UC Berkeley
Haas School of Business
Economic Analysis for Business Decisions
(EWMBA 201A)

Game Theory I (PR 5)
The main ideas

Lectures 5-6
Aug. 29, 2009

Prologue

- Game theory is about what happens when decision makers (spouses, workers, managers, presidents) interact.

- In the past fifty years, game theory has gradually became a standard language in economics.

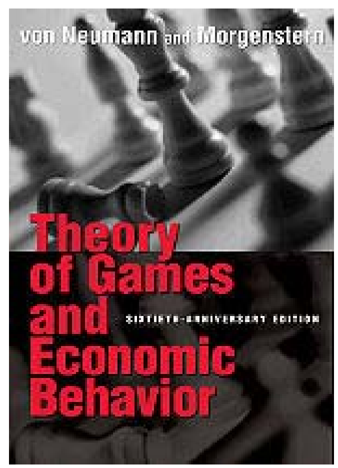
- The power of game theory is its generality and (mathematical) precision.

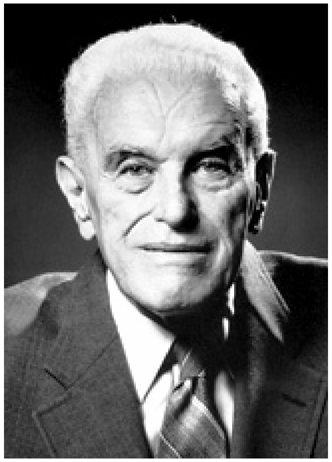
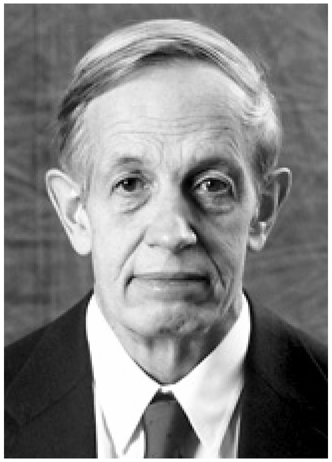
- Because game theory is rich and crisp, it could unify many parts of social science.

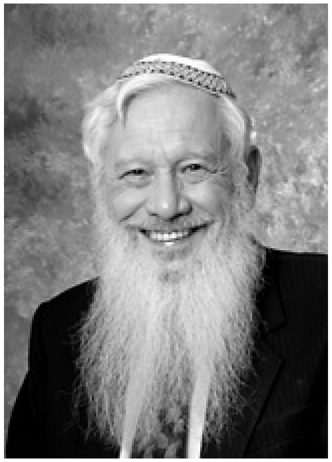
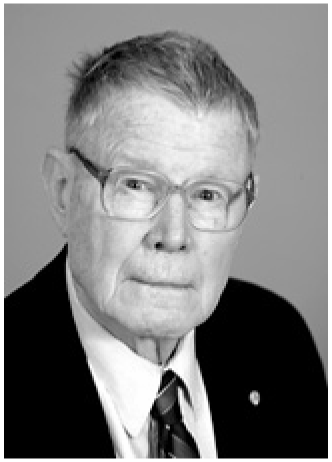
- The spread of game theory outside of economics has suffered because of the misconception that it requires a lot of fancy math.

- Game theory is also a natural tool for understanding complex social and economic phenomena in the real world.

The paternity of game theory



What is game theory good for?

Q Is game theory meant to predict what decision makers do, to give them advice, or what?

A The tools of analytical game theory are used to predict, postdict (explain), and prescribe.

Remember: even if game theory is not always accurate, descriptive failure is prescriptive opportunity!

Game theory and MBAs

- Adam Brandenburger (NYU) and Barry Nalebuff (Yale) explain how to use game theory to shape strategy (Co-Opetition).

- Both are brilliant game theorists who could have written a more theoretical book.

- They choose not to because teaching MBAs and working with managers is more useful.

Aumann (1987):

> "*Game theory is a sort of umbrella or 'unified field' theory for the rational side of social science, where 'social' is interpreted broadly, to include human as well as non-human players (computers, animals, plants).*"

Three examples

Example I: Hotelling's electoral competition game

- There are two candidates and a continuum of voters, each with a favorite position on the interval $[0, 1]$.

- Each voter's distaste for any position is given by the distance between the position and her favorite position.

- A candidate attracts the votes off all citizens whose favorite positions are closer to her position.

Example II: Keynes's beauty contest game

- Simultaneously, everyone choose a number (integer) in the interval $[0, 100]$.

