

# **Confronting Theory with Experimental Data and vice versa**

## **Risk Preferences**

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Department of Economics  
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## The basic assumptions about preferences

The theory begins with two (not three!) assumptions about preferences

### [1] Completeness

$$x \succsim y \text{ or } y \succsim x$$

for any pair of bundles  $x$  and  $y$ .

### [2] Transitivity

$$\text{if } x \succsim y \text{ and } y \succsim z \text{ then } x \succsim z$$

or any three bundles  $x$ ,  $y$  and  $z$ .

Together, completeness and transitivity constitute the formal definition of *rationality* as the term is used in economics.

Rational economic agents are ones who [1] have the ability to make choices, and [2] whose choices display a logical consistency.

The preferences of a rational agent can be represented, or summarized, by a *utility function* (more later).

## Preferences toward risk

The standard model of decisions under risk (known probabilities) is based on von Neumann and Morgenstern Expected Utility Theory.

Let  $X$  be a set of *lotteries*, or gambles, (outcomes and probabilities). A fundamental assumption about preferences toward risk is *independence*:

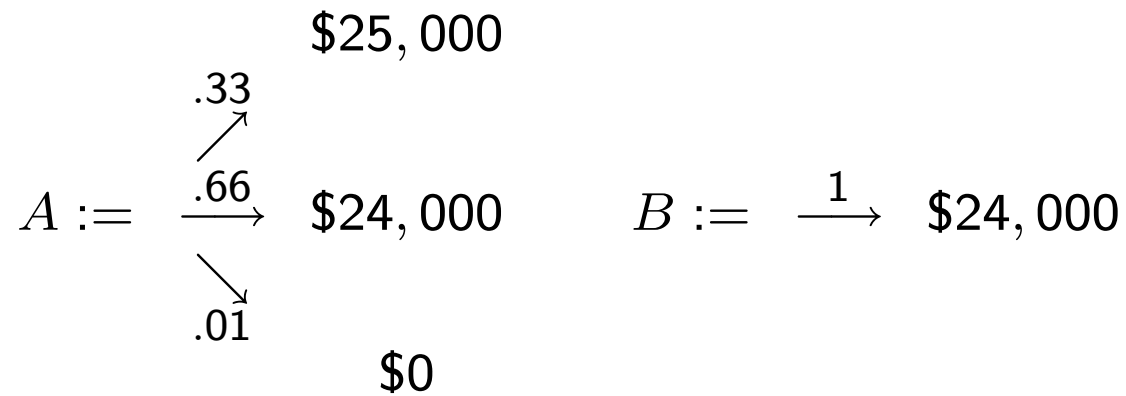
For any lotteries  $x, y, z$  and  $0 < \alpha < 1$

$$x \succ y \text{ implies } \alpha x + (1 - \alpha)r \succ \alpha y + (1 - \alpha)r.$$

## Experiments à la Allais

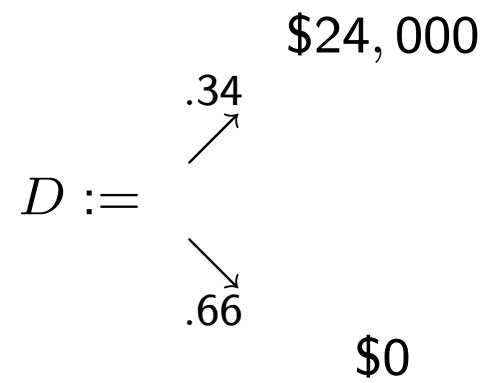
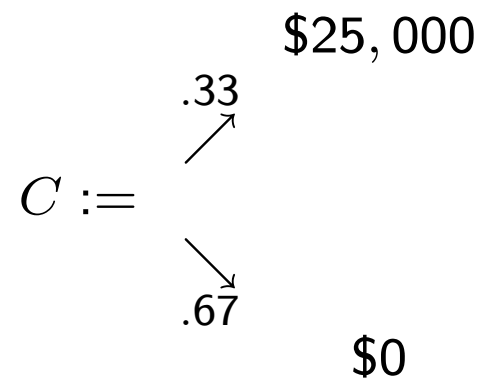
### Allais (1953) I

- Choose between the two gambles:

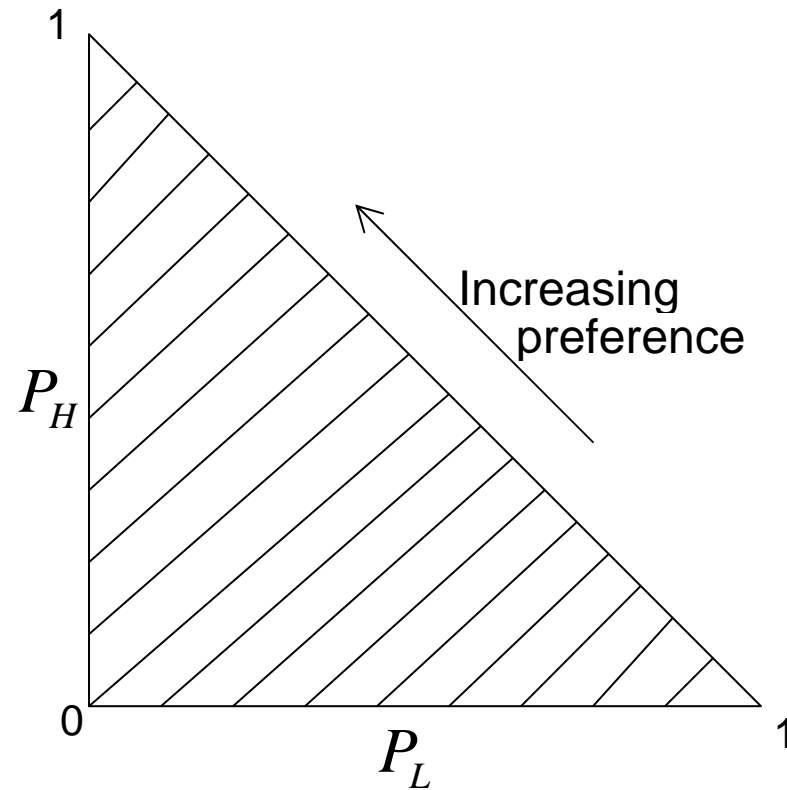


## Allais (1953) II

– Choose between the two gambles:

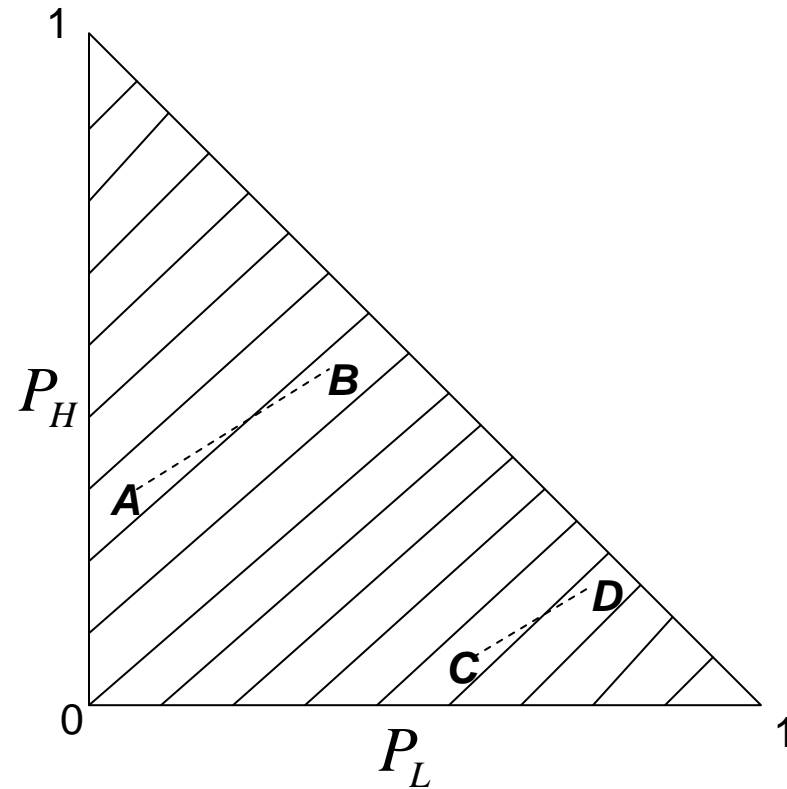


## The Marschak-Machina probability triangle



$H$ ,  $M$ , and  $L$  are three degenerate gambles with certain outcomes  $H > M > L$

## A test of Expected Utility Theory (EUT)



EUT requires that indifference lines are parallel so one must choose either **A** and **C**, or **B** and **D**.



## Contributions

Results have generated the most impressive dialogue between observation and theorizing (Camerer, 1995):

- Violations of EUT raise criticisms about the status of the Savage axioms as the touchstone of rationality.
- These criticisms have generated the development of various alternatives to EUT, such as Prospect Theory.

