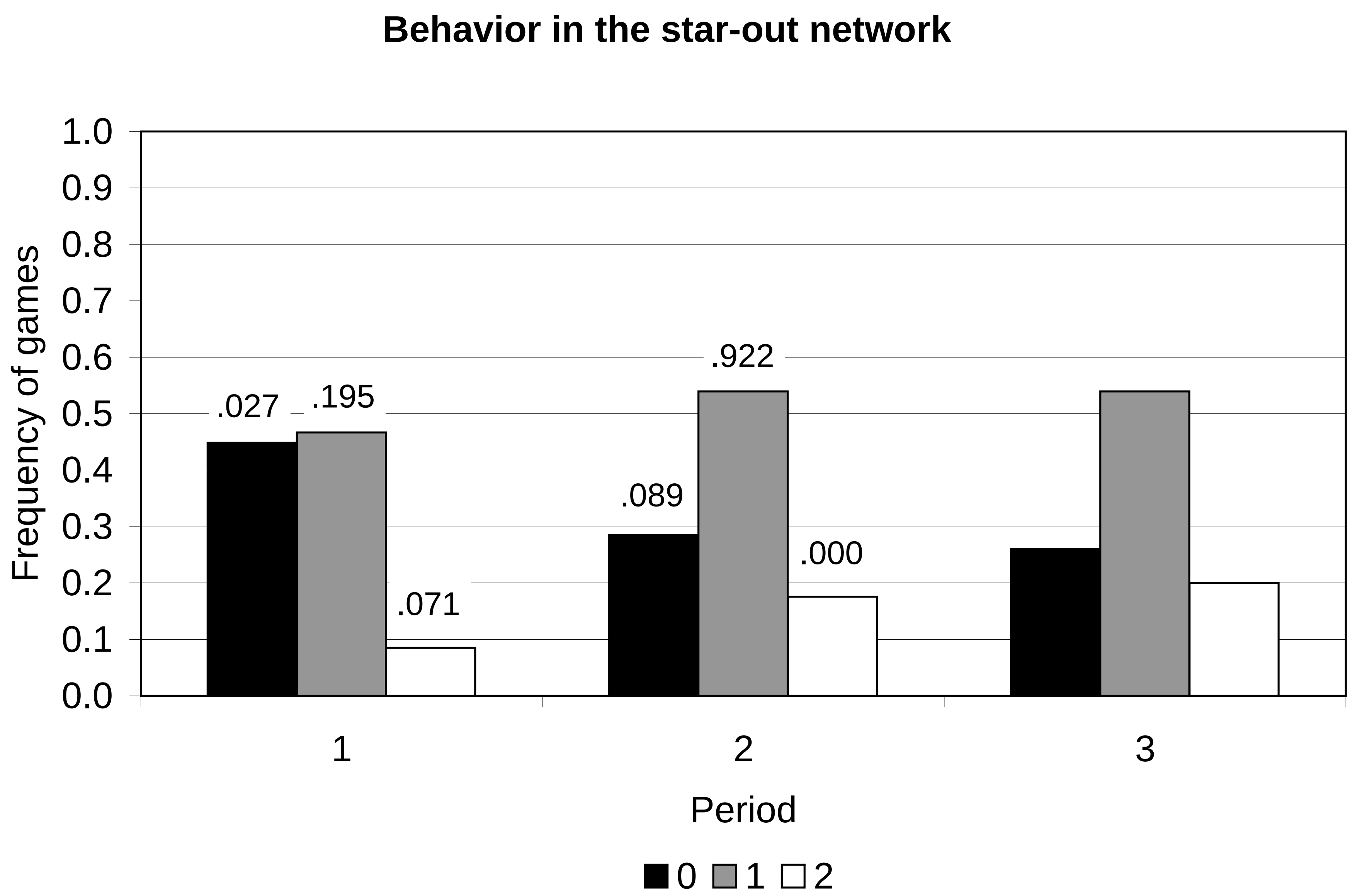
A=2, E=1, K=2, N=3, T=3

		A, B		C
1	n_i	--		--
	Freq.	0.300 (300)		0.100 (150)
2	n_i	0	1	--
	Freq.	0.327 (156)	0.426 (54)	0.022 (135)
3	n_i	0	1	--
	Freq.	0.333 (105)	0.419 (31)	0.053 (132)

