UC Berkeley
Haas School of Business
Economic Analysis for Business Decisions
(EWMBA 201A)
Fall 2013

Oligopolistic markets
(PR 12.2-12.5)

Module 5

Sep. 27, 2014

Pricing

While there is some involved analysis required, the important takeaways about optimal pricing are

- At the optimal quantity produced q^*, marginal revenue equals marginal cost

$$MR(q^*) = MC(q^*)$$

- Marginal revenue comes from the underlying demands curve. Demand curves themselves come from consumer preferences.

Simple (nondiscriminatory) pricing

A firm engages in simple pricing for a particular product if that product is sold for the same price per unit no matter who the buyer is or how many units the buyer purchases.

The profit-maximizing quantity for the firm to produce (if it should be in business at all) q^* satisfies:

$$\begin{array}{rl} (i) & MR(q^*) = MC(q^*) \\ (ii) & MR(q) > MC(q) \text{ for all } q < q^* \\ (iii) & MR(q) < MC(q) \text{ for all } q > q^* \end{array}$$

Note that marginal profit,

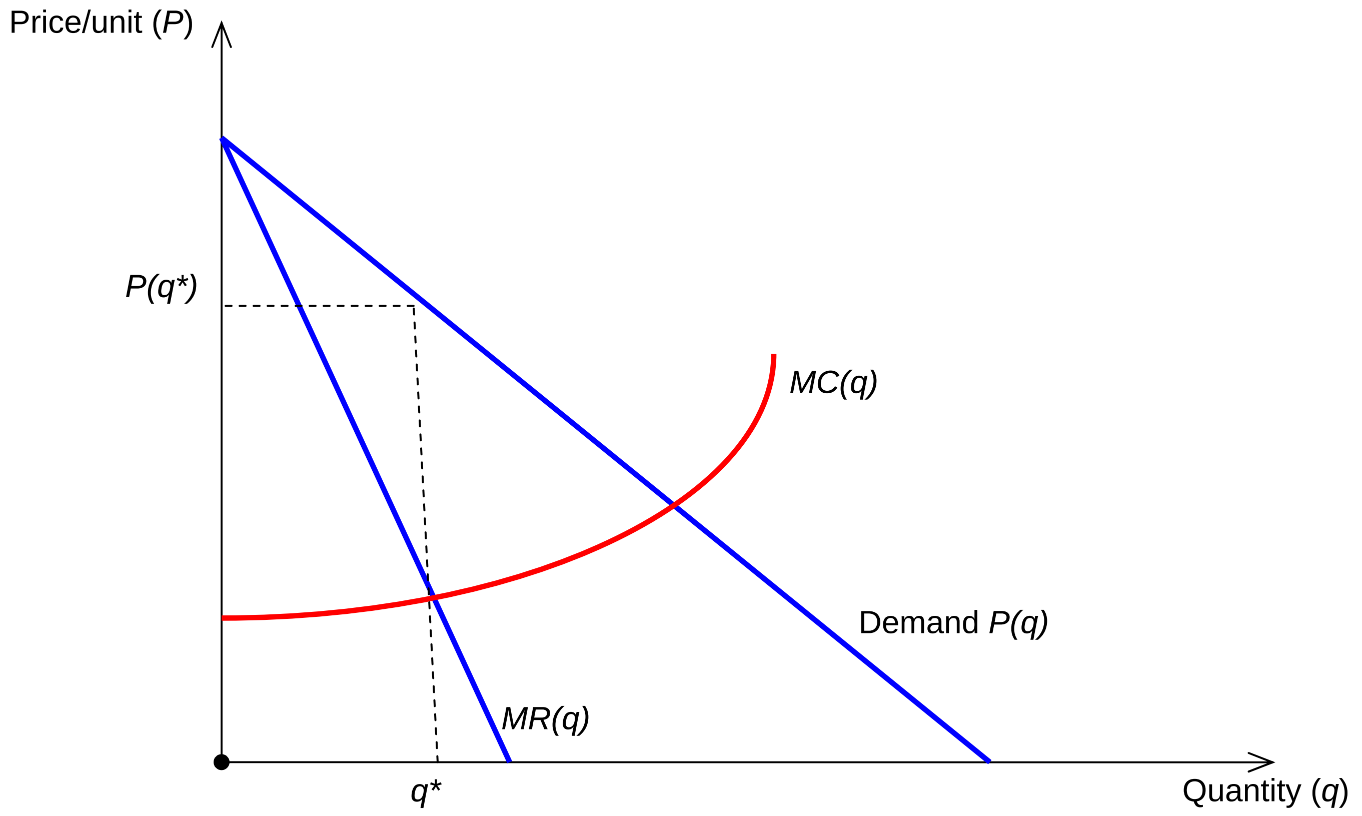
$$MR(q) - MC(q)$$

is positive for all $q < q^*$, that is, every additional unit in this region contributes positively to total profit.

