UC Berkeley
Haas School of Business
Economic Analysis for Business Decisions
(EWMBA 201A)

Choice under uncertainty (PR 5)
Expected Utility Theory vs. Prospect Theory

Lectures 3-4
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Prologue

- Uncertainty is a fact of life so people's attitudes towards risk enter every realm of economic decision-making.

- We *must* study individual behavior with respect to choice involving uncertainty.

- Models of decision making under uncertainty play a key role in every field of economics.

Preliminaries (PR 5.1)

First, we define some key concepts to which we refer throughout the lecture:

- Objective and subjective probability, and probability distribution.
- Risk (known probabilities) and ambiguity (unknown probabilities).
- Variability (expected value, standard deviation, and variance).

Probability is the likelihood that a given outcome will occur. Suppose there are two possible outcomes having payoffs x_1 and x_2 and that the probabilities of these outcomes are p_1 and p_2, then the *expected value* is

$$E(x) = p_1 x_1 + p_2 x_2$$

where $p_1 + p_2 = 1$ (a probability distribution). More generally, when there are n outcomes the *expected value* is

$$E(x) = p_1 x_1 + p_2 x_2 + \cdots + p_n x_n.$$

When there are two outcomes x_1 and x_2 occurring with probabilities p_1 and p_2 the *variance* is given by

$$\sigma^2 = p_1[x_1 - E(x)]^2 + p_2[x_2 - E(x)]^2,$$

and when there are n outcomes $x_1, x_2, ..., x_n$ occurring with probabilities p_1, $p_2, ...p_n$ the variance is given by

$$\sigma^2 = p_1[x_1 - E(x)]^2 + p_2[x_2 - E(x)]^2 + \cdots + p_n[x_n - E(x)]^2.$$

The *standard deviation* is the square root of the variance σ and it is a standard measure of variability.

An example (PR Tables 5.1-5.3)

	Outcome 1		Outcome 2		
	p_1	x_1	p_2	x_2	$E(x)$
Job 1	.5	2000	.5	1000	1500
Job 2	.99	1510	.01	510	1500

	Outcome 1		Outcome 2		
	x_1	$[x_1 - E(x)]^2$	x_2	$[x_2 - E(x)]^2$	σ^2
Job 1	2000	$250,000$	1000	$250,000$	500
Job 2	1510	100	510	$9,900$	99.5

A modified example (PR Table 5.4)

	Outcome 1		Outcome 2		
	p_1	x_1	p_2	x_2	$E(x)$
Job 1	.5	2100	.5	1100	1600
Job 2	.99	1510	.01	510	1500

	Outcome 1		Outcome 2		
	x_1	$[x_1 - E(x)]^2$	x_2	$[x_2 - E(x)]^2$	σ^2
Job 1	2000	$250,000$	1000	$250,000$	500
Job 2	1510	100	510	$9,900$	99.5

Preferences toward risk (PR 5.2)

The standard model of decisions under risk (known probabilities) is based on von Neumann and Morgenstern Expected Utility Theory.

Let X be a set of *lotteries*, or gambles, (outcomes and probabilities). A fundamental assumption about preferences toward risk is *independence*:

For any lotteries x, y, z and $0 < \alpha < 1$

$$x \succ y \text{ implies } \alpha x + (1-\alpha)r \succ \alpha y + (1-\alpha)r.$$

Expected Utility Theory has some very convenient properties for analyzing choice under uncertainty.

To clarify, we will consider the *utility* that a consumer gets from her or his income.

More precisely, from the consumption bundle that the consumer's income can buy.

Expected utility is the sum of utilities associated with all possible outcomes, weighted by the probability that each outcome will occur.

In the job example above the expected utility from job 1 is given by

$$E(u) = .5u(\$2000) + .5u(\$1000),$$

and the expected utility from job 1 is given by

$$E(u) = .5u(\$1510) + .5u(\$510).$$

– PR Figures 5.1-5.2 here –

Asset markets (PR 5.4)

An *asset* provides a flow of money (or services) to its owner. A *risky asset* provides a monetary flow that is at least in part random.

Suppose there are two assets – risk-free (treasury bill) with known return R_f and risky asset (stock) with expected return $R_m > R_f$.

We can describe the risk-return tradeoff in terms of indifference curves and solve the investor's choice problem.

– PR Figures 5.5-5.6 here –

Behavioral economics (PR 5.5)

Allais (1953) I

- Choose between the two gambles:

$$A := \begin{array}{l} \overset{.33}{\nearrow} \ \$25,000 \\ \xrightarrow{.66} \ \$24,000 \\ \underset{.01}{\searrow} \ \$0 \end{array} \qquad B := \xrightarrow{1} \$24,000$$

Allais (1953) II

– Choose between the two gambles:

$$C := \begin{array}{l} \overset{.33}{\nearrow} \quad \$25,000 \\ \underset{.067}{\searrow} \quad \$0 \end{array} \qquad D := \begin{array}{l} \overset{.34}{\nearrow} \quad \$24,000 \\ \underset{.066}{\searrow} \quad \$0 \end{array}$$

Ellsberg (1961)

An urn contains 300 marbles; 100 of the marbles are red, and 200 are some mixture of blue and green. We will reach into this urn and select a marble at random:

– You receive $\$25,000$ if the marble selected is of a specified color. Would you rather the color be red or blue?

– You receive $\$25,000$ if the marble selected is *not* of a specified color. Would you rather the color be red or blue?

Prospect Theory (Kahneman and Tversky, 1979)

Explains several key expected utility violations, including the classical paradox of Allais (1953).

Prospect theory has two ingredients:

- *Reference point* – the point from which the agent makes a decision.

- Loss aversion – the aggravation from losing is greater than the pleasure from gaining.

$\Longrightarrow$ The *endowment effect* – the tendency to value an asset more when you own it...

Takeaways

- Consumers and managers frequently make decisions with uncertain consequences.

- Facing uncertain choices, Expected Utility consumers maximize the average expected utility associated with each outcome.

- Individual behavior is often contrary to the assumptions of Expected Utility Theory (an important frontier of choice theory).