

**UC Berkeley
Haas School of Business
Economic Analysis for Business Decisions
(EWMBA 201A)**

Leftovers, review and takeaways

Lectures 13-14

Oct. 3, 2009

Firm behavior

- The first thing to do is to examine the constraints on the firm's behavior.
- The constraints are imposed by its customers, by its competitors, by its employees, and by its technology.
- Production theory is very easy because it uses the same tools as consumer theory.
- In fact, it is much simpler because the output of a production process is generally observable.

Production

- The technological constraints facing a firm are described through the use of isoquants.
- The isoquants are curves that indicate all the combinations of inputs capable of producing a given level of output.
- We generally assume that isoquants are convex and monotonic, just like well-behaved consumer preferences.
- The technical rate of substitution (TRS) measures the slope of the isoquants.

If, for example, we consider the case of two inputs, the production function

$$f(x_1, x_2)$$

would measure the maximum of output that we could get from x_1 units of factor 1 and x_2 units of factor 2.

Since we already know a lot about indifference curves, it is easy to understand how isoquants work.

Technologies and isoquants – perfect complements, perfect substitutes, and Cobb-Douglas.

Cobb-Douglas production function

The Cobb-Douglas production function is given by

$$f(x_1, x_2) = Ax_1^\alpha x_2^\beta$$

where the parameter $A > 0$ measures the scale of production and the parameters α and β measure the returns to scale.

It exhibits constant returns to scale when $\alpha + \beta = 1$, increasing when $\alpha + \beta > 1$, and decreasing when $\alpha + \beta < 1$.

Pricing

While there is some involved analysis required, the important takeaways about optimal pricing are

- At the optimal quantity produced q^* , marginal revenue equals marginal cost

$$MR(q^*) = MC(q^*)$$

- Marginal revenue comes from the underlying demands curve. Demand curves themselves come from consumer preferences.

Simple (nondiscriminatory) pricing

A firm engages in simple pricing for a particular product if that product is sold for the same price per unit no matter who the buyer is or how many units the buyer purchases.

The profit-maximizing quantity for the firm to produce (if it should be in business at all) q^* satisfies:

- (i) $MR(q^*) = MC(q^*)$
- (ii) $MR(q) > MC(q)$ for all $q < q^*$
- (iii) $MR(q) < MC(q)$ for all $q > q^*$

