UC Berkeley
Haas School of Business
Economic Analysis for Business Decisions
(EWMBA 201A)
Fall 2014

Game theory
(PR 13)

Module 6
Oct. 4, 2014

Prologue

- Game theory is about what happens when decision makers (spouses, workers, managers, presidents) interact.

- In the past fifty years, game theory has gradually became a standard language in economics.

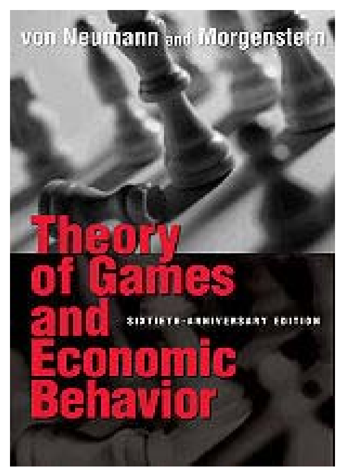
- The power of game theory is its <u>generality</u> and (mathematical) <u>precision</u>.

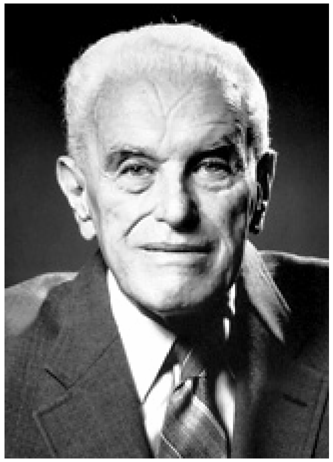
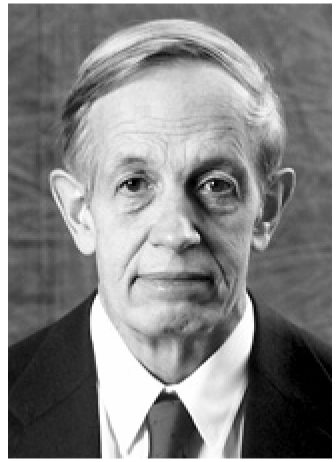
- Because game theory is rich and crisp, it could unify many parts of social science.

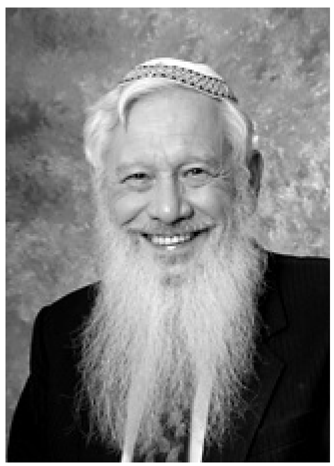
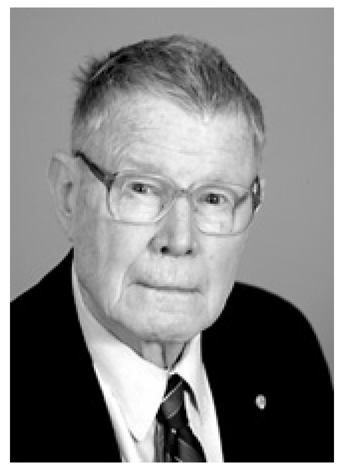
- The spread of game theory outside of economics has suffered because of the misconception that it requires a lot of fancy math.

- Game theory is also a natural tool for understanding complex social and economic phenomena in the real world.

The paternity of game theory



What is game theory good for?

Q Is game theory meant to predict what decision makers do, to give them advice, or what?

A The tools of analytical game theory are used to predict, postdict (explain), and prescribe.

Remember: even if game theory is not always accurate, descriptive failure is prescriptive opportunity!

Game theory and MBAs

- Adam Brandenburger (NYU) and Barry Nalebuff (Yale) explain how to use game theory to shape strategy (Co-Opetition).

- Both are brilliant game theorists who could have written a more theoretical book.

- They choose not to because teaching MBAs and working with managers is more useful.

Aumann (1987):

> "*Game theory is a sort of umbrella or 'unified field' theory for the rational side of social science, where 'social' is interpreted broadly, to include human as well as non-human players (computers, animals, plants).*"