- The person whose number is closest to $2/3$ of the average number wins a fixed prize.

John Maynard Keynes (1936):

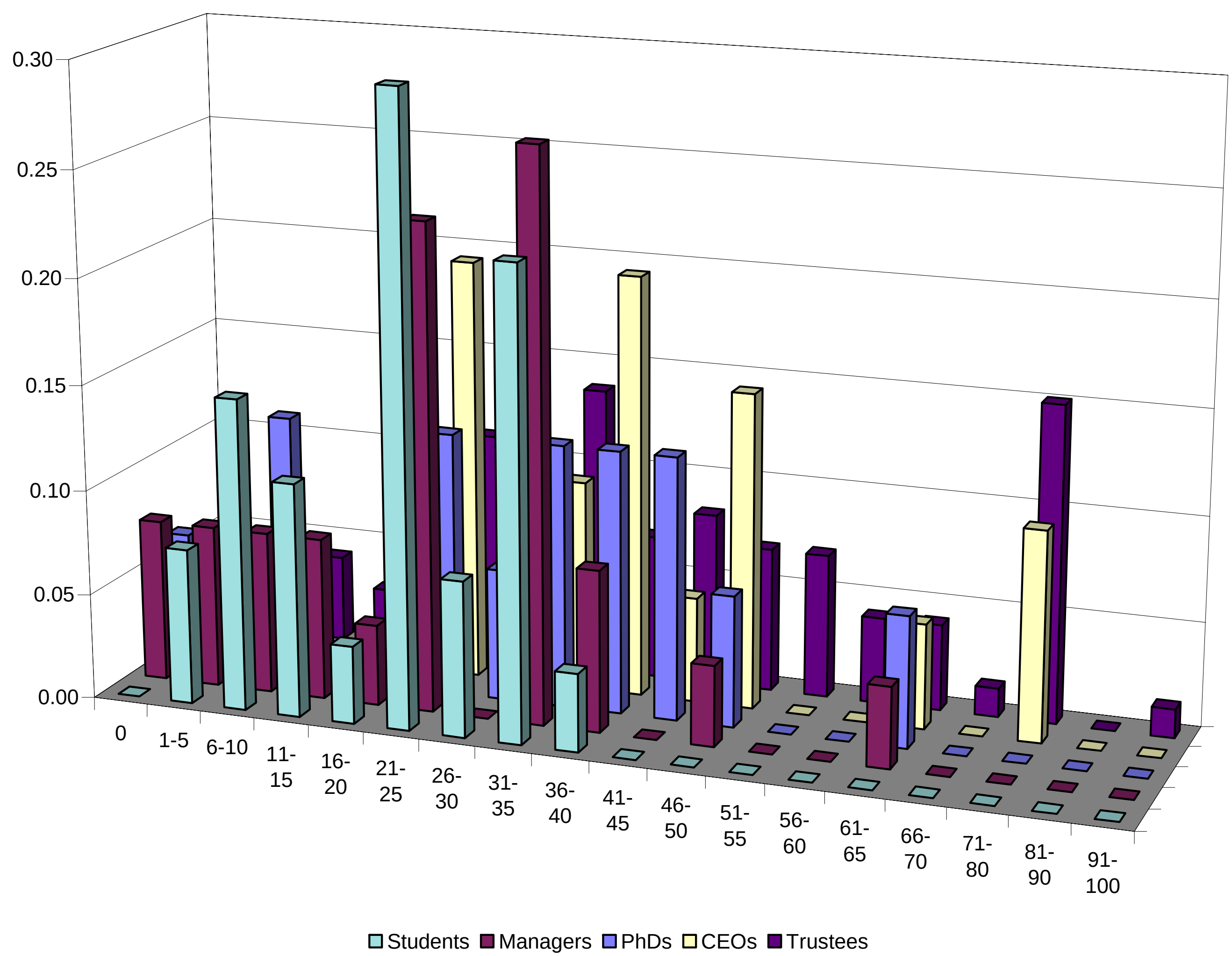
> "*It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.*"

$\Longrightarrow$ self-fulfilling price bubbles!

