UC Berkeley Department of Economics Foundations of Psychology and Economics (219A)

Module I: Choice under Uncertainty

#### Reading list

- 1. Camerer, C. (1995) "Individual Decision Making," in *Handbook of Experimental Economics*. J. Kagel and A. Roth, eds. Princeton U. Press.
- Starmer, C. (2000) "Developments in Non-Expected Utility Theory: The Hunt for a descriptive Theory of Choice under Risk," *Journal of Economic Literature*, 38, pp. 332-382.
- 3. Harless, D. and C. Camerer (1994) "The Predictive Utility of Generalized Expected Utility Theories," *Econometrica*, 62, pp. 1251-1289.

- 4. Hey, J. and C. Omre (1994) "Investigating Generalizations of Expected Utility Theory Using Experimental Data," *Econometrica*, 62, pp. 1291-1326.
- 5. Holt, C. and S. Laury (2002) "Risk Aversion and Incentive Effects," *American Economic Review*, 92, pp. 1644-1655.
- 6. Choi, S., R. Fisman, D. Gale, and S. Kariv (2007) "Consistency and Heterogeneity of Individual Behavior under Uncertainty," *American Economic Review*, 97, pp. 1921-1938.
- 7. Halevy, Y. (2007) "Ellsberg Revisited: An Experimental Study", *Econometrica*, 75, pp. 503-536.

## Background

- Decisions under uncertainty enter every realm of economic decision-making.
- Models of choice under uncertainty play a key role in every field of economics.
- Test the empirical validity of particular axioms or to compare the predictive abilities of competing theories.

#### Experiments à la Allais

- Each theory predicts indifference curves with distinctive shapes in the probability triangle.
- By choosing alternatives that theories rank differently, each theory can be tested against the others.
- The criterion typically used to evaluate a theory is the fraction of choices it predicts correctly.

# Allais (1953) I

- Choose between the two gambles:

$$\begin{array}{ccc} \$25,000\\ A := & \stackrel{.33}{\nearrow} \\ A := & \stackrel{.66}{\longrightarrow} & \$24,000 \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & &$$

# Allais (1953) II

- Choose between the two gambles:



The Marschak-Machina probability triangle



H, M, and L are three degenerate gambles with certain outcomes H>M>L

A violation of Expected Utility Theory (EUT)



EUT requires that indifference lines are parallel so one must choose either **A** and **C**, or **B** and **D**.

## Contributions

Results have generated the most impressive dialogue between observation and theorizing:

- Violations of EUT raise criticisms about the status of the Savage axioms as the touchstone of rationality.
- These criticisms have generated the development of various alternatives to EUT, such as Prospect Theory.

Results have generated the most impressive dialogue between observation and theorizing (Camerer, 1995):

- ...EU violations are much smaller (though still statistically significant) when subjects choose between gambles that all lie inside the triangle...
- ...due to nonlinear weighting of the probabilities near zero (as the rank dependent weighting theories and prospect theory predict)...
- ...the only theories that can explain the evidence of mixed fanning, violation of betweeness, and approximate EU maximization inside the triangle...

#### Rank-dependent expected utility (Quiggin, 1982)

Index consequences  $(x_1, ..., x_n)$  such that  $x_1$  is the worst and  $x_n$  is the best.

The weights for i = 1, ..., n - 1 are give by:

$$\omega_i = \pi(p_i + \cdots + p_n) - \pi(p_{i+1} + \cdots + p_n)$$

and  $\omega_n = \pi(p_n)$ .

The predictions of the model depend crucially on the form of  $\pi(\cdot)$ . One possibility is an (inverted) S-shaped probability weighting function.

## Limitations

Choice scenarios narrowly tailored to reveal *anomalies* limits the usefulness of data for other purposes:

- Subjects face *extreme* rather than *typical* decision problems designed to encourage violations of specific axioms.
- Small data sets force experimenters to pool data and to ignore individual heterogeneity.

#### **Research questions**

#### Consistency

– Is behavior under uncertainty consistent with the utility maximization model?

#### <u>Structure</u>

– Is behavior consistent with a utility function with some special structural properties?

## Recoverability

- Can the underlying utility function be recovered from observed choices?

## <u>Heuristics</u>

- Can heuristic procedures be identified when they occur?