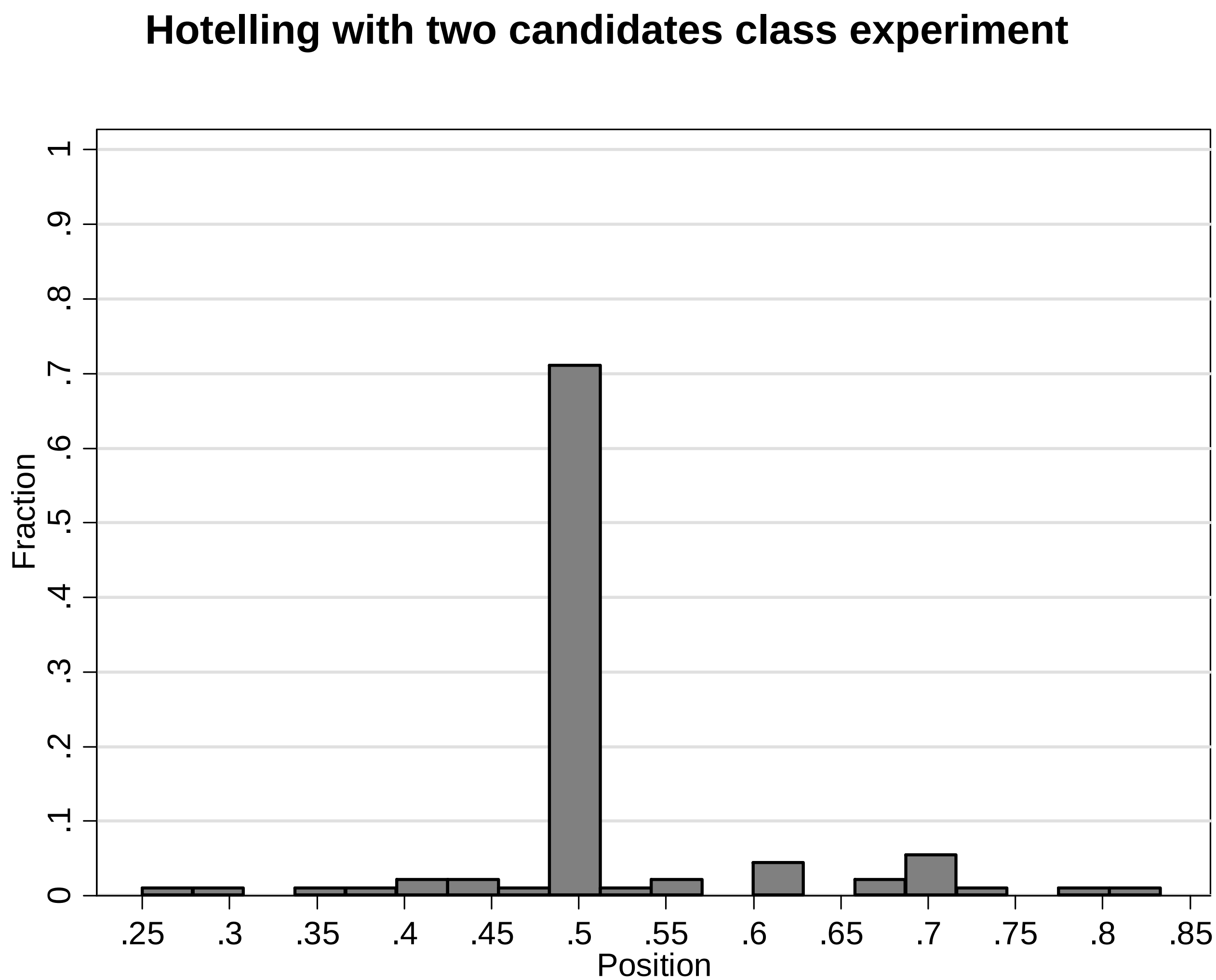
Three examples

Example I: Hotelling's electoral competition game

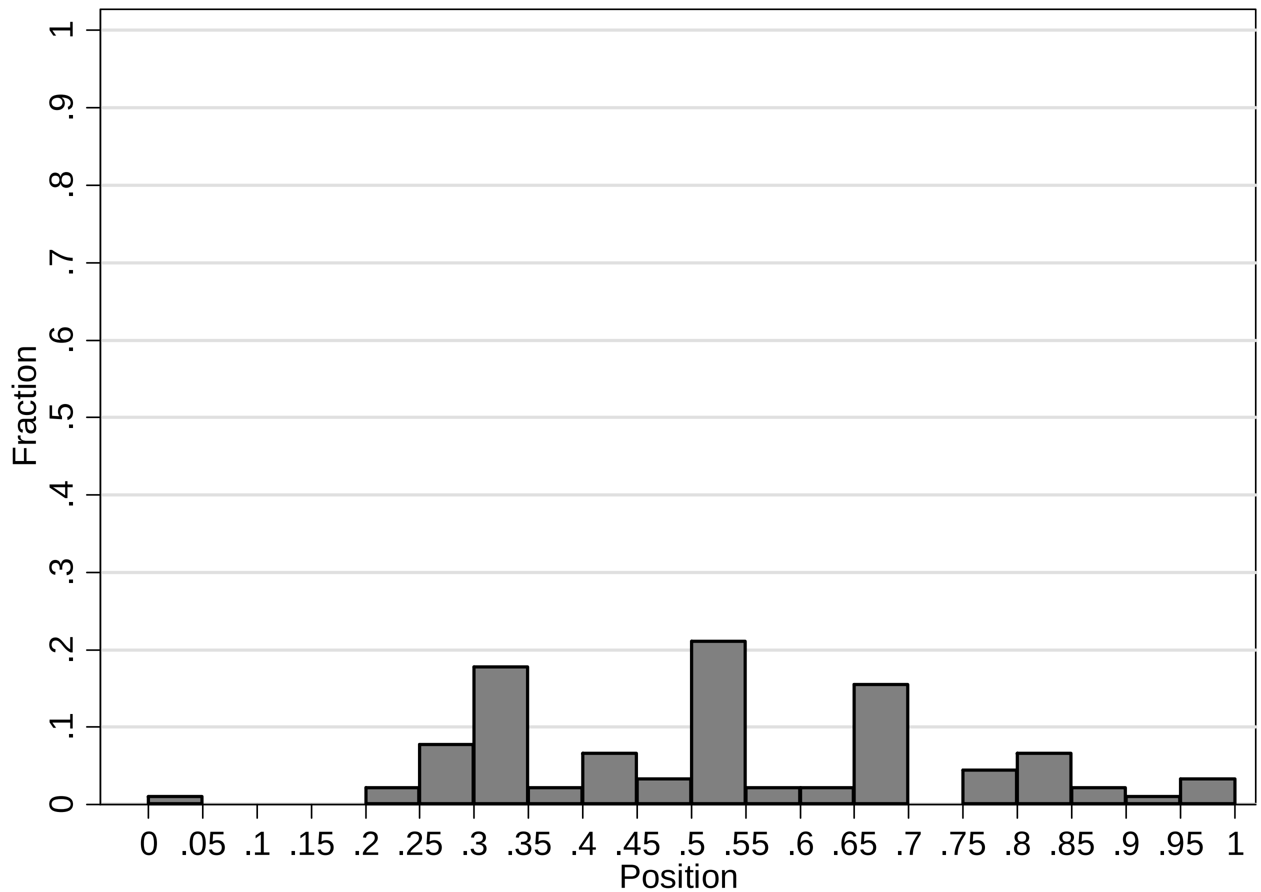
– There are two candidates and a continuum of voters, each with a favorite position on the interval $[0, 1]$.