Note that marginal profit,

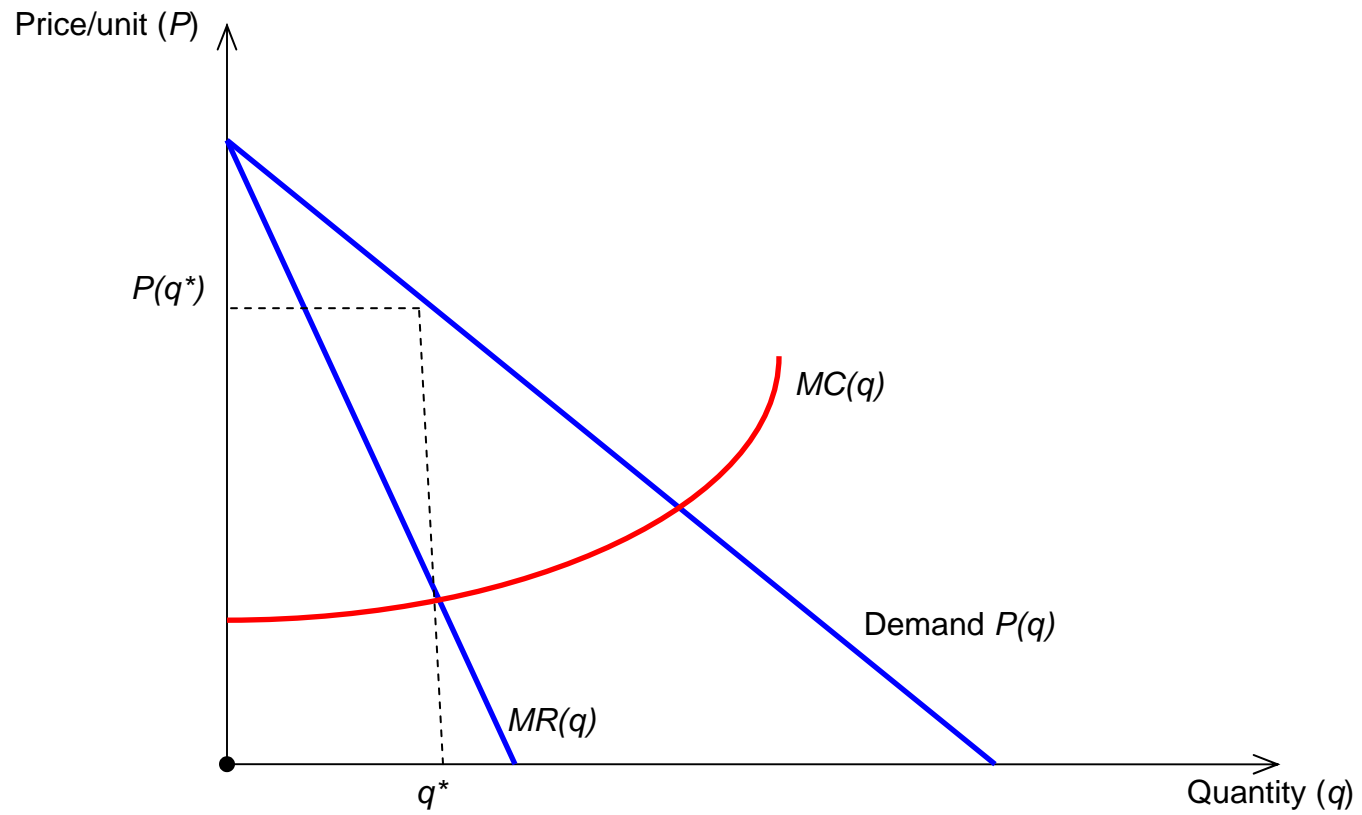
$$MR(q) - MC(q)$$

is positive for all $q < q^*$, that is, every additional unit in this region contributes positively to total profit.