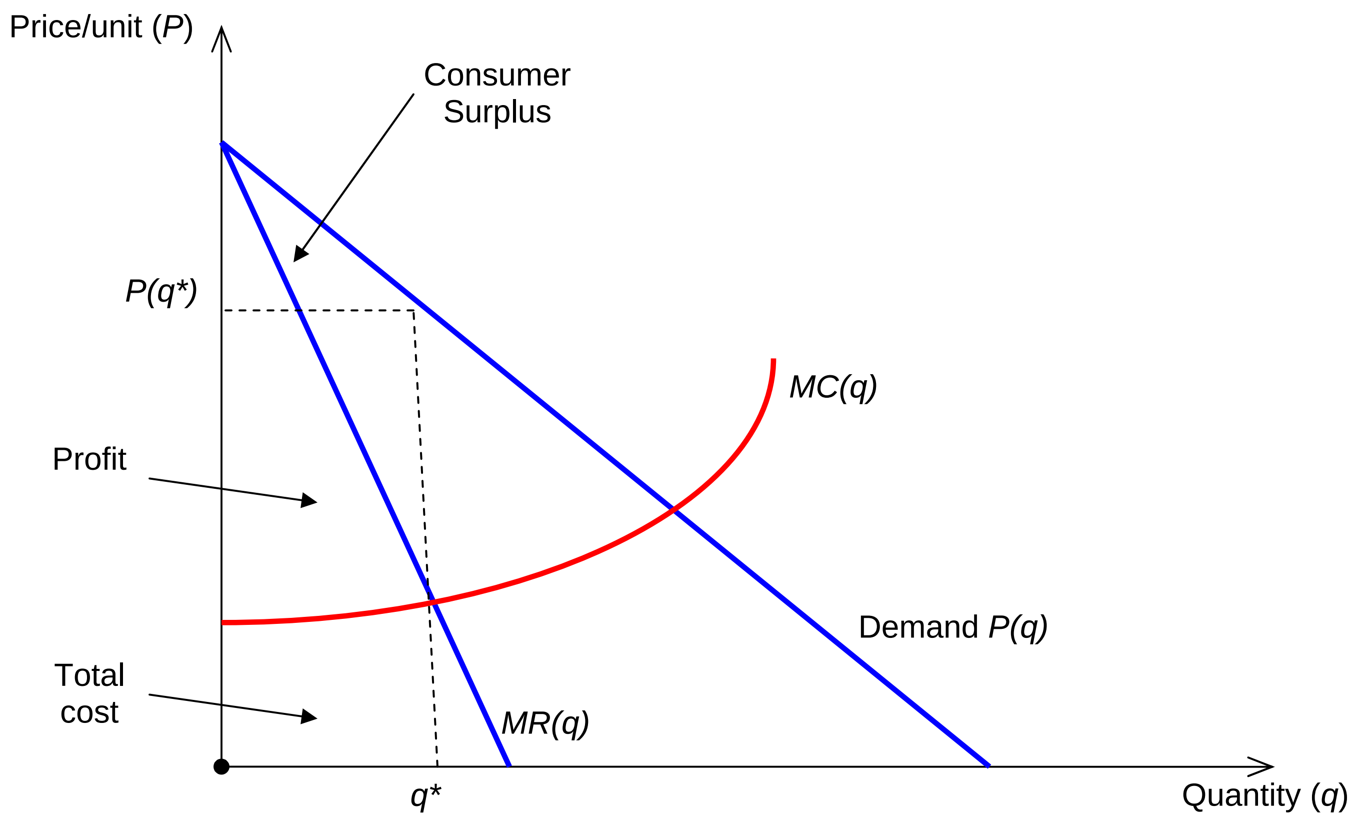
On the other hand, marginal profit is negative for all $q > q^*$, that is, every additional unit in this region reduces total profit.

$\Longrightarrow$ Increasing the total profit in the region $q < q^*$ and descending the total profit in the region $q > q^*$.

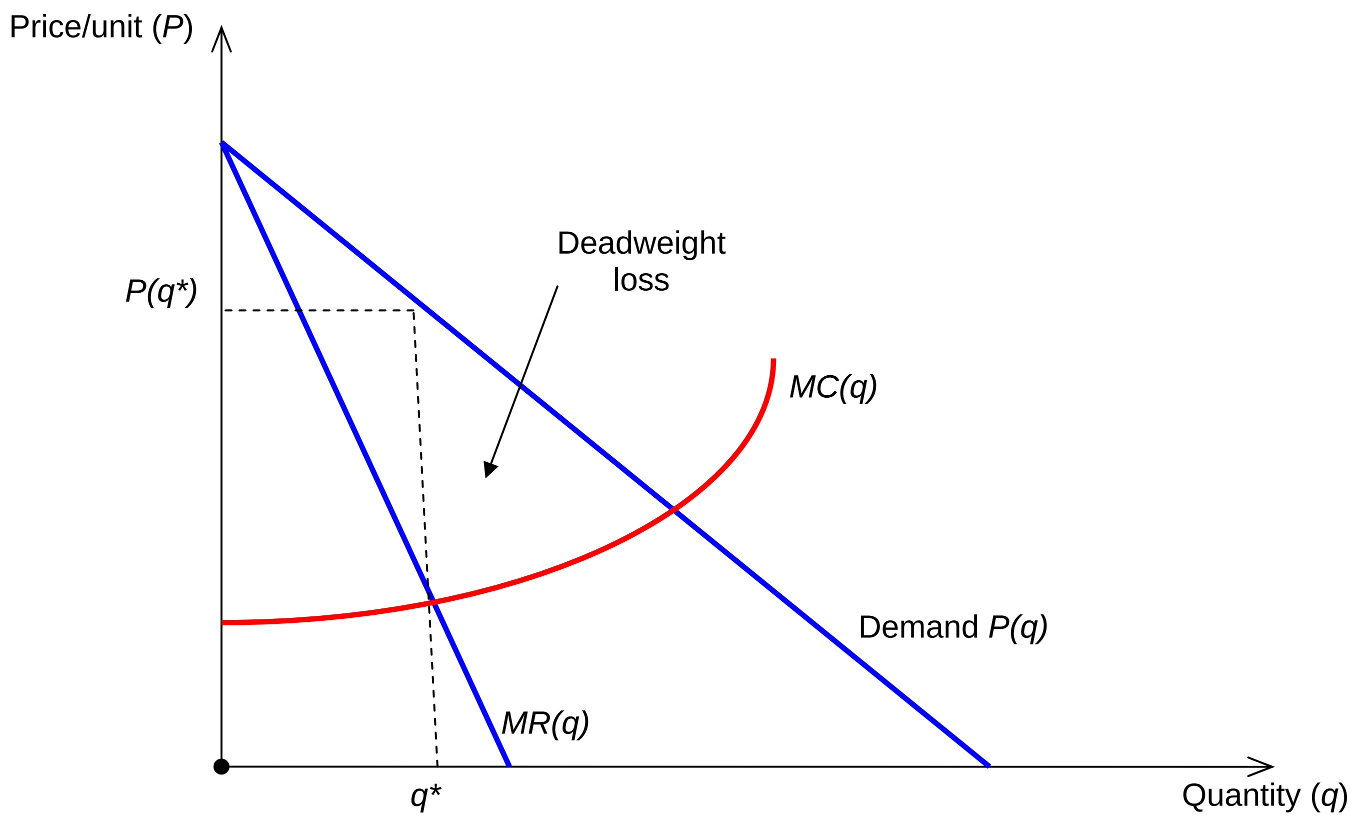
Profit-maximizing price and quantity



Total costs, profit, and consumer surplus



What simple pricing loses?