– Each voter's distaste for any position is given by the distance between the position and her favorite position.

– A candidate attracts the votes off all citizens whose favorite positions are closer to her position.


Hotelling with two candidates class experiment
Fraction
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
.25
.3
.35
.4
.45
.5
.55
.6
.65
.7
.75
.8
.85
Position

Hotelling with three candidates class experiment



Example II: Keynes's beauty contest game

- Simultaneously, everyone choose a number (integer) in the interval $[0, 100]$.

- The person whose number is closest to $2/3$ of the average number wins a fixed prize.

John Maynard Keynes (1936):

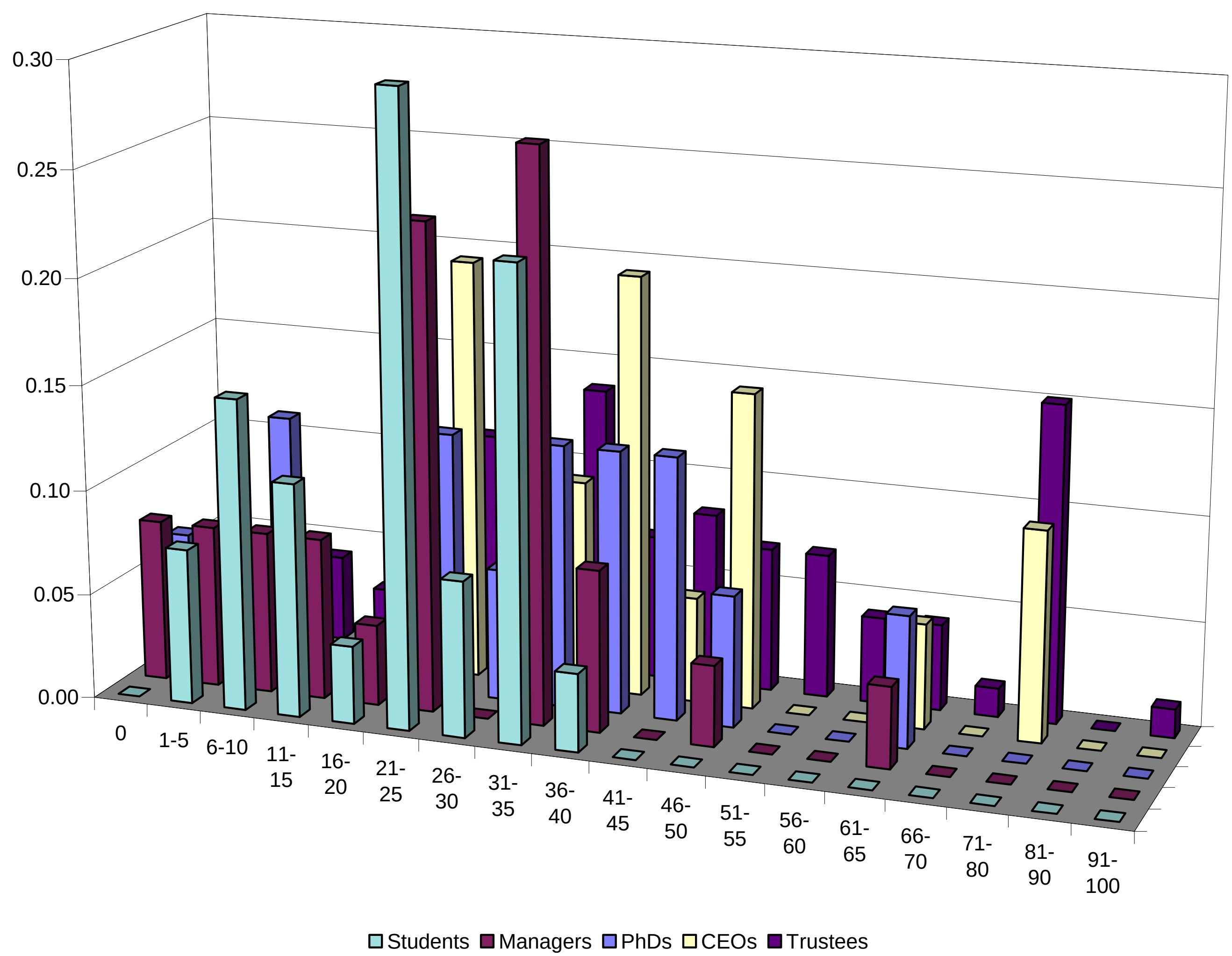
> *"It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."*

