UC Berkeley
Haas School of Business
Economic Analysis for Business Decisions
(EWMBA 201A)
Fall 2012

Production and the costs production (PR 6.1-6.4 7.1-7.2)
Cost concepts and the relations among costs

Lectures 3-4

Aug. 18, 2012

Production (PR 6.1)

- In the production process, firms turn inputs (labor, capital, materials) into output.
- The theory of the firm explains how a firm makes (optimal) production decisions and how its costs vary with its output.
- Like the theory of the consumer, the theory of the firm can be used to predict, postdict (explain), and prescribe.

The production function

The production function indicates the (highest) output q that the firm can produce for a given combination of inputs.

For our purposes it is convenient to consider only the case of two inputs – capital K and labor L.

We can the write the production function as

$$q = F(K, L).$$

The Cobb-Douglas production function is widely used to represent the technology of production

$$q = K^{\alpha} \cdot L^{\beta}$$

where $\alpha, \beta > 0$.

The parameters of the Cobb-Douglas function, α and β, determine the marginal product of capital and labor.

The law of diminishing marginal returns $\alpha, \beta < 1$ holds for most production processes:

$$F'_K = \alpha K^{\alpha-1} L^{\beta} > 0 \text{ and } F'_L = \beta L^{\beta-1} K^{\alpha} > 0$$

and

$$F''_K = (\alpha - 1)\alpha K^{\alpha-2} L^{\beta} < 0 \text{ and } F''_L = (\beta - 1)\beta L^{\beta-2} K^{\alpha} < 0.$$

Returns to scale (PR 6.4)

Returns to scale is the rate is the rate at which output q increases as inputs, K and L, are increased proportional.

$\longrightarrow$ Increasing – output *more* than doubles when inputs are doubled.

$\longrightarrow$ Decreasing – output *less* than doubles when inputs are doubled.

$\longrightarrow$ Constant – output doubles when inputs are doubled.

Returns to scale vary considerably across and within industries. Other things being equal, the greater the return to scale in an industry, the larger the firms are likely to be (a natural monopoly!).

The Cobb-Douglas production function exhibits increasing, constant, or decreasing returns to scale if

$$\begin{aligned} \alpha + \beta &> 1 \\ \alpha + \beta &= 1 \\ \alpha + \beta &< 1 \end{aligned}$$

respectively.

– PR Figure 6.6-6.9 and 7.3-7.7 here –

The short run vs. the long run (PR 6.1)

- The "short" run is the period of time in which the quantities of one or more inputs cannot be changed.

- The "long" run is the period of time in which the quantities of *all* inputs can be changed.

- Typically, we assume that in the sort run labor is variable and capital is fixed (more later!).

$\Rightarrow$ John Keynes: "The long run is a misleading guide to current affairs. In the long run we are all dead..."

Costs (PR 7.1)

Question: What is the cost of an activity (producing a good or service)?

- The obvious answer might seem "the money spent on that activity."

Like so many "obvious" answers, this is not (necessarily) the correct answer.

- The correct way to consider the cost to be the value of the most highly valued forgone activity the value of the best alternative decision).

$\Rightarrow$ Note: Economists describe this way of viewing cost as considering the *opportunity cost* of an activity or decision.

Accounting cost

– Actual expenses *plus* depreciation charges to capital (this naive view, focusing on expenditures can lead one to make bad decisions).

Economic cost

– Cost of utilizing inputs in production, including opportunity cost (for example, the imputed value of the forgone rent on owned office space).

$\Rightarrow$ Note: A sunk expenditure (cannot be recovered or avoided over the relevant decision-making horizon) is not an economic cost.

Cost concepts

Fixed cost (FC) does not vary with the level of production and can be eliminated *only* by shutting down (not sunk cost). Variable cost (VC) varies as output varies. Total cost (TC or C) is the total economic cost of production

$$TC(q) = FC + VC(q).$$

Marginal cost (MC) is the increase in cost resulting from the production of one extra unit of output

$$MC = \frac{\Delta TC}{\Delta q} = \frac{\Delta VC}{\Delta q}.$$

Relations among costs in the short run (PR 7.2)

We next explore the relations among total cost, average cost, and marginal cost from an algebraic and graphical perspectives.

Note that

$$MC(q) = VC(q) - VC(q-1)$$

and observe that we can express variable cost as follows

$$VC(q) = MC(1) + \cdots + MC(q) = \sum_{j=1}^{q} MC(j).$$

The variable total of producing q units is the sum of the marginal costs of producing the first q units.

The average variable cost (AVC) of producing q units is

$$AVC(q) = \frac{VC(q)}{q} = \frac{\sum_{j=1}^{q} MC(j)}{q}$$

and the average total cost (ATC) of producing q units is

$$\begin{aligned} ATC(q) &= \frac{TC(q)}{q} \\ &= \frac{VC(q)}{q} + \frac{FC}{q}. \end{aligned}$$

Result Marginal cost and average cost

(i) If average variable cost (AVC) is decreasing, then marginal cost (MC) is less than average variable cost.

(ii) If average variable cost (AVC) is increasing, then marginal cost (MC) is greater than average average variable cost.

Another way to state this conclusion is

$$MC(q+1) < AVC(q) \text{ then } AVC(q+1) < AVC(q)$$

and

$$MC(q+1) > AVC(q) \text{ then } AVC(q+1) > AVC(q)$$

– PR Figure 7.1 here –

Proof Case (ii):

$$AVC(q+1) = \frac{MC(q+1) + \sum_{j=1}^{q} MC(j)}{q+1} = \frac{MC(q+1)}{q+1} + \frac{q}{q+1} AVC(q).$$

If $MC(q+1) > AVC(q)$, then

$$AVC(q+1) = \frac{MC(q+1)}{q+1} + \frac{q}{q+1} AVC(q) > \frac{1+q}{q+1} AVC(q) = AVC(q).$$

Case (i) is established similarly.

Costs in the short run and the long run

The relationship between short-run and long-run cost can be fairly complicated.

In the long run, the firm can change the quantity of any of its input, whereas in the short run the quantity of at least one input is fixed.

As a result, the long-run average cost curve never lies above any of the short-run average cost curves (an envelope).

– PR Figure 7.8 and 7.10 here –

Problem set II

- PR 6 - exercises 8-11.

- PR 7 - exercises 1-5.