The Holy Grail of pricing

- If the firm can capture *all* the welfare generated from selling q units, then the firm will want to produce $q^{**} > q^*$ such that $P(q^{**}) = MC(q^{**})$.

- Because this outcome is so good, any form of pricing that achieves this Holy Grail is known as perfect price discrimination.

- For historic reasons, perfect price discrimination is also known as first-degree price discrimination.

$\Longrightarrow$ Can the firm ever obtain the Holy Grail? Generally, the answer is no!!!

Two-part tariffs

A two-part tariff is pricing with an entry fee and per-unit charge. It can help get a firm closer to the Grail than can simple pricing.

Formally, a two-part tariff consists of an entry fee F and a per-unit charge p. A consumer's expenditure if she buys q units is given by

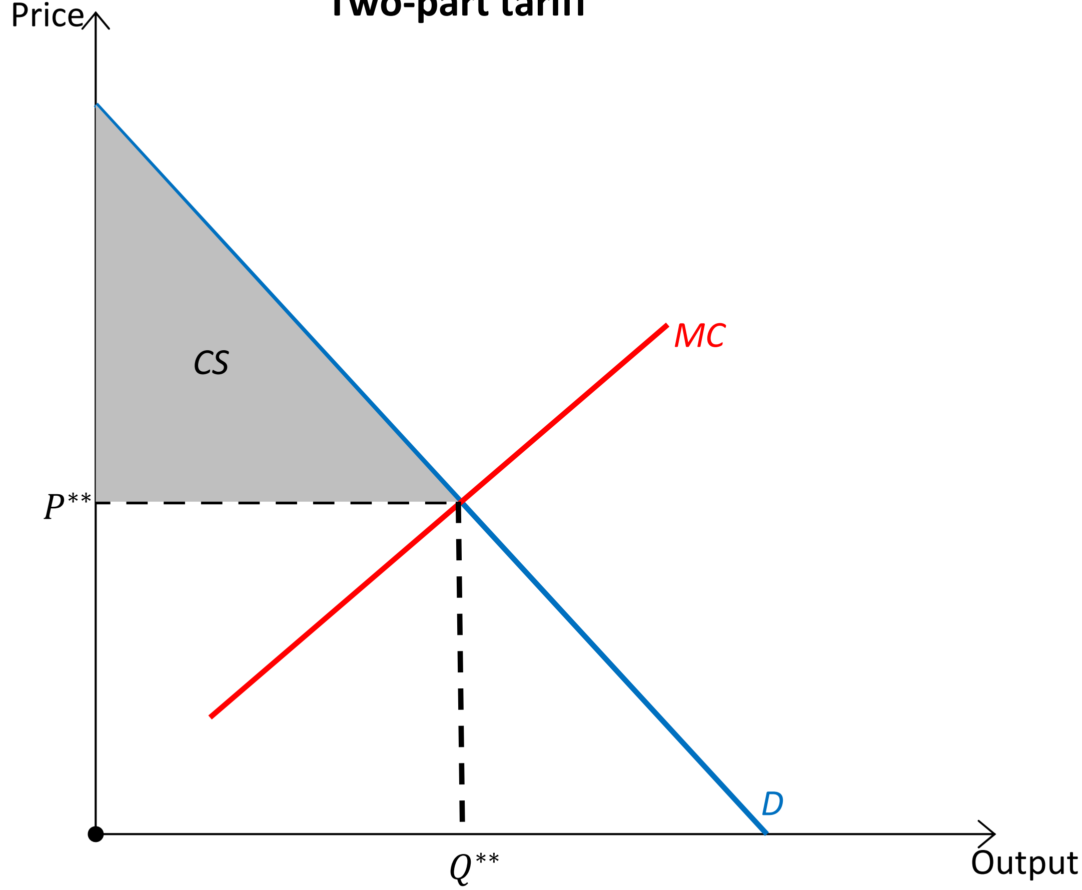
$$T(q) = \begin{cases} 0 & \text{if} \quad q = 0 \\ F + pq & \text{if} \quad q > 0 \end{cases}$$

If there are N homogeneous (have identical demands) consumers, then under the profit-maximizing two-part tariff, the firm

- produces q^{**} units, where $P(q^{**}) = MC(q^{**})$
- sets the per-unit charge p to equal $P(q^{**})$
- sets the entry fee F to equal average consumer surplus CS/N.

If consumers are heterogeneous, the firm can still profit from using a two-part tariff, but designing the optimal tariff is much more complicated...

Two-part tariff



Third- and second-degree price discrimination

Third-degree price discrimination is charging different prices on the basis of observed group membership.

– Examples: Senior-citizen/child/student discounts, and geography-based third-degree price discrimination.

Second-degree price discrimination is price discrimination via induced revelation of preferences.

– Examples: quantity discounts, quality distortions (an adverse selection problem!).

Oligopoly
(preface to game theory)

- Another form of market structure is **oligopoly** – a market in which only a few firms compete with one another, and entry of new firms is impeded.

- The situation is known as the Cournot model after Antoine Augustin Cournot, a French economist, philosopher and mathematician (1801-1877).