#### A new experimental design

An experimental design that has a couple of fundamental innovations over previous work:

- A selection of a bundle of contingent commodities from a budget set (a portfolio choice problem).
- A graphical experimental interface that allows for the collection of a rich individual-level data set.

#### The experimental computer program dialog windows





# • The choice of a portfolio subject to a budget constraint provides more information about preferences than a binary choice.

- A large menu of decision problems that are representative, in the statistical sense and in the economic sense.
- A rich dataset that provides the opportunity to interpret the behavior at the level of the individual subject.

#### Rationality

Let  $\{(p^i, x^i)\}_{i=1}^{50}$  be some observed individual data  $(p^i \text{ denotes the } i\text{-th observation of the price vector and } x^i \text{ denotes the associated portfolio}).$ 

A utility function u(x) rationalizes the observed behavior if it achieves the maximum on the budget set at the chosen portfolio

 $u(x^i) \ge u(x)$  for all x s.t.  $p^i \cdot x^i \ge p^i \cdot x$ .

#### **Revealed** preference

A portfolio  $x^i$  is directly revealed preferred to a portfolio  $x^j$  if  $p^i \cdot x^i \ge p^i \cdot x^j$ , and  $x^i$  is strictly directly revealed preferred to  $x^j$  if the inequality is strict.

The relation *indirectly revealed preferred* is the transitive closure of the directly revealed preferred relation.

**Generalized Axiom of Revealed Preference (GARP)** If  $x^i$  is indirectly revealed preferred to  $x^j$ , then  $x^j$  is not strictly directly revealed preferred (i.e.  $p^j \cdot x^j \le p^j \cdot x^i$ ) to  $x^i$ .

GARP is tied to utility representation through a theorem, which was first proved by Afriat (1967).

**Afriat's Theorem** *The following conditions are equivalent:* 

- The data satisfy GARP.
- There exists a non-satiated utility function that rationalizes the data.
- There exists a concave, monotonic, continuous, non-satiated utility function that rationalizes the data.

### Goodness-of-fit

- Verifying GARP is conceptually straightforward but it can be difficult in practice.
- Since GARP offers an exact test, it is necessary to measure the extent of GARP violations.
- Measures of GARP violations based on three indices: Afriat (1972), Varian (1991), and Houtman and Maks (1985).

**Afriat's critical cost efficiency index (CCEI)** The amount by which each budget constraint must be relaxed in order to remove all violations of GARP.

The CCEI is bounded between zero and one. The closer it is to one, the smaller the perturbation required to remove all violations and thus the closer the data are to satisfying GARP.

The construction of the CCEI for a simple violation of GARP



The agent is 'wasting' as much as A/B < C/D of his income by making inefficient choices.

#### A benchmark level of consistency

A random sample of hypothetical subjects who implement the power utility function

$$u(x) = \frac{x^{1-\rho}}{1-\rho},$$

commonly employed in the empirical analysis of choice under uncertainty, with error.

The likelihood of error is assumed to be a decreasing function of the utility cost of an error.

More precisely, we assume an idiosyncratic preference shock that has a logistic distribution

$$\mathsf{Pr}(x^*) = rac{e^{\gamma \cdot u(x^*)}}{\int e^{\gamma \cdot u(x)}},$$

where the precision parameter  $\gamma$  reflects sensitivity to differences in utility.

If utility maximization is not the correct model, is our experiment sufficiently powerful to detect it?



#### The distributions of GARP violations – $\rho$ =1/2 and different $\gamma$

#### Bronnars' (1987) test (γ=0)



## Recoverability

- GARP imposes on the data the complete set of conditions implied by utility-maximization.
- Revealed preference relations in the data thus contain the information that is necessary for recovering preferences.
- Varian's (1982) algorithm serves as a partial solution to this so-called *re-coverability problem*.





Infinite risk aversion







■ Actual ■ Random

### The distributions of GARP violations - Varian (1991)




# The distribution of GARP violations - Houtman and Maks (1985)

Actual

# Granularity

- A measure of the size of the components, or descriptions of components, that make up a system (Wikipedia).
- There is no taxonomy that allows us to classify all subjects unambiguously.
- A review of the full data set reveals striking regularities *within* and marked heterogeneity *across* subjects.





















#### **Risk** aversion

A "low-tech" approach of estimating an individual-level power utility function directly from the data:

$$u(x) = \frac{x^{1-\rho}}{(1-\rho)}.$$

 $\rho$  is the Arrow-Pratt measure of relative risk aversion. The aversion to risk increases as  $\rho$  increases.