## Limitations

Choice scenarios narrowly tailored to reveal “anomalies” limits the usefulness of data for other purposes:

- Subjects face “extreme” rather than “typical” decision problems designed to encourage violations of specific axioms.
- Small data sets force experimenters to pool data and to ignore individual heterogeneity.

## Research questions

### Consistency

- Is behavior under uncertainty consistent with the utility maximization model?

### Structure

- Is behavior consistent with a utility function with some special structural properties?

### Recoverability

- Can the underlying utility function be recovered from observed choices?

### Extrapolation

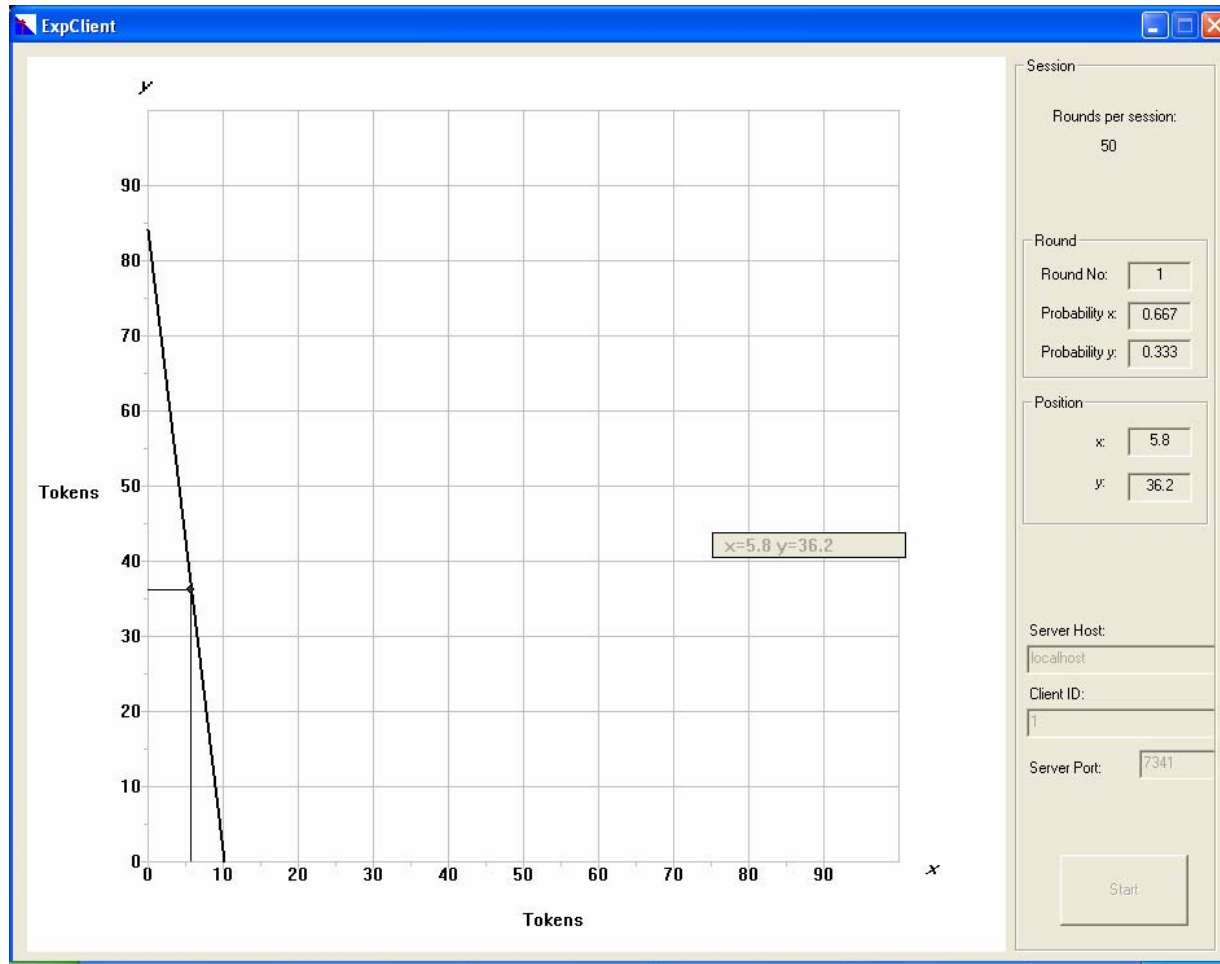
- Given behavior in the laboratory, can we forecast behavior in other environments?