$\Longrightarrow$ self-fulfilling price bubbles!

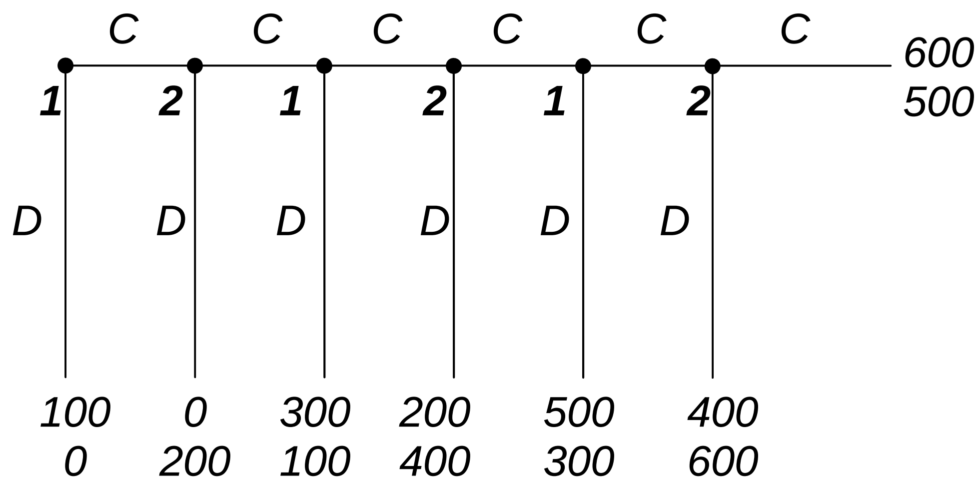
Beauty contest results

	Portfolio Managers	Economics PhDs	CEOs	Caltech students	Caltech trustees
Mean	24.3	27.4	37.8	21.9	42.6
Median	24.4	30.0	36.5	23.0	40.0
Fraction choosing zero	7.7%	12.5%	10.0%	7.4%	2.7%

	Germany	Singapore	UCLA	Wharton	High school (US)
Mean	36.7	46.1	42.3	37.9	32.4
Median	33.0	50.0	40.5	35.0	28.0
Fraction choosing zero	3.0%	2.0%	0.0%	0.0%	3.8%


0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1-5
6-10
11-15
16-20
21-25
26-30
31-35
36-40
41-45
46-50
51-55
56-60
61-65
66-70
71-80
81-90
91-100
Students
Managers
PhDs
CEOs
Trustees

Example III: the centipede game (graphically resembles a centipede insect)



The centipede game class experiment

Down	*0.311*
Continue, Down	*0.311*
Continue, Continue, Down	*0.267*
Continue, Continue, Continue	*0.111*

Games

We study four groups of game theoretic models:

I strategic games

II extensive games (with and without perfect information)

III repeated games

IV coalitional games

Strategic games

A strategic game consists of

– a set of players (decision makers)

– for each player, a set of possible actions

– for each player, preferences over the set of action profiles (outcomes).

In strategic games, players move simultaneously. A wide range of situations may be modeled as strategic games.

A two-player (finite) strategic game can be described conveniently in a so-called bi-matrix.

For example, a generic 2×2 (two players and two possible actions for each player) game

$$\begin{array}{c|c|c|} & L & R \\ \hline T & A_1, A_2 & B_1, B_2 \\ \hline B & C_1, C_2 & D_1, D_2 \\ \hline \end{array}$$

where the two rows (resp. columns) correspond to the possible actions of player 1 (resp. 2).

For example, rock-paper-scissors (over a dollar):

	R	P	S
R	$0, 0$	$-1, 1$	$1, -1$
P	$1, -1$	$0, 0$	$-1, 1$
S	$-1, 1$	$1, -1$	$0, 0$

Each player's set of actions is $\{Rock, Papar, Scissors\}$ and the set of action profiles is

$$\{RR, RP, RS, PR, PP, PS, SR.SP, SS\}.$$

Classical 2×2 games

- The following simple 2×2 games represent a variety of strategic situations.
- Despite their simplicity, each game captures the essence of a type of strategic interaction that is present in more complex situations.
- These classical games "span" the set of almost *all* games (strategic equivalence).