- In the basic example, a single good is produced by two firms (the industry is a "duopoly").

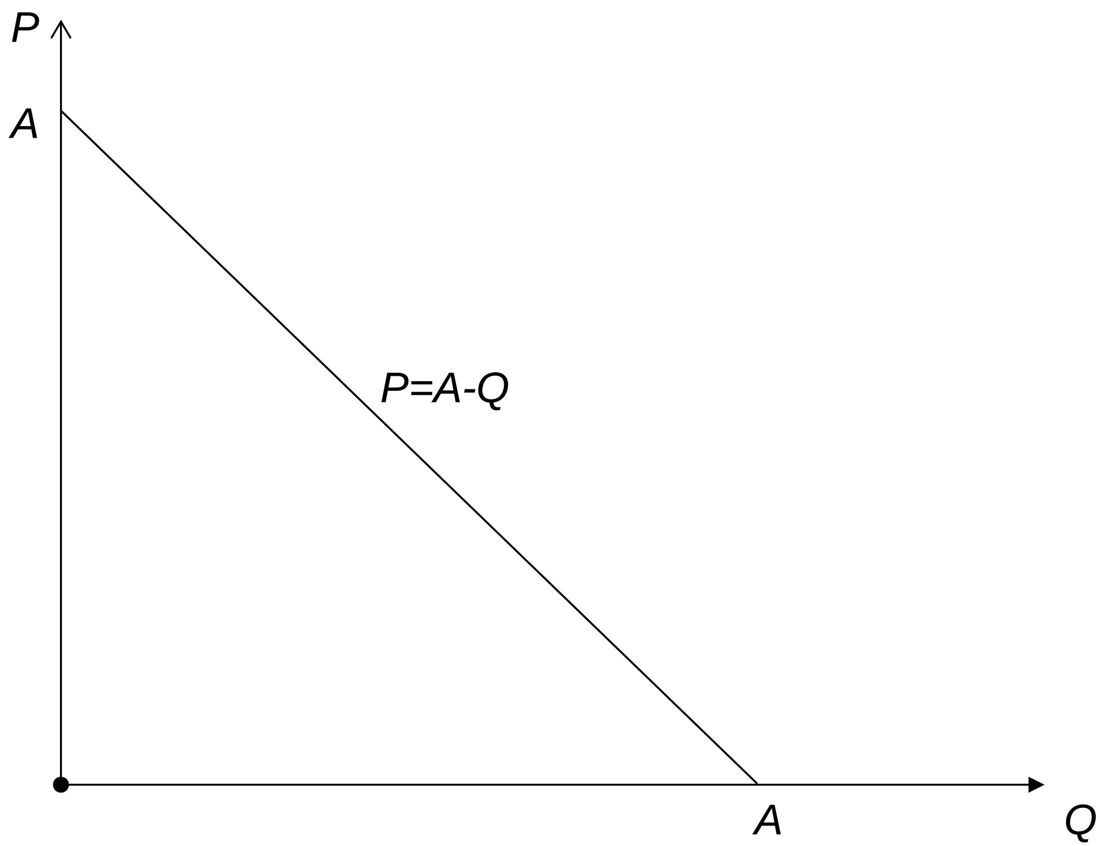
Cournot's oligopoly model (1838)

- A single good is produced by two firms (the industry is a "duopoly").
- The cost for firm $i = 1, 2$ for producing q_i units of the good is given by $c_i q_i$ ("unit cost" is constant equal to $c_i > 0$).
- If the firms' total output is $Q = q_1 + q_2$ then the market price is

  $$P = A - Q$$

  if $A \geq Q$ and zero otherwise (linear inverse demand function). We also assume that $A > c$.

The inverse demand function



To find the Nash equilibria of the Cournot's game, we can use the procedures based on the firms' best response functions.

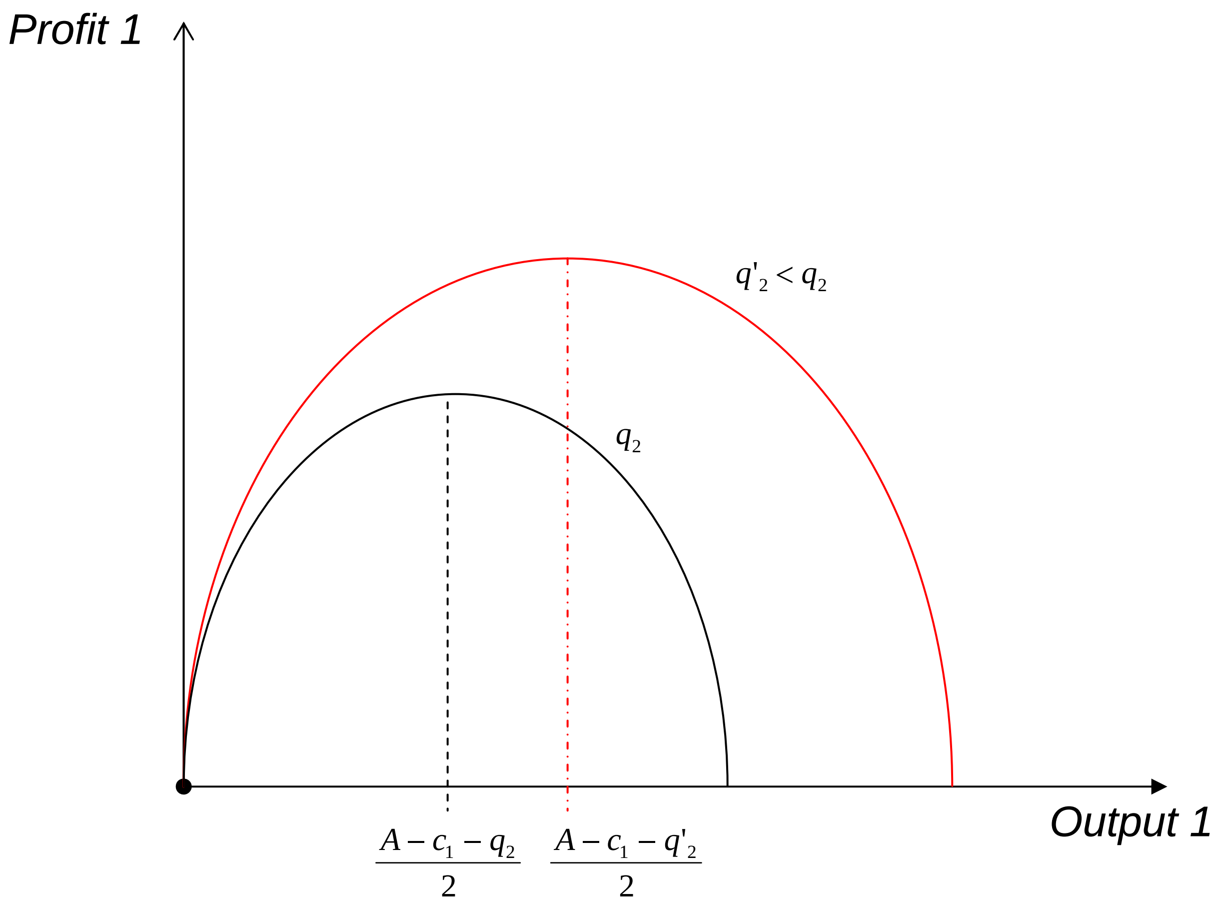
But first we need the firms payoffs (profits):