This generates the following individual-level econometric specification for each subject n:

$$\log\left(\frac{x_{2n}^i}{x_{1n}^i}\right) = \alpha_n + \beta_n \log\left(\frac{p_{1n}^i}{p_{2n}^i}\right) + \epsilon_n^i$$
 where  $\epsilon_n^i \sim N(\mathbf{0}, \sigma_n^2)$ .

We generate estimates of  $\hat{\rho}_n = 1/\hat{\beta}_n$  which allows us to test for heterogeneity of risk preferences.



The distribution of the individual Arrow-Pratt measures (OLS)

■ OLS

## Loss/disappointment aversion

The theory of Gul (1991) implies that the utility function over portfolios takes the form

$$\min \{ \alpha u(x_1) + u(x_2), u(x_1) + \alpha u(x_2) \},\$$

where  $\alpha \geq 1$  measures loss/disappointment aversion and  $u(\cdot)$  is the utility of consumption in each state.

If  $\alpha > 1$  there is a kink at the point where  $x_1 = x_2$  and if  $\alpha = 1$  we have the standard EUT representation.

### An illustration of the derived demand



The indifference map of Gul (1991)





## Scatterplot of the estimated CRRA parameters



The risk premium r(1) for different values of  $\alpha$  and  $\rho$ 



## Scatterplot of the risk measures – power and loss/disappointment

# Ambiguity

- The distinction between settings with risk and ambiguity dates back to at least the work of Knight (1921).
- Ellsberg (1961) countered the reduction of subjective uncertainty to risk with several thought experiments.
- A large theoretical literature (axioms over preferences) has developed models to accommodate this behavior.
- But what matters most is the implications of the models for choice behavior.

# Ellsberg (1961)

An urn contains 300 marbles; 100 of the marbles are red, and 200 are some mixture of blue and green. We will reach into this urn and select a marble at random:

- You receive \$25,000 if the marble selected is of a specified color.
  Would you rather the color be red or blue?
- You receive \$25,000 if the marble selected is *not* of a specified color.
  Would you rather the color be red or blue?

Consider the following two-color Ellsberg-type urns (Halevy, 2007):

- I. 5 red balls and 5 black balls
- II. an unknown number of red and black
- III. a bag containing 11 tickets with the numbers 0-10; the number written on the drawn ticket determines the number of red balls
- IV. a bag containing 2 tickets with the numbers 0 and 10; the number written on the drawn ticket determines the number of red balls.

- A clever experiment to verify the connection between the reduction of objective compound lotteries and attitudes to ambiguity.
- Four different urns are used to elicit choices in the presence of risk, ambiguity, and two degrees of compound uncertainty.
- Different models generate different predictions about how the reservation values (BDM) for these four urns will be ordered.
- The experiment can therefore classify each subject according to which model predicts his ordering of reservation values.

- Now, consider three states of nature and corresponding Arrow security (pays one dollar in one state and nothing in the other states).
- One state has an objectively known probability, whereas the probabilities of the other states are ambiguous.
- The presence of ambiguity could cause not just a departure from EU, but a more fundamental departure from rationality.
- Our analysis suggests otherwise choices under ambiguity are at least as rationalizable as choices under risk.

#### The distributions of CCEI scores



■ Risk □ Ambiguity ☑ Random

## Structure, recoverability and extrapolation

The conventional parametric approach:

- Choose a parametric form for the underlying utility function and fit the associated demand function to the data.
- Test to see if they conform to the special restrictions imposed by hypotheses concerning functional structure.
- Construct an estimate of the underlying utility function and forecast demand behavior in new situations.