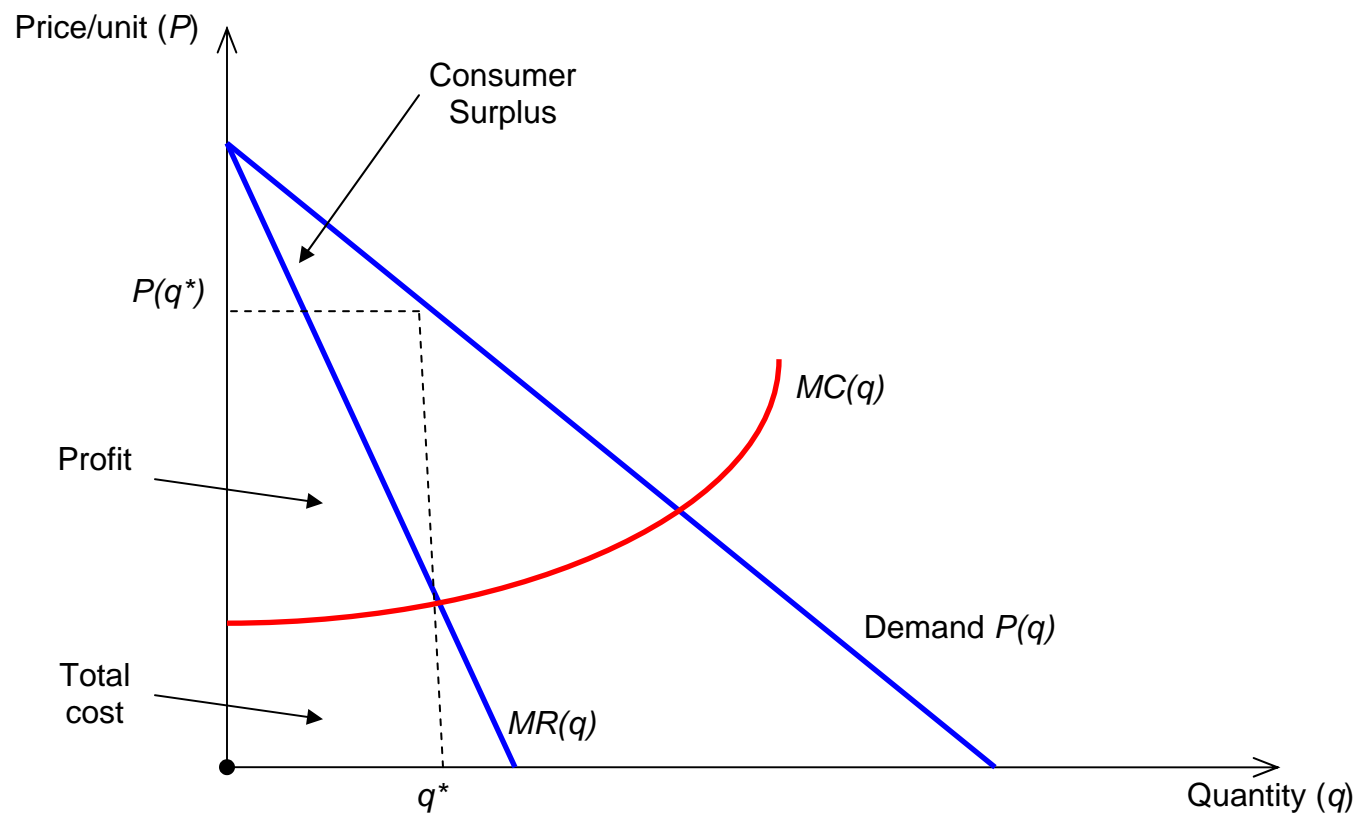
On the other hand, marginal profit is negative for all $q > q^*$, that is, every additional unit in this region reduces total profit.