Game I: Prisoner's Dilemma

	$Work$	$Goof$
$Work$	3, 3	0, 4
$Goof$	4, 0	1, 1

A situation where there are gains from cooperation but each player has an incentive to "free ride."

Examples: team work, duopoly, arm/advertisement/R&D race, public goods, and more.

Game II: Battle of the Sexes (BoS)

	$Ball$	$Show$
$Ball$	$2, 1$	$0, 0$
$Show$	$0, 0$	$1, 2$

Like the Prisoner's Dilemma, Battle of the Sexes models a wide variety of situations.

Examples: political stands, mergers, among others.

Game III-V: Coordination, Hawk-Dove, and Matching Pennies

	Ball	*Show*
Ball	2, 2	0, 0
Show	0, 0	1, 1

	Dove	*Hawk*
Dove	3, 3	1, 4
Hawk	1, 4	0, 0

	Head	*Tail*
Head	1, −1	−1, 1
Tail	−1, 1	1, −1

Nash equilibrium

Nash equilibrium (NE) is a steady state of the play of a strategic game – no player has a profitable deviation given the actions of the other players.

Put differently, a NE is a set of actions such that all players are doing their best given the actions of the other players.

Mixed strategy Nash equilibrium

- A mixed strategy of a player in a strategic game is a *probability distribution* over the player's actions.

- Mixed strategy Nash equilibrium is a valuable tool for studying the equilibria of any game.

- Existence: *any* (finite) game has a pure and/or mixed strategy Nash equilibrium.

Three Matching Pennies games in the laboratory

		.48	.52
		a_2	b_2
.48	a_1	80, 40	40, 80
.52	b_1	40, 80	80, 40

		.16	.84
		a_2	b_2
.96	a_1	320, 40	40, 80
.04	b_1	40, 80	80, 40

		.80	.20
		a_2	b_2
.08	a_1	44, 40	40, 80
.92	b_1	40, 80	80, 40

Extensive games with perfect information

- The model of a strategic suppresses the sequential structure of decision making.
  - All players simultaneously choose their plan of action once and for all.

- The model of an extensive game, by contrast, describes the sequential structure of decision-making explicitly.
  - In an extensive game of perfect information all players are fully informed about all previous actions.

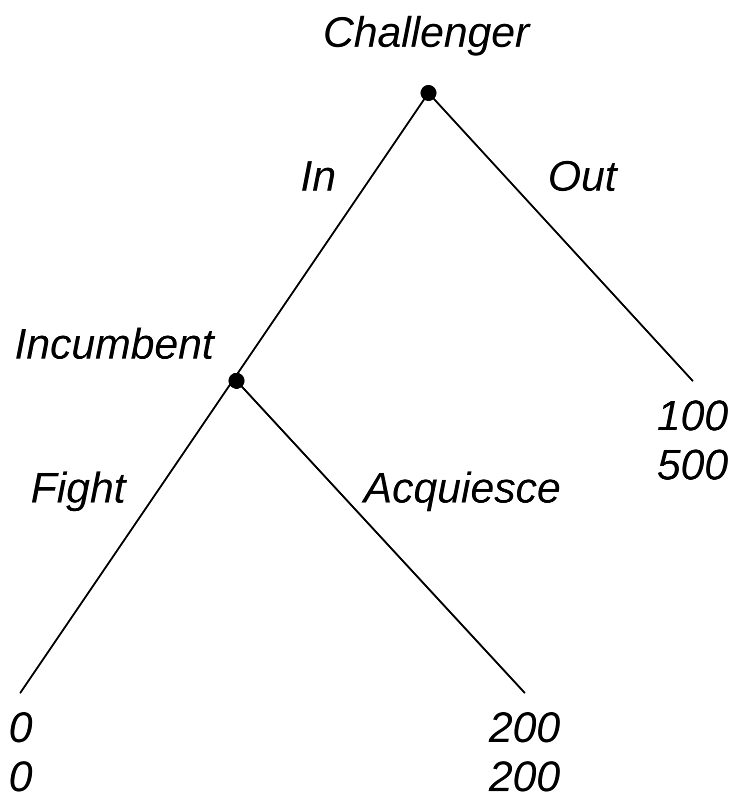
Subgame perfect equilibrium

- The notion of Nash equilibrium ignores the sequential structure of the game.