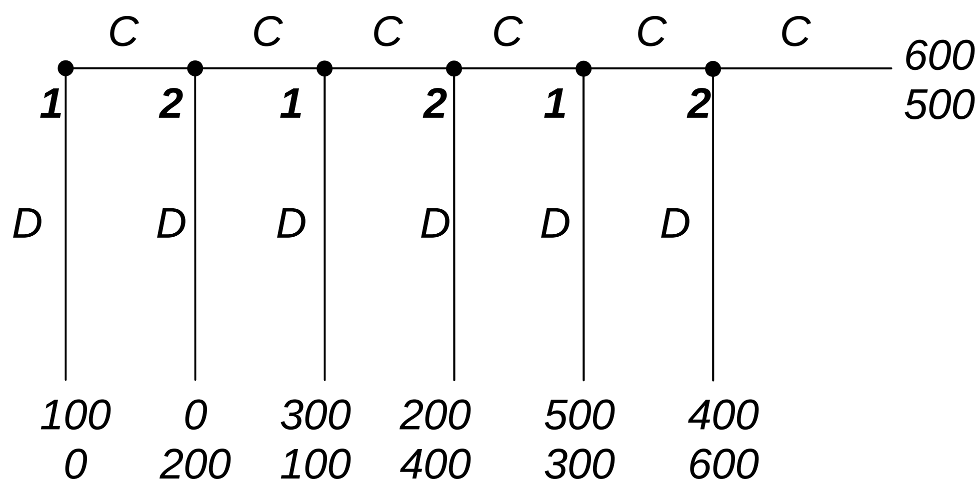
Beauty contest results

	Portfolio Managers	Economics PhDs	CEOs	Caltech students	Caltech trustees
Mean	24.3	27.4	37.8	21.9	42.6
Median	24.4	30.0	36.5	23.0	40.0
Fraction choosing zero	7.7%	12.5%	10.0%	7.4%	2.7%

	Germany	Singapore	UCLA	Wharton	High school (US)
Mean	36.7	46.1	42.3	37.9	32.4
Median	33.0	50.0	40.5	35.0	28.0
Fraction choosing zero	3.0%	2.0%	0.0%	0.0%	3.8%


0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1-5
6-10
11-15
16-20
21-25
26-30
31-35
36-40
41-45
46-50
51-55
56-60
61-65
66-70
71-80
81-90
91-100
Students
Managers
PhDs
CEOs
Trustees

Example III: the centipede game (graphically resembles a centipede insect)



Games

We study four groups of game theoretic models:

I strategic games

II extensive games (with and without perfect information)

III repeated games

IV coalitional games

Strategic games

A strategic game consists of

– a set of players (decision makers)

– for each player, a set of possible actions

– for each player, preferences over the set of action profiles (outcomes).

In strategic games, players move simultaneously. A wide range of situations may be modeled as strategic games.

A two-player (finite) strategic game can be described conveniently in a so-called bi-matrix.

For example, a generic 2×2 (two players and two possible actions for each player) game

	L	R
T	A_1, A_2	B_1, B_2
B	C_1, C_2	D_1, D_2

where the two rows (resp. columns) correspond to the possible actions of player 1 (resp. 2).

For example, rock-paper-scissors (over a dollar):

	R	P	S
R	$0,0$	$-1,1$	$1,-1$
P	$1,-1$	$0,0$	$-1,1$
S	$-1,1$	$1,-1$	$0,0$

Each player's set of actions is $\{Rock, Papar, Scissors\}$ and the set of action profiles is

$$\{RR, RP, RS, PR, PP, PS, SR.SP, SS\}.$$

Classical 2×2 games

- The following simple 2×2 games represent a variety of strategic situations.

- Despite their simplicity, each game captures the essence of a type of strategic interaction that is present in more complex situations.

- These classical games "span" the set of almost *all* games (strategic equivalence).

Game I: Prisoner's Dilemma

	$Work$	$Goof$
$Work$	$3, 3$	$0, 4$
$Goof$	$4, 0$	$1, 1$

A situation where there are gains from cooperation but each player has an incentive to "free ride."

Examples: team work, duopoly, arm/advertisement/R&D race, public goods, and more.

Game II: Battle of the Sexes (BoS)

	$Ball$	$Show$
$Ball$	$2, 1$	$0, 0$
$Show$	$0, 0$	$1, 2$

Like the Prisoner's Dilemma, Battle of the Sexes models a wide variety of situations.

Examples: political stands, mergers, among others.

Game III-V: Coordination, Hawk-Dove, and Matching Pennies

	Ball	*Show*
Ball	2, 2	0, 0
Show	0, 0	1, 1

	Dove	*Hawk*
Dove	3, 3	1, 4
Hawk	1, 4	0, 0

	Head	*Tail*
Head	1, −1	−1, 1
Tail	−1, 1	1, −1

Best response functions

Action a_i is player i's *best response* to a_{-i} if it is the optimal choice when i conjectures that others will play a_{-i}.