⇒ Increasing the total profit in the region $q < q^*$ and descending the total profit in the region $q > q^*$.

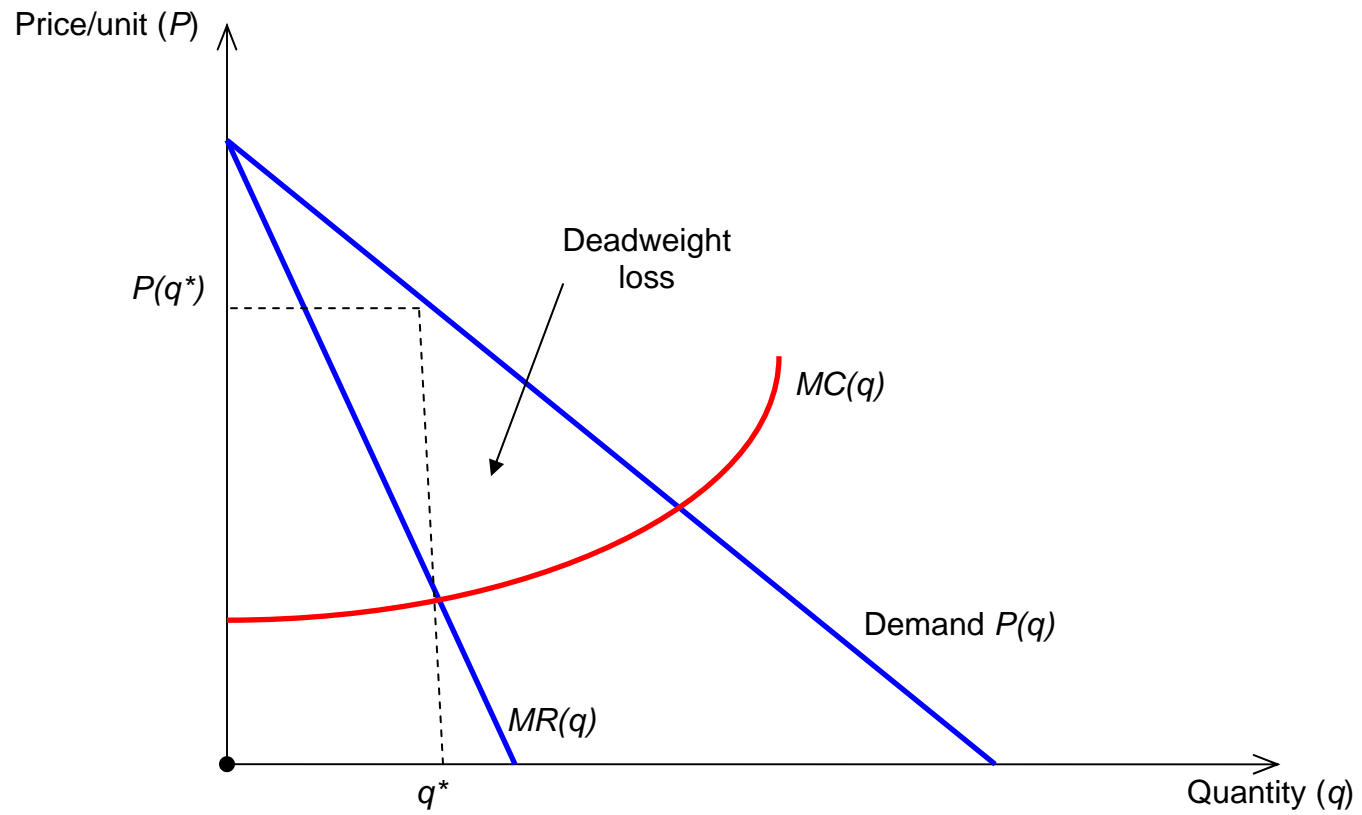
Profit-maximizing price and quantity



Total costs, profit, and consumer surplus



What simple pricing loses?



The Holy Grail of pricing

- If the firm can capture *all* the welfare generated from selling q units, then the firm will want to produce $q^{**} > q^*$ such that $P(q^{**}) = MC(q^{**})$.
- Because this outcome is so good, any form of pricing that achieves this Holy Grail is known as perfect price discrimination.
- For historic reasons, perfect price discrimination is also known as first-degree price discrimination.

⇒ Can the firm ever obtain the Holy Grail? Generally, the answer is no!!!

Two-part tariffs

A two-part tariff is pricing with an entry fee and per-unit charge. It can help get a firm closer to the Grail than can simple pricing.

Formally, a two-part tariff consists of an entry fee F and a per-unit charge p . A consumer's expenditure if she buys q units is given by

$$T(q) = \begin{cases} 0 & \text{if } q = 0 \\ F + pq & \text{if } q > 0 \end{cases}$$

If there are N homogeneous (have identical demands) consumers, then under the profit-maximizing two-part tariff, the firm

- produces q^{**} units, where $P(q^{**}) = MC(q^{**})$
- sets the per-unit charge p to equal $P(q^{**})$
- sets the entry fee F to equal average consumer surplus CS/N .

If consumers are heterogeneous, the firm can still profit from using a two-part tariff, but designing the optimal tariff is much more complicated...