$$\begin{aligned}
\pi_1 &= Pq_1 - c_1q_1 \\
&= (A - Q)q_1 - c_1q_1 \\
&= (A - q_1 - q_2)q_1 - c_1q_1 \\
&= (A - q_1 - q_2 - c_1)q_1
\end{aligned}$$

and similarly,

$$\pi_2 = (A - q_1 - q_2 - c_2)q_2$$

Firm 1's profit as a function of its output (given firm 2's output)



To find firm 1's best response to any given output q_2 of firm 2, we need to study firm 1's profit as a function of its output q_1 for given values of q_2.

Using calculus, we set the derivative of firm 1's profit with respect to q_1 equal to zero and solve for q_1:

$$q_1 = \frac{1}{2}(A - q_2 - c_1).$$

We conclude that the best response of firm 1 to the output q_2 of firm 2 depends on the values of q_2 and c_1.

Because firm 2's cost function is $c_2 \neq c_1$, its best response function is given by

$$q_2 = \frac{1}{2}(A - q_1 - c_2).$$

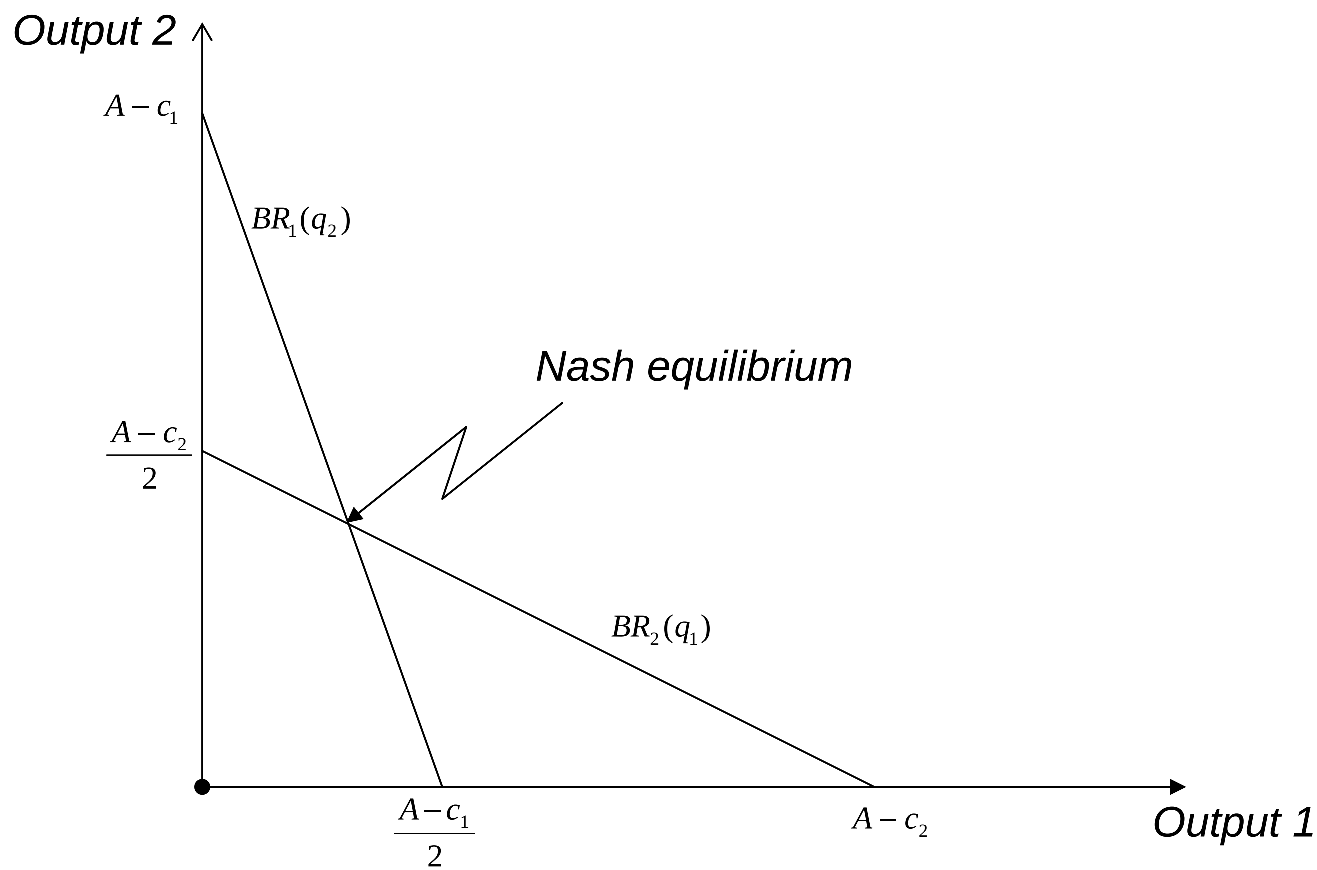
A Nash equilibrium of the Cournot's game is a pair (q_1^*, q_2^*) of outputs such that q_1^* is a best response to q_2^* and q_2^* is a best response to q_1^*.