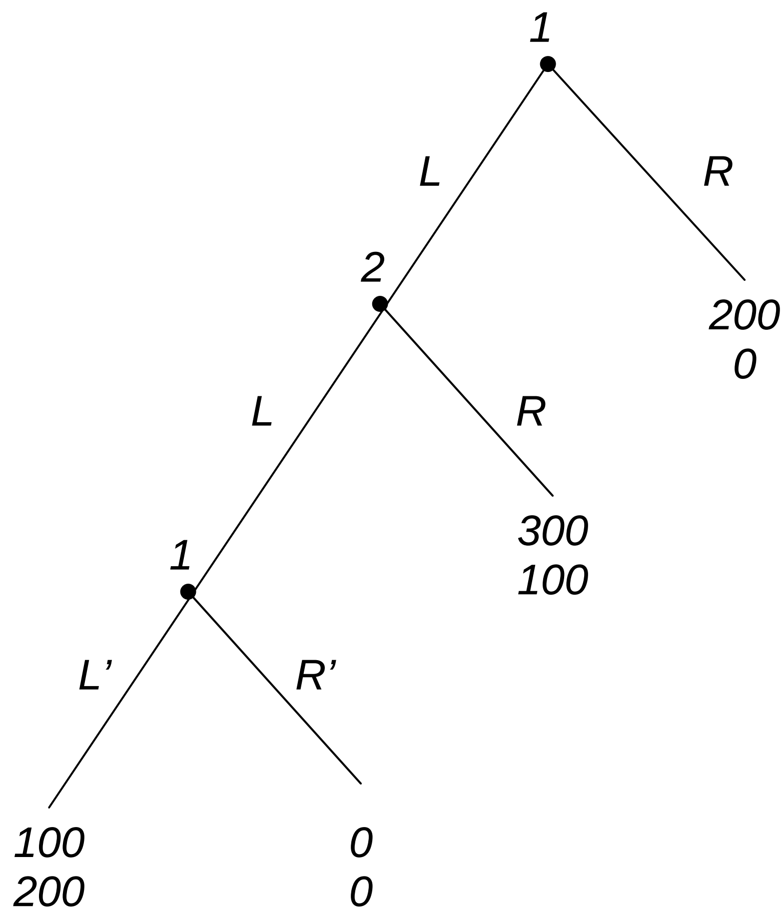
- Consequently, the steady state to which a Nash Equilibrium corresponds may not be robust.

- A *subgame perfect equilibrium* is an action profile that induces a Nash equilibrium in every *subgame* (so every subgame perfect equilibrium is also a Nash equilibrium).