Third- and second-degree price discrimination

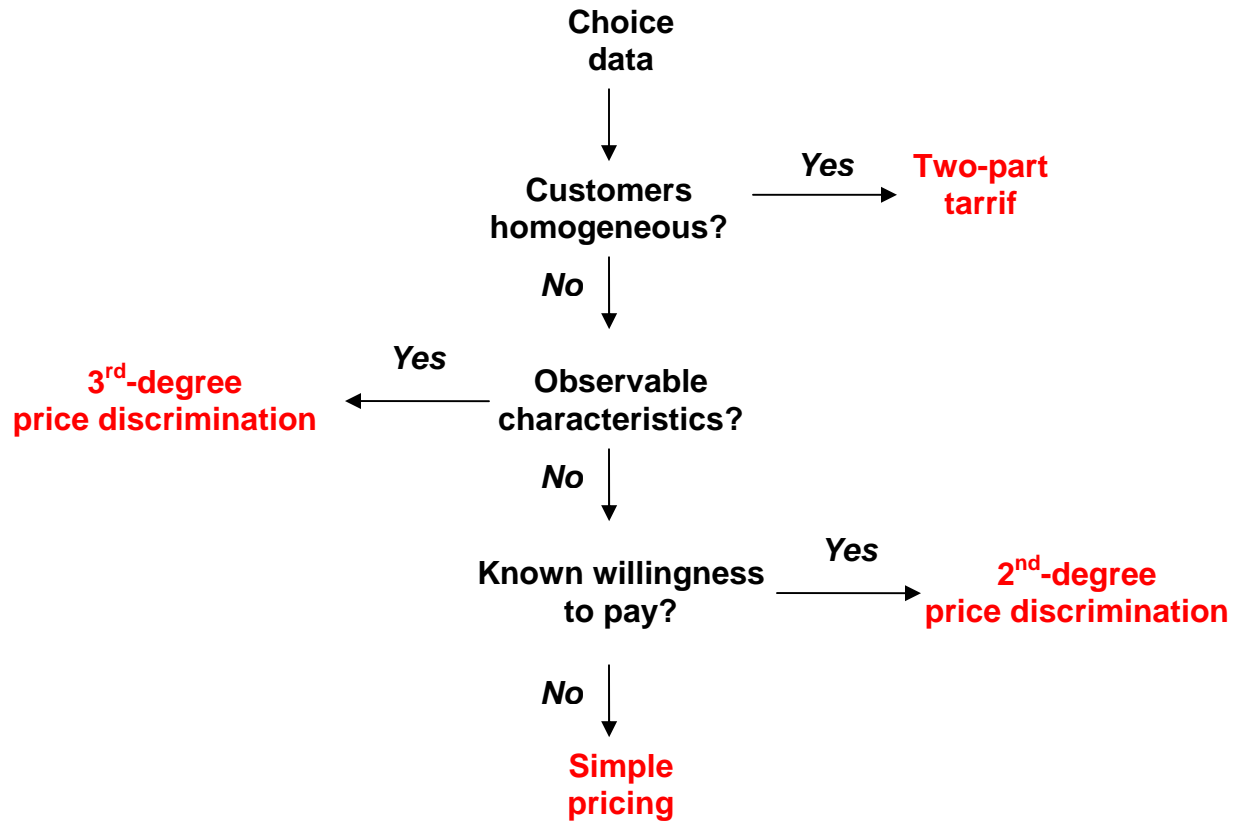
Third-degree price discrimination is charging different prices on the basis of observed group membership.

- Examples: Senior-citizen/child/student discounts, and geography-based third-degree price discrimination.

Second-degree price discrimination is price discrimination via induced revelation of preferences.

- Examples: quantity discounts, quality distortions (an adverse selection problem!).

How to choose a pricing strategy?!



Bargaining / negotiation

The strategic approach (Rubinstein, 1982)

- Two players $i = 1, 2$ bargain over a “pie” of size 1.
- An agreement is a pair (x_1, x_2) where x_i is player i 's share of the pie.
- The set of possible agreements is $x_1 + x_2 = 1$ where for any two possible agreements x and y

$$x \succsim_i y \text{ if and only if } x_i \geq y_i$$

The bargaining procedure

- The players can take actions only at times in an (infinite) set of dates.
- In each period t player i , proposes an agreement (x_1, x_2) and player $j \neq i$ either accepts (Y) or rejects (N).
- If (x_1, x_2) is accepted (Y) then the bargaining ends and (x_1, x_2) is implemented. If it is rejected (N) then the play passes to period $t + 1$ in which j proposes an agreement (alternating offers).

Preferences

The preferences over outcomes alone may not be sufficient to determine a solution. Time preferences (toward agreements at different points in time) are the driving force of the model:

- Disagreement is the worst outcome.
- The pie is desirable and time is valuable.
- Increasing loss to delay.

Under this assumptions, the preferences of player i are represented by

$$\delta_i^t u_i(x_i)$$

for any $0 < \delta_i < 1$ and u_i is increasing and concave function

Any two-player bargaining game of alternating offers in which players' preferences satisfy the assumptions above has a unique (!) subgame perfect equilibrium.