Let A_i be the set of actions of player i then

$$B_i(a_{-i}) = \{a_i \in A_i : (a_{-i}, a_i) \succsim_i (a_{-i}, a_i') \text{ for all } a_i' \text{ in } A_i\}$$

is the set of players i's best actions given a_{-i}.

We will next use best response functions to define Nash equilibrium.

Dominated actions

In any game, player i's action a_i is *strictly* dominated if it is never a best response (inferior no matter what the other players do):

a_i is *not* in $B_i(a_{-i})$ for any a_{-i} in A_{-i}.

In the Prisoner's Dilemma, for example, action $Work$ is strictly dominated by action $Goof$.

As we will see, a strictly dominated action is not used in any Nash equilibrium.

Nash equilibrium

Nash equilibrium (NE) is a steady state of the play of a strategic game – no player has a profitable deviation given the actions of the other players.

Let a be an action profile in which the actions of player i is a_i. A NE of a strategic game is a profile of actions a^* such that

$$(a^*_{-i}, a^*_i) \succsim_i (a^*_{-i}, a_i)$$

for all i and for any a_i in A_i, or equivalently,

$$a^*_i \text{ is in } B_i(a^*_{-i})$$

for all i.

Mixed strategy Nash equilibrium

- A mixed strategy of a player in a strategic game is a *probability distribution* over the player's actions.

- Mixed strategy Nash equilibrium is a valuable tool for studying the equilibria of any game.

- Existence: *any* (finite) game has a pure and/or mixed strategy Nash equilibrium.

Three Matching Pennies games in the laboratory

		.48	.52
		a_2	b_2
.48	a_1	80, 40	40, 80
.52	b_1	40, 80	80, 40

		.16	.84
		a_2	b_2
.96	a_1	320, 40	40, 80
.04	b_1	40, 80	80, 40

		.80	.20
		a_2	b_2
.08	a_1	44, 40	40, 80
.92	b_1	40, 80	80, 40

Extensive games with perfect information

- The model of a strategic suppresses the sequential structure of decision making.
  - All players simultaneously choose their plan of action once and for all.

- The model of an extensive game, by contrast, describes the sequential structure of decision-making explicitly.
  - In an extensive game of perfect information all players are fully informed about all previous actions.

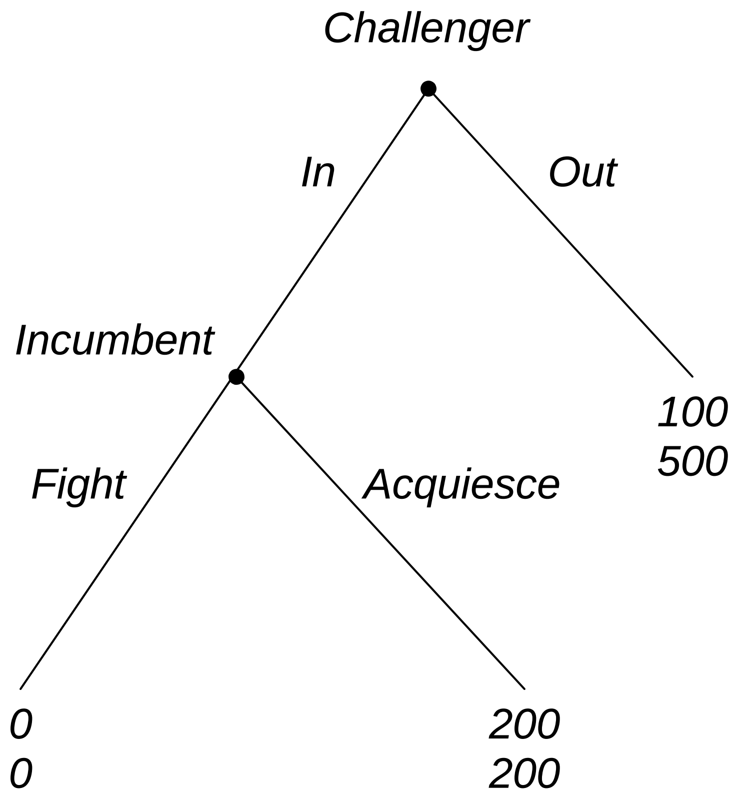
Subgame perfect equilibrium

- The notion of Nash equilibrium ignores the sequential structure of the game.