From the figure below, we see that there is exactly one such pair of outputs

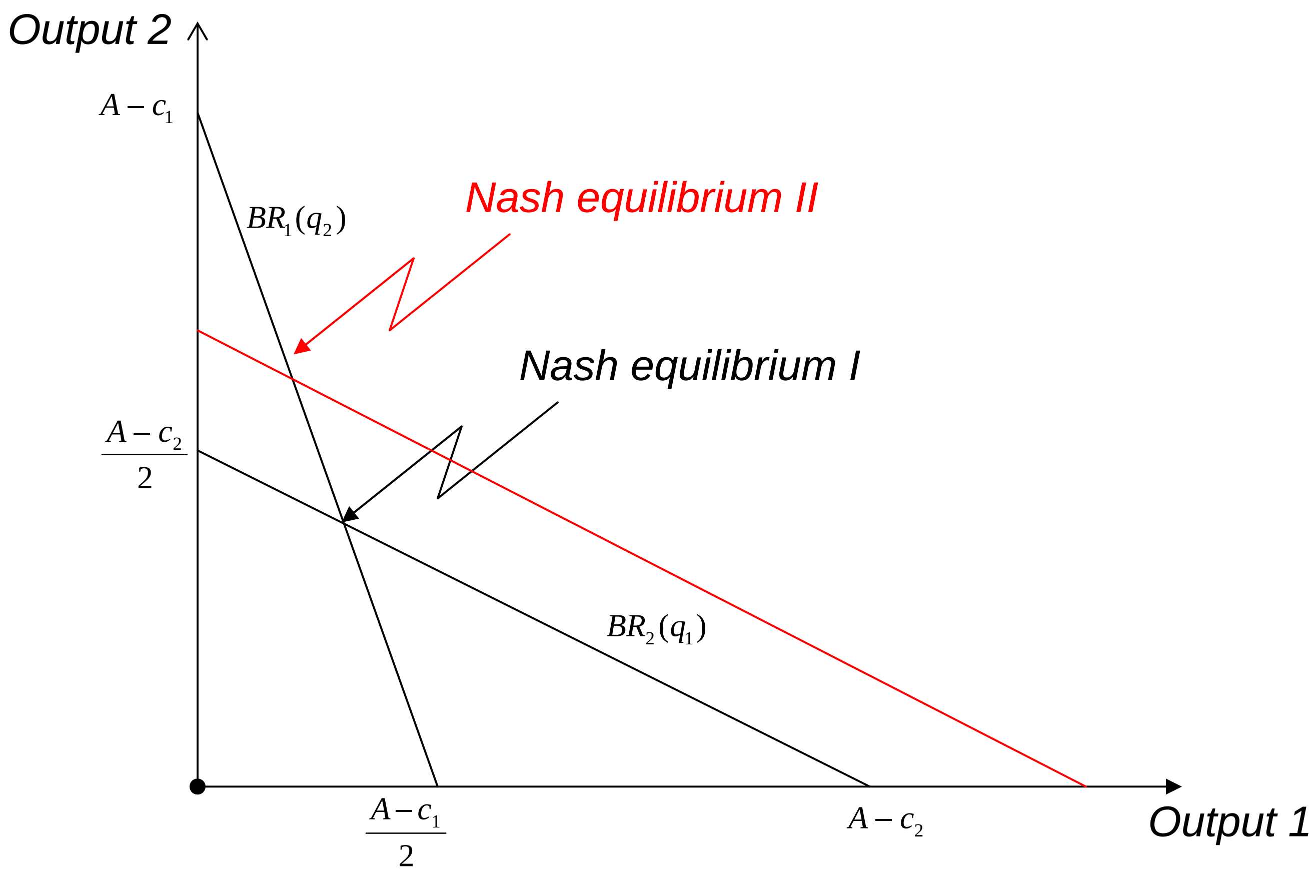
$$q_1^* = \tfrac{A+c_2-2c_1}{3} \quad \text{and} \quad q_2^* = \tfrac{A+c_1-2c_2}{3}$$

which is the solution to the two equations above.

The best response functions in the Cournot's duopoly game



Nash equilibrium comparative statics
(a decrease in the cost of firm 2)



A question: what happens when consumers are willing to pay more (*A* increases)?

In summary, this simple Cournot's duopoly game has a unique Nash equilibrium.

Two economically important properties of the Nash equilibrium are (to economic regulatory agencies):

[1] The relation between the firms' equilibrium profits and the profit they could make if they act collusively.

[2] The relation between the equilibrium profits and the number of firms.

[1] Collusive outcomes: in the Cournot's duopoly game, there is a pair of outputs at which *both* firms' profits exceed their levels in a Nash equilibrium.

[2] Competition: The price at the Nash equilibrium if the two firms have the *same* unit cost $c_1 = c_2 = c$ is given by

$$\begin{aligned} P^* &= A - q_1^* - q_2^* \\ &= \frac{1}{3}(A + 2c) \end{aligned}$$

which is above the unit cost c. But as the number of firm increases, the equilibrium price deceases, approaching c (zero profits!).