⇒ Player 1 (moves first) always proposes

$$(x_1^*, x_2^*) = \left(\frac{1 - \delta_2}{1 - \delta_1 \delta_2}, \frac{\delta_2(1 - \delta_1)}{1 - \delta_1 \delta_2} \right),$$

and accepts an offer (y_1, y_2) of player 2 if and only if $y_1 \geq y_1^*$.

⇒ Player 2 always proposes

$$(y_1^*, y_2^*) = \left(\frac{\delta_1(1 - \delta_2)}{1 - \delta_1 \delta_2}, \frac{1 - \delta_1}{1 - \delta_1 \delta_2} \right).$$

and accepts an offer $(x_1; x_2)$ of player 1 if and only if $x_2 \geq x_2^*$.

The unique outcome is that player 1 proposes (x_1^*, x_2^*) at the first period and player 2 accepts (no delay!).

When players have the same discount rate $\delta_1 = \delta_2 = \delta$ then

$$(x_1^*, x_2^*) = \left(\frac{1}{1 + \delta}, \frac{\delta}{1 + \delta} \right),$$

and

$$(y_1^*, y_2^*) = \left(\frac{\delta}{1 + \delta}, \frac{1}{1 + \delta} \right).$$

\implies Properties of subgame equilibrium: efficiency (no delay), first-mover advantage (perfect information), effects of changes in patience.

The axiomatic approach (Nash, 1950)

Nash's (1950) work is the starting point for formal bargaining theory.

- Bargaining problem: a set of utility pairs $(s_1; s_2)$ that can be derived from possible agreements, and a pair of utilities (d_1, d_2) which is designated to be a disagreement point.
- Bargaining solution: a function that assigns a *unique* outcome to every bargaining problem.

Let S be the set of all utility pairs $(s_1; s_2)$. $\langle S, d \rangle$ is the primitive of Nash's bargaining problem.

Nash's axioms

One states as axioms several properties that it would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely – Nash 1953 –

[1] Invariance to equivalent utility representations (INV)

If $\langle S', d' \rangle$ is obtained from $\langle S, d \rangle$ by “monotonic” transformations then $\langle S', d' \rangle$ and $\langle S, d \rangle$ represent the same situation.

INV requires that the utility outcome of the bargaining problem co-vary with representation of preferences. The physical outcome predicted by the bargaining solution is the same for $\langle S', d' \rangle$ and $\langle S, d \rangle$.

[2] Symmetry (SYM)

A bargaining problem $\langle S, d \rangle$ is *symmetric* if

$$d_1 = d_2$$

and

(s_1, s_2) is in S if and only if (s_2, s_1) is in S .

If the bargaining problem $\langle S, d \rangle$ is symmetric then the bargaining solution must assign the same utility.

Nash does not describe differences between the players. All asymmetries (in the bargaining abilities) must be captured by $\langle S, d \rangle$.

[3] Independence of irrelevant alternatives (IIA)

If $\langle S, d \rangle$ and $\langle T, d \rangle$ are bargaining problems, S is a strict subset of T , and the solution to $\langle T, d \rangle$ is in $\langle S, d \rangle$ then it is also the solution to $\langle S, d \rangle$.

Put differently, if T is available and players agree on (s_1, s_2) in S then they also agree on the same (s_1, s_2) if only S is available.

IIA excludes situations in which the fact that a certain agreement is available influences the outcome.

Pareto efficiency (PAR)

If $\langle S, d \rangle$ is a bargaining problem where (s_1, s_2) and (t_1, t_2) are in S and $t_i > s_i$ for $i = 1, 2$ then the solution is *not* (s_1, s_2) .

Players never agree on an outcome (s_1, s_2) when there is an outcome (t_1, t_2) in which both are better off.

After agreeing on the outcome (s_1, s_2) , players can always “renegotiate” and agree on (t_1, t_2) .

Nash's solution

There is precisely *one* bargaining solution, satisfying SYM, PAR, INV and IIA.

The unique bargaining solution is the utility pair that maximizes the product of the players' utilities

$$\arg \max_{(s_1, s_2)} s_1 s_2$$