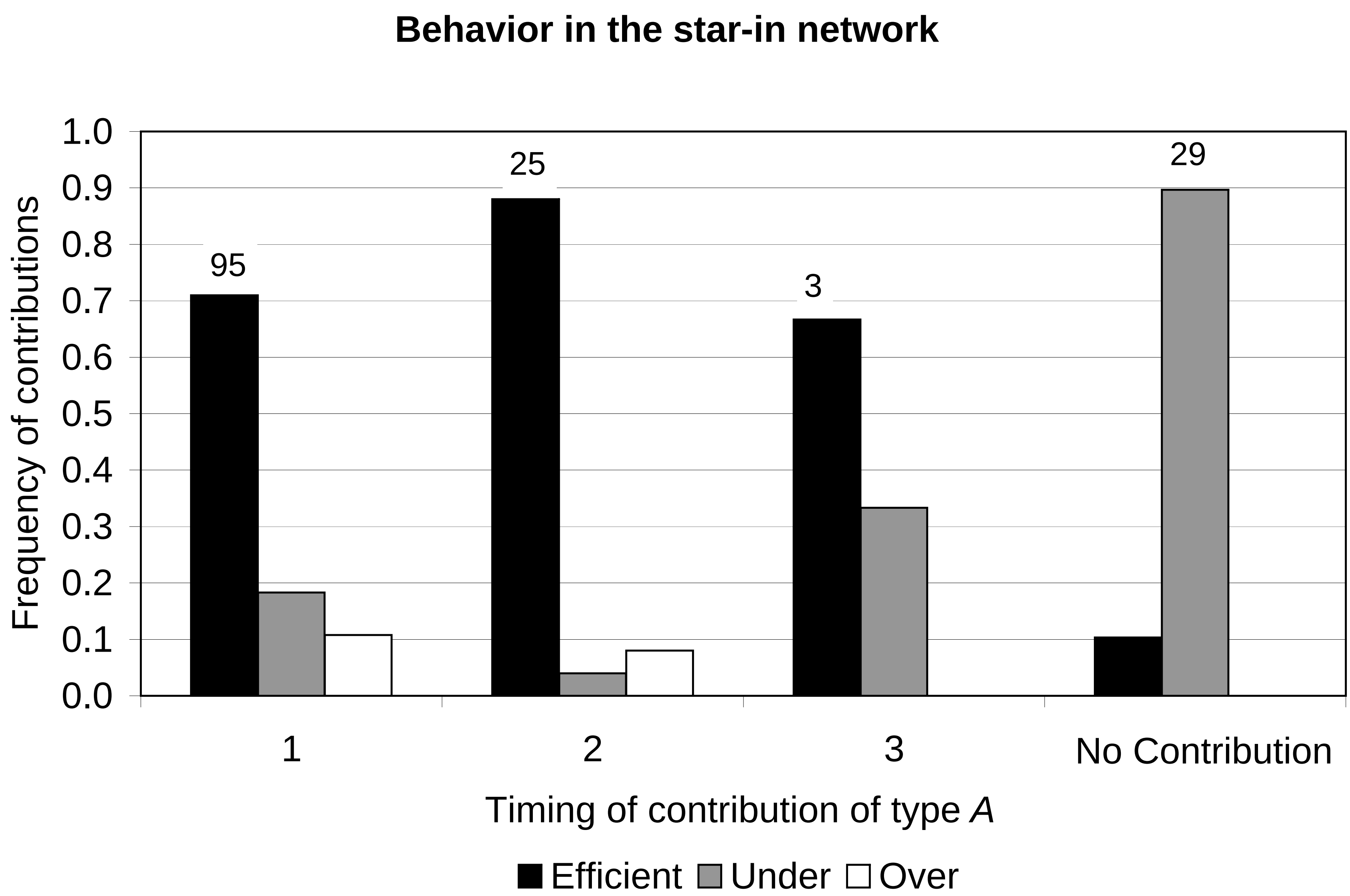
( ) - # of obs.


Strategic commitment
Frequency of contributions
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
Period
one-link B
line C
star-in A


Strategic delay
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Frequency of contributions
0
1
State
one-link A
line A
line B
star-out A



Behavior in the star-out network
Frequency of games
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
.027
.195
.071
.922
.089
.000
1
2
3
Period
0
1
2



Behavior in the star-in network
Frequency of contributions
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
95
25
3
29
1
2
3
No Contribution
Timing of contribution of type A
Efficient
Under
Over