Stackelberg's duopoly model (1934)

How do the conclusions of the Cournot's duopoly game change when the firms move sequentially? Is a firm better off moving before or after the other firm?

Suppose that $c_1 = c_2 = c$ and that firm 1 moves at the start of the game. We may use backward induction to find the subgame perfect equilibrium.

– First, for *any* output q_1 of firm 1, we find the output q_2 of firm 2 that maximizes its profit. Next, we find the output q_1 of firm 1 that maximizes its profit, *given the strategy* of firm 2.

Firm 2

Since firm 2 moves after firm 1, a strategy of firm 2 is a *function* that associate an output q_2 for firm 2 for each possible output q_1 of firm 1.

We found that under the assumptions of the Cournot's duopoly game Firm 2 has a unique best response to each output q_1 of firm 1, given by

$$q_2 = \frac{1}{2}(A - q_1 - c)$$

(Recall that $c_1 = c_2 = c$).

Firm 1

Firm 1's strategy is the output q_1 the maximizes

$$\pi_1 = (A - q_1 - q_2 - c)q_1 \quad \text{subject to} \quad q_2 = \tfrac{1}{2}(A - q_1 - c)$$

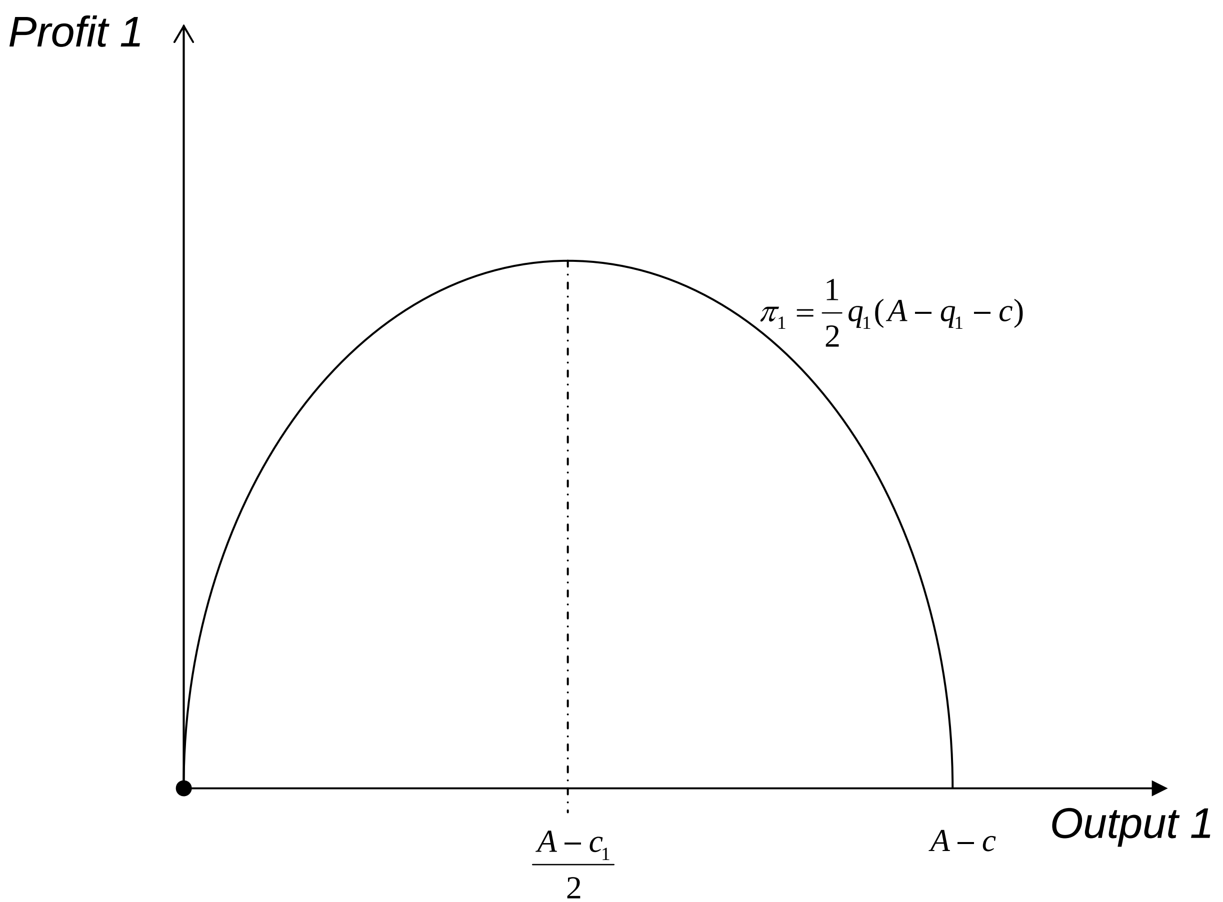
Thus, firm 1 maximizes

$$\pi_1 = (A - q_1 - (\frac{1}{2}(A - q_1 - c)) - c)q_1 = \frac{1}{2}q_1(A - q_1 - c).$$

This function is quadratic in q_1 that is zero when $q_1 = 0$ and when $q_1 = A - c$. Thus its maximizer is

$$q_1^* = \frac{1}{2}(A - c).$$

Firm 1's (first-mover) profit in Stackelberg's duopoly game



We conclude that Stackelberg's duopoly game has a unique subgame perfect equilibrium, in which firm 1's strategy is the output