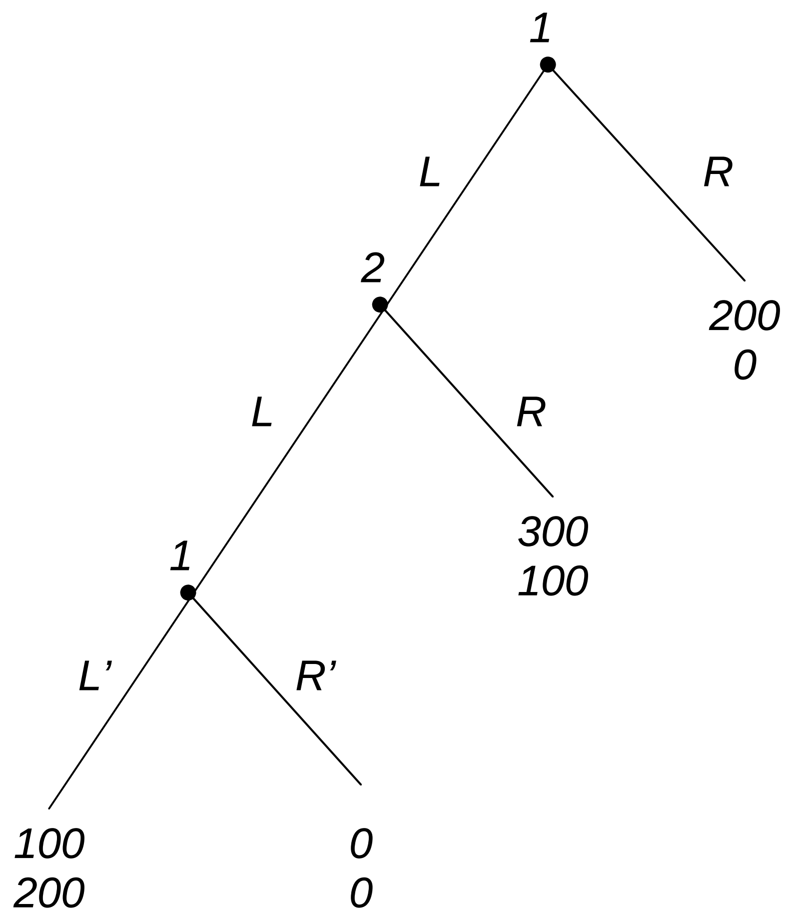
- Consequently, the steady state to which a Nash Equilibrium corresponds may not be robust.

- A *subgame perfect equilibrium* is an action profile that induces a Nash equilibrium in every *subgame* (so every subgame perfect equilibrium is also a Nash equilibrium).