### Recursive Expected Utility (REU)



2.5

2.5



The Simulated  $x_1/(x_1+x_2)$  and  $log(p_1/p_2)$  in the REU Model when alpha = 1.75

2.5

2.5

### $\alpha$ -Maxmin Expected Utility ( $\alpha$ -MEU)





The Simulated  $x_1/(x_1+x_3)$  and  $log(p_1/p_3)$  in the Alpha-MEU Model when alpha = 0.6





The Simulated  $x_1/(x_1+x_2)$  and  $log(p_1/p_2)$  in the Alpha-MEU Model when alpha = 0.6



The Simulated  $x_1/(x_1+x_2)$  and  $log(p_1/p_2)$  in the Alpha-MEU Model when alpha = 0.55





The Simulated  $x^{}_1/(x^{}_1+x^{}_2)$  and log(p^{}\_1/p^{}\_2) in the Alpha-MEU Model when alpha = 0.8

0.9

0.8

0.7

0.6

rho = 0.05

•• rho = 0.10

rho = 0.25

rho = 0.5

--- rho = 1

• rho = 2

•










Estimated  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  in the Ambiguity Treatment for Subject ID 339



### **Procedural rationality**

- How subjects come to make decisions that are consistent with an underlying preference ordering?
- Boundedly rational individuals use *heuristics* in their attempt to maximize an underlying preference ordering.
  - There is a distinction between true *underlying* preferences and *revealed* preferences.
  - Preferences have an EU representation, even though revealed preferences appear to be non-EU.

## A type-mixture model (TMM)

A *unified* account of both procedural rationality and substantive rationality.

- Allow EU maximization to play the role of the underlying preference ordering.
- Account for subjects' underlying preferences and their choice of decision rules.

### Ingredients

- The "true" underlying preferences are represented by a power utility function.
- A discrete choice among the fixed set of prototypical heuristics, D, S and  $B(\omega)$ .
- The probability of choosing each particular heuristic is a function of the budget set.

- Subjects could make mistakes when trying to maximize EU by employing heuristic S.
- In contrast, when following heuristic D or  $B(\omega)$  subjects' hands do not tremble.
- A subject may prefer to choose heuristic  $B(\omega)$  or D instead of the noisy version of heuristic S.

#### **Specification**

The underlying preferences of each subject are assumed to be represented by

$$u(x) = \frac{x^{1-\rho}}{(1-\rho)}$$

(power utility function as long as consumption in each state meets the secure level  $\omega$ ).

Let  $\varphi(p)$  be the portfolio which gives the subject the maximum (expected) utility achievable at given prices p.

The *ex ante* expected payoff from attempting to maximize EU by employing heuristic S is given by

$$U_S(p) = \mathbb{E}[\pi u \left( \tilde{\varphi}_1(p) \right) + (1 - \pi) u \left( \tilde{\varphi}_2(p) \right)]$$

 $\tilde{\varphi}(p)$  is a random portfolio s.t.  $p \cdot \tilde{\varphi}(p) = 1$  for every  $p = (p_1, p_2)$ , and  $p_1[\tilde{\varphi}_1(p) - \varphi_1(p)] = \varepsilon$  and  $\epsilon_n^i \sim N(0, \sigma_n^2)$ .

When following heuristic D or B subjects' hands do not tremble. We therefore write

$$U_D(p) = u(1/(p_1 + p_2))$$

 $\mathsf{and}$ 

$$U_B(p) = \max\{\pi u(0) + (1 - \pi)u(1/p_2), \pi u(1/p_1) + (1 - \pi)u(0)\}$$

#### Estimation

The probability of choosing heuristic  $k = D, S, B(\omega)$  is given by a standard logistic discrete choice model:

$$\mathsf{Pr}(\mathsf{heuristic} \ \tau \ | p; \beta, \rho, \sigma) = \frac{e^{\beta U_{\tau}}}{\sum\limits_{k=D,S,B} e^{\beta U_{k}}}$$

where  $U_D$ ,  $U_S$  and  $U_B$  is the payoff specification for heuristic D, S and  $B(\omega)$ , respectively.

- The  $\hat{\beta}$  estimates are significantly positive, implying that the TMM has some predictive power.
- Most subjects exhibit moderate to high levels of risk preferences around  $\hat{\rho} = 0.8.$
- There is a strong correlation between the estimated  $\hat{\rho}$  parameters from "low-tech" OLS and TMM estimations.



### The distribution of the individual Arrow-Pratt measures (TMM)

■ TMM

#### Goodness-of-fit

- Compare the choice probabilities predicted by the TMM and empirical choice probabilities.
- Nadaraya-Watson nonparametric estimator with a Gaussian kernel function.
- The empirical data are supportive of the TMM model (fits best in the symmetric treatment).

### **Procedural rationality**

- How subjects come to make decisions that are consistent with an underlying preference ordering?
- Boundedly rational individuals use *heuristics* in their attempt to maximize an underlying preference ordering.
  - There is a distinction between true *underlying* preferences and *revealed* preferences.
  - Preferences have an EU representation, even though revealed preferences appear to be non-EU.