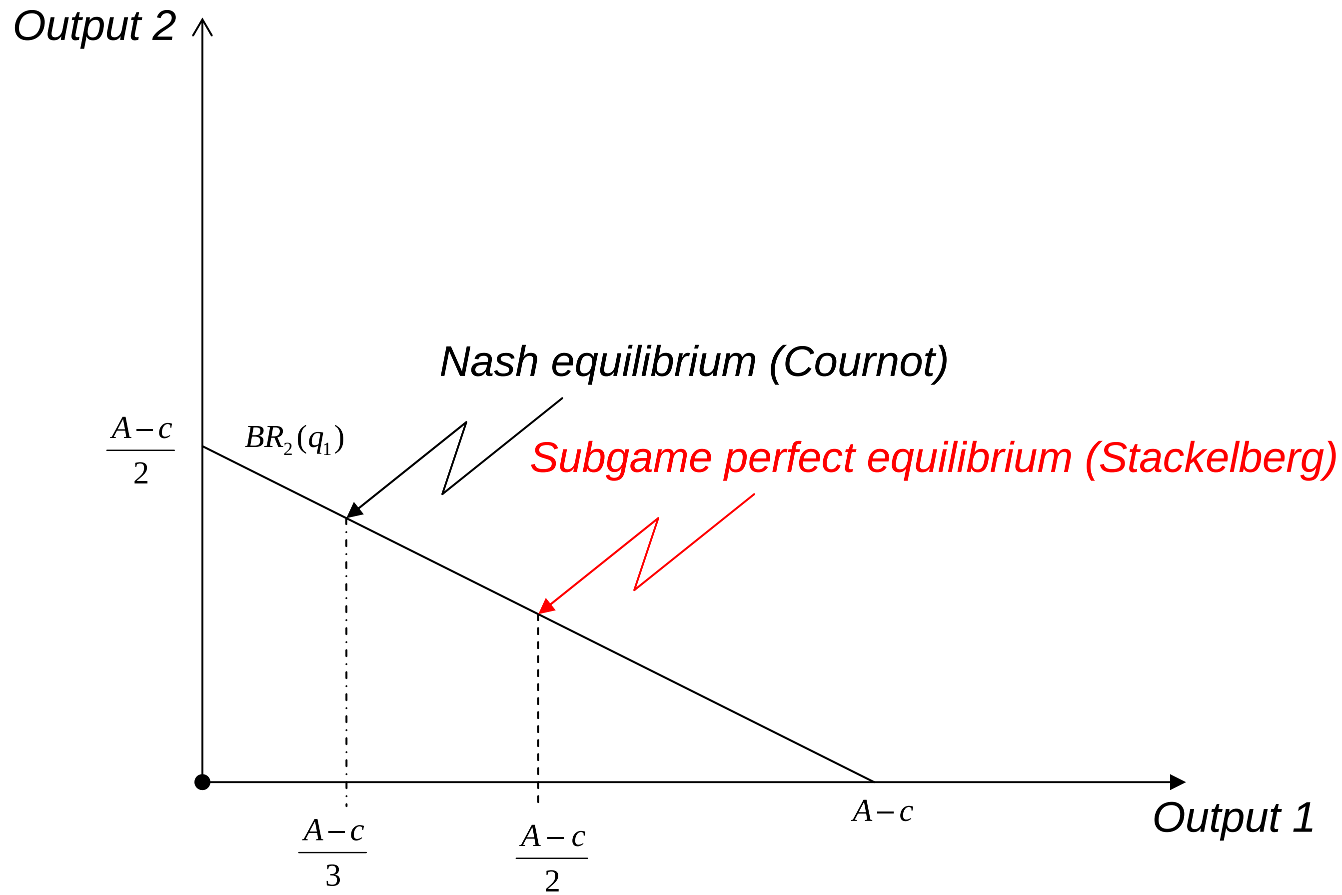
$$q_1^* = \frac{1}{2}(A - c)$$

and firm 2's output is

$$\begin{aligned} q_2^* &= \frac{1}{2}(A - q_1^* - c) \\ &= \frac{1}{2}(A - \frac{1}{2}(A - c) - c) \\ &= \frac{1}{4}(A - c). \end{aligned}$$

By contrast, in the unique Nash equilibrium of the Cournot's duopoly game under the same assumptions ($c_1 = c_2 = c$), each firm produces $\frac{1}{3}(A - c)$.

The subgame perfect equilibrium of Stackelberg's duopoly game



Bertrand's oligopoly model (1883)

In Cournot's game, each firm chooses an output, and the price is determined by the market demand in relation to the total output produced.

An alternative model, suggested by Bertrand, assumes that each firm chooses a price, and produces enough output to meet the demand it faces, given the prices chosen by *all* the firms.

$\Longrightarrow$ As we shell see, some of the answers it gives are different from the answers of Cournot.

Suppose again that there are two firms (the industry is a "duopoly") and that the cost for firm $i = 1, 2$ for producing q_i units of the good is given by cq_i (equal constant "unit cost").

Assume that the demand function (rather than the inverse demand function as we did for the Cournot's game) is

$$D(p) = A - p$$

for $A \geq p$ and zero otherwise, and that $A > c$ (the demand function in PR 12.3 is different).

Because the cost of producing each until is the same, equal to c, firm i makes the profit of $p_i - c$ on every unit it sells. Thus its profit is

$$\pi_i = \begin{cases} (p_i - c)(A - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(A - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

where j is the other firm.

In Bertrand's game we can easily argue as follows: $(p_1, p_2) = (c, c)$ is the unique Nash equilibrium.

Using intuition,

- If one firm charges the price c, then the other firm can do no better than charge the price c.

- If $p_1 > c$ and $p_2 > c$, then each firm i can increase its profit by lowering its price p_i slightly below p_j.

$\Longrightarrow$ In Cournot's game, the market price decreases toward c as the number of firms increases, whereas in Bertrand's game it is c (so profits are zero) even if there are only two firms (but the price remains c when the number of firm increases).

Avoiding the Bertrand trap

If you are in a situation satisfying the following assumptions, then you will end up in a Bertrand trap (zero profits):

[1] Homogenous products

[2] Consumers know all firm prices

[3] No switching costs

[4] No cost advantages

[5] No capacity constraints

[6] No future considerations

Problem set V

PR 12 – exercises 3-7.