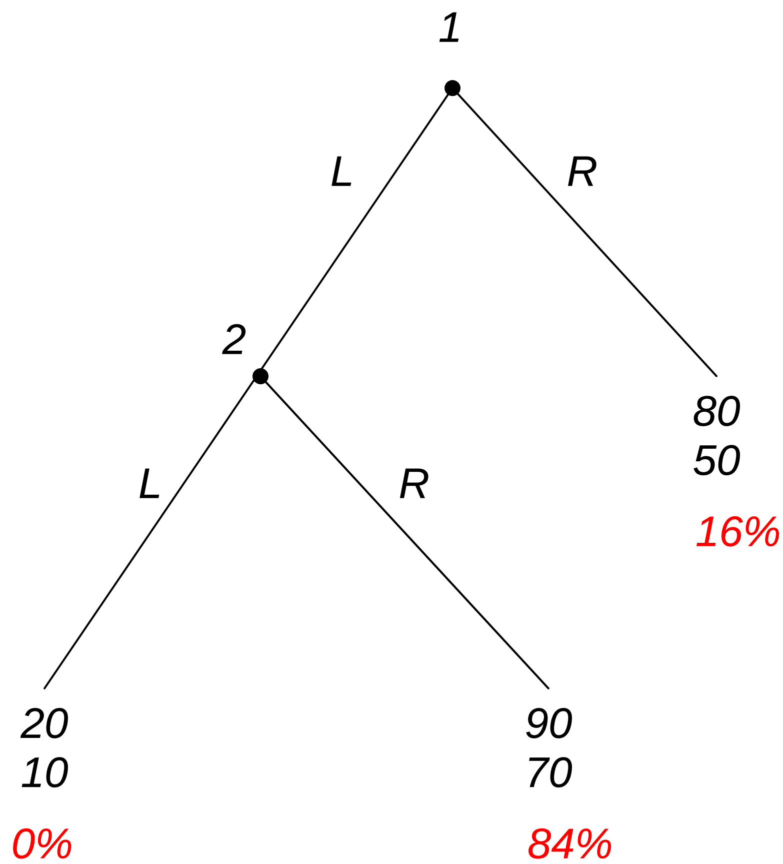
An example: entry game

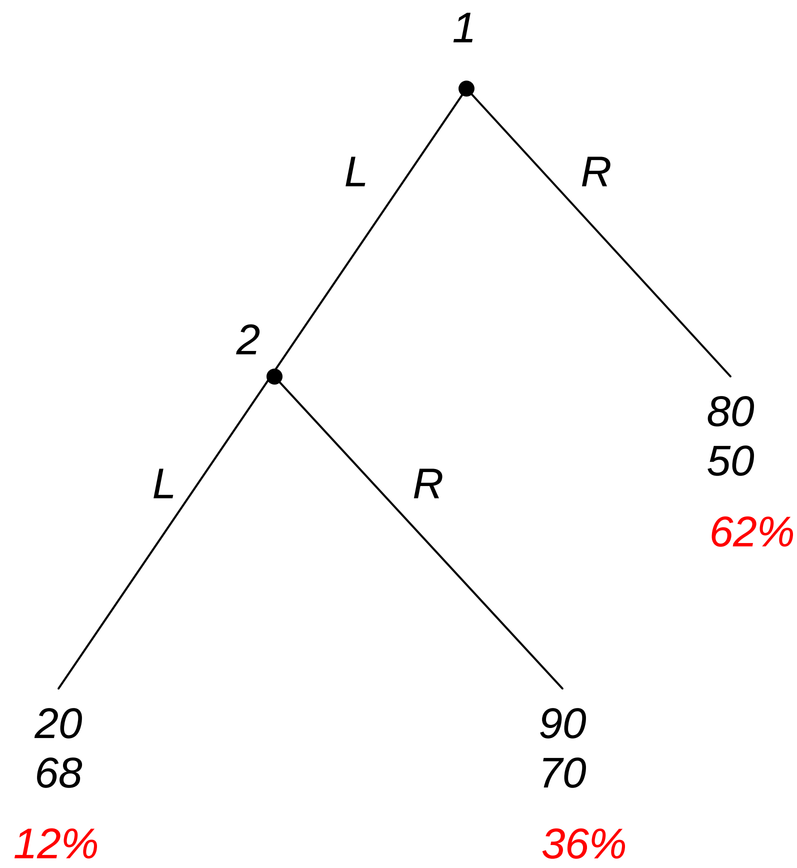


Subgame perfect and backward induction



Two entry games in the laboratory





1
L
R
2
L
R
80
50
62%
20
68
12%
90
70
36%

Auctions

From Babylonia to eBay, auctioning has a very long history.

- Babylon:
  - women at marriageable age.
- Athens, Rome, and medieval Europe:
  - rights to collect taxes,
  - dispose of confiscated property,
  - lease of land and mines,

  and more...

- Auctions, broadly defined, are used to allocate significant economics resources.

  Examples: works of art, government bonds, offshore tracts for oil exploration, radio spectrum, and more.

- Auctions take many forms. A game-theoretic framework enables to understand the consequences of various auction designs.

- Game theory can suggest the design likely to be most effective, and the one likely to raise the most revenues.

Types of auctions

Sequential / simultaneous

Bids may be called out sequentially or may be submitted simultaneously in sealed envelopes:

- English (or oral) – the seller actively solicits progressively higher bids and the item is soled to the highest bidder.
- Dutch – the seller begins by offering units at a "high" price and reduces it until all units are soled.
- Sealed-bid – all bids are made simultaneously, and the item is sold to the highest bidder.

First-price / second-price

The price paid may be the highest bid or some other price:

– First-price – the bidder who submits the highest bid wins and pay a price equal to her bid.

– Second-prices – the bidder who submits the highest bid wins and pay a price equal to the second highest bid.

Variants: all-pay (lobbying), discriminatory, uniform, Vickrey (William Vickrey, Nobel Laureate 1996), and more.

Private-value / common-value

Bidders can be certain or uncertain about each other's valuation:

- In private-value auctions, valuations differ among bidders, and each bidder is certain of her own valuation and can be certain or uncertain of every other bidder's valuation.

- In common-value auctions, all bidders have the same valuation, but bidders do not know this value precisely and their estimates of it vary.

First-price auction (with perfect information)

To define the game precisely, denote by v_i the value that bidder i attaches to the object. If she obtains the object at price p then her payoff is $v_i - p$.

Assume that bidders' valuations are all different and all positive. Number the bidders 1 through n in such a way that

$$v_1 > v_2 > \cdots > v_n > 0.$$

Each bidder i submits a (sealed) bid b_i. If bidder i obtains the object, she receives a payoff $v_i - b_i$. Otherwise, her payoff is zero.

Tie-breaking – if two or more bidders are in a tie for the highest bid, the winner is the bidder with the highest valuation.

In summary, a first-price sealed-bid auction with perfect information is the following strategic game:

- Players: the n bidders.

- Actions: the set of possible bids b_i of each player i (nonnegative numbers).

- Payoffs: the preferences of player i are given by

$$u_i = \begin{cases} v_i - \bar{b} & \text{if} \quad b_i = \bar{b} \text{ and } v_i > v_j \text{ if } b_j = \bar{b} \\ 0 & \text{if} \quad b_i < \bar{b} \end{cases}$$

  where $\bar{b}$ is the highest bid.

The set of Nash equilibria is the set of profiles $(b_1, ..., b_n)$ of bids with the following properties:

$$
\begin{array}{ll}
[1] & v_2 \leq b_1 \leq v_1 \\
[2] & b_j \leq b_1 \text{ \underline{for all} } j \neq 1 \\
[3] & b_j = b_1 \text{ \underline{for some} } j \neq 1
\end{array}
$$

It is easy to verify that all these profiles are Nash equilibria. It is harder to show that there are no other equilibria. We can easily argue, however, that there is no equilibrium in which player 1 does not obtain the object.

$\Longrightarrow$ The first-price sealed-bid auction is socially efficient, but does not necessarily raise the most revenues.

Second-price auction (with perfect information)

A second-price sealed-bid auction with perfect information is the following strategic game:

- Players: the n bidders.

- Actions: the set of possible bids b_i of each player i (nonnegative numbers).

- Payoffs: the preferences of player i are given by

$$u_i = \begin{cases} v_i - \bar{b} & \text{if} \quad b_i > \bar{b} \text{ or } b_i = \bar{b} \text{ and } v_i > v_j \text{ if } b_j = \bar{b} \\ 0 & \text{if} \quad b_i < \bar{b} \end{cases}$$

where $\bar{b}$ is the highest bid submitted by a player other than i.