#### Archetypes and polytypes

- We identify a finite number of stylized behaviors, which collectively pose a challenge to decision theory.
- We call these basic behaviors *archetypes*. We also find mixtures of archetypal behaviors, which we call *polytypes*.
- The archetypes account for a large proportion of the data set and play a role in the behavior of most subjects.
- The combinations of types defy any of the standard models of risk aversion.











# The aggregate distribution of archetypes for different token confidence intervals

	Center	Vertex	Centroid	Edge	<b>Bisector</b>	All
0.1	0.005	0.003	0.000	0.019	0.083	0.110
0.25	0.061	0.004	0.002	0.083	0.187	0.337
0.5	0.093	0.011	0.007	0.139	0.215	0.466
1	0.134	0.032	0.019	0.165	0.252	0.602
2.5	0.185	0.064	0.049	0.186	0.285	0.769

# The distribution of archetypes, by subject (half token confidence interval)



■ Center □ Vertex ■ Centroid □ Edge ■ Bisector

# The distribution of archetypes, by subject (one token confidence interval)



■ Center □ Vertex ■ Centroid □ Edge ■ Bisector



Token Shares in 3-asset experiment for Subject ID 3







Token Shares in 3-asset experiment for Subject ID 47



Token Shares in 3-asset experiment for Subject ID 25



Token Shares in 3-asset experiment for Subject ID 61



Token Shares in 3-asset experiment for Subject ID 65

## Type-mixture model (TMM)

- A subject chooses among the fixed set of *types* (heuristics), in order to approximate the behavior that is optimal for his true underlying preferences.
- Consistent behavior requires subjects to choose among heuristics in a consistent manner as well as behaving consistently in applying a given heuristic.
- A TMM (CFGK, 2006) combines the distinctive types of behavior observed in the raw data in a coherent theory based on underlying preferences.
- It exhibits significant explanatory power and produces reasonable estimates of risk aversion.

#### Discussion

Suppose there are  $\ell$  states of nature and  $\ell$  associated Arrow securities and that the agent's behavior is represented by the decision problem

 $\begin{array}{ll} \mathsf{max} & u\left(\mathbf{x}\right) \\ \mathsf{s.t.} & \mathbf{x} \in \mathbf{B}\left(\mathbf{p}\right) \cap \mathcal{A} \end{array}$ 

where B(p) is the budget set and A is the set of portfolios corresponding to the various archetypes the agent uses to simplify his choice problem.

The only restriction we have to impose is that  $\mathcal{A}$  is a pointed cone (closed under multiplication by positive scalars), which is satisfied if  $\mathcal{A}$  is composed of any selection of archetypes except the *Centroid*.

We can derive the following properties of the agent's demand:

1. Let  $\mathbf{p}^k$  denotes the k-th observation of the price vector and  $\mathbf{x}^{k} \in {
m arg\,max}\left\{ u\left(\mathbf{x}
ight): \mathbf{x} \in \mathbf{B}\left(\mathbf{p}^{k}
ight) \cap \mathcal{A}
ight\}$ denotes the associated portfolio. Then the data  $\left\{ \mathbf{p}^k, \mathbf{x}^k 
ight\}$  satisfy

GARP.

2. There exists a utility function  $u^*(\mathbf{x})$  such that for any price vector  $\mathbf{p}$ ,

$$\begin{split} \mathbf{x}^{*} \in \arg \max \left\{ u\left(\mathbf{x}\right) : \mathbf{x} \in \mathbf{B}\left(\mathbf{p}\right) \cap \mathcal{A} \right\} \\ \Leftrightarrow \\ \mathbf{x}^{*} \in \arg \max \left\{ u^{*}\left(\mathbf{x}\right) : \mathbf{x} \in \mathbf{B}\left(\mathbf{p}\right) \right\}. \end{split}$$

#### Takeaways

- [1] Classical economics assumes that decisions are based on substantive rationality, and has little to say about the procedures by which decisions are reached.
- [2] Rather than focusing on the consistency of behavior with non-EUT theories, we study the fine-grained details of individual behaviors in search of clues to procedural rationality.
- [3] The "switching" behavior that is evident in the data leads us to prefer an "alternative" approach – one that emphasizes standard preferences and procedural rationality.