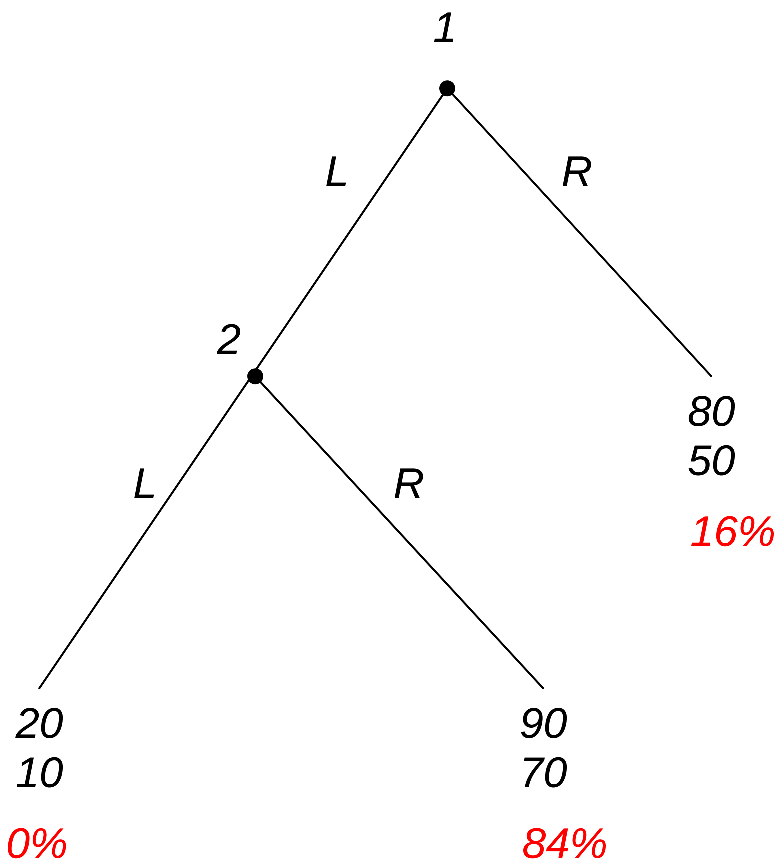
An example: entry game

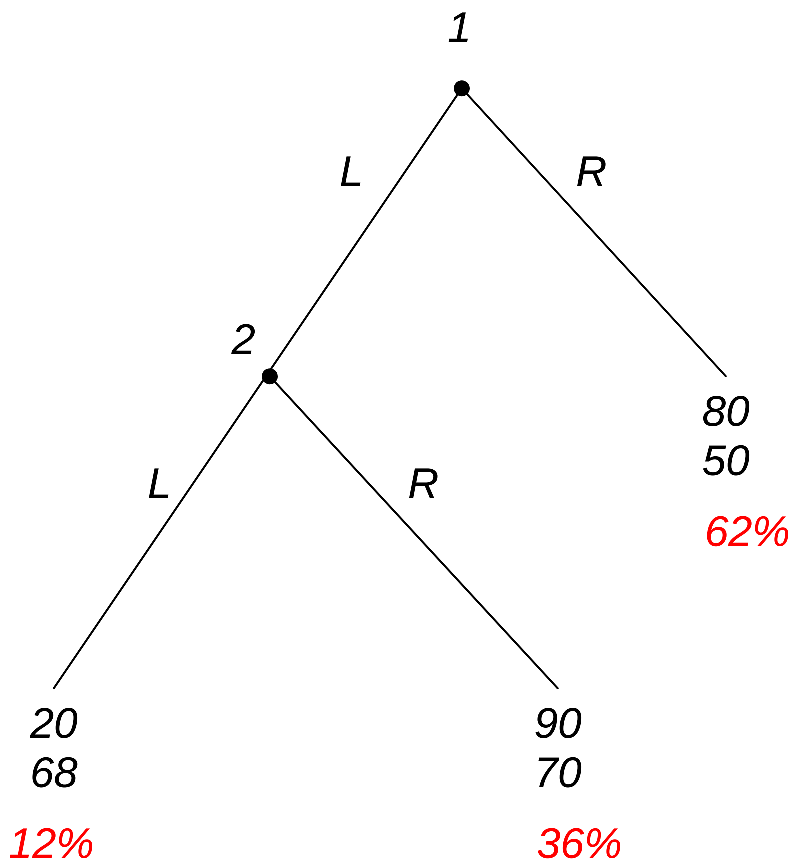


Subgame perfect and backward induction

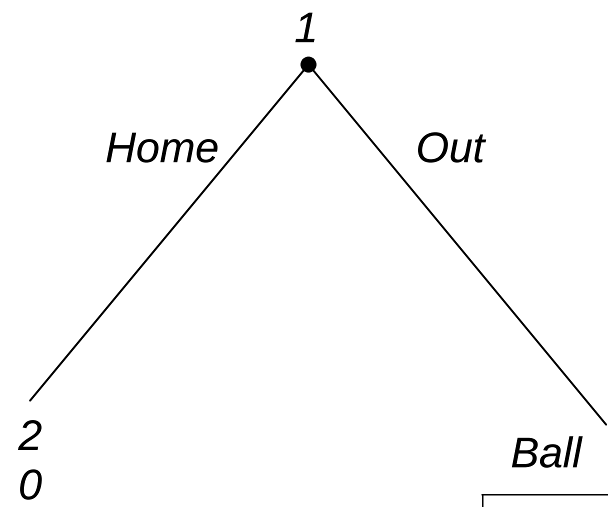


Two entry games in the laboratory




1
L
R
2
L
R
80
50
62%
20
68
12%
90
70
36%

Forward induction



	Ball	*Show*
Ball	3,1	0,0
Show	0,0	1,3

Conclusions

Adam Brandenburger:

> *There is nothing so practical as a good [game] theory. A good theory confirms the conventional wisdom that "less is more." A good theory does less because it does not give answers. At the same time, it does a lot more because it helps people organize what they know and uncover what they do not know. A good theory gives people the tools to discover what is best for them.*

- Read The Right Game: Use Game Theory to Shape Strategy (Brandenburger and Nalebuff, Harvard Business Review)

- Watch Game Theory with Ben Polak (one of my PhD advisors) at Open Yale Courses.

- Next we will apply the science of game theory to the art of management.

Have a great Labor Day holiday