First note that for any player i the bid $b_i = v_i$ is a (weakly) dominant action (a "truthful" bid), in contrast to the first-price auction.

The second-price auction has many equilibria, but the equilibrium $b_i = v_i$ for all i is distinguished by the fact that every player's action dominates all other actions.

Another equilibrium in which player $j \neq 1$ obtains the good is that in which

[1] $b_1 < v_j$ and $b_j > v_1$
[2] $b_i = 0$ <u>for all</u> $i \neq \{1, j\}$

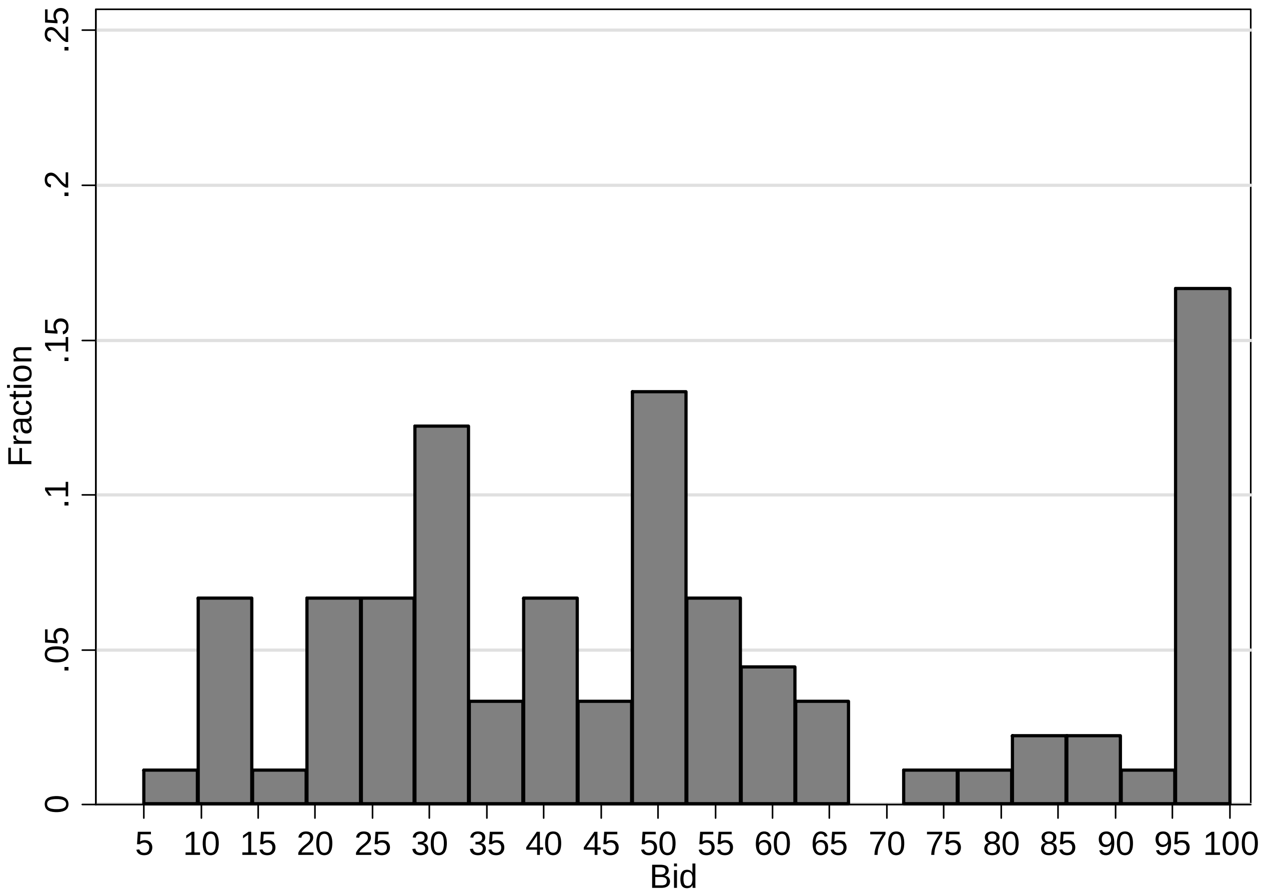
Common-value auctions and the winner's curse

Suppose we all participate in a sealed-bid auction for a jar of coins. Once you have estimated the amount of money in the jar, what are your bidding strategies in first- and second-price auctions?

The winning bidder is likely to be the bidder with the largest positive error (the largest overestimate).

In this case, the winner has fallen prey to the so-called the winner's curse. Auctions where the winner's curse is significant are oil fields, spectrum auctions, pay per click, and more.

First-price auction class experiment




Second-price auction class experiment
Fraction
0
.05
.1
.15
.2
.25
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
Bid

Conclusions

Adam Brandenburger:

> *There is nothing so practical as a good [game] theory. A good theory confirms the conventional wisdom that "less is more." A good theory does less because it does not give answers. At the same time, it does a lot more because it helps people organize what they know and uncover what they do not know. A good theory gives people the tools to discover what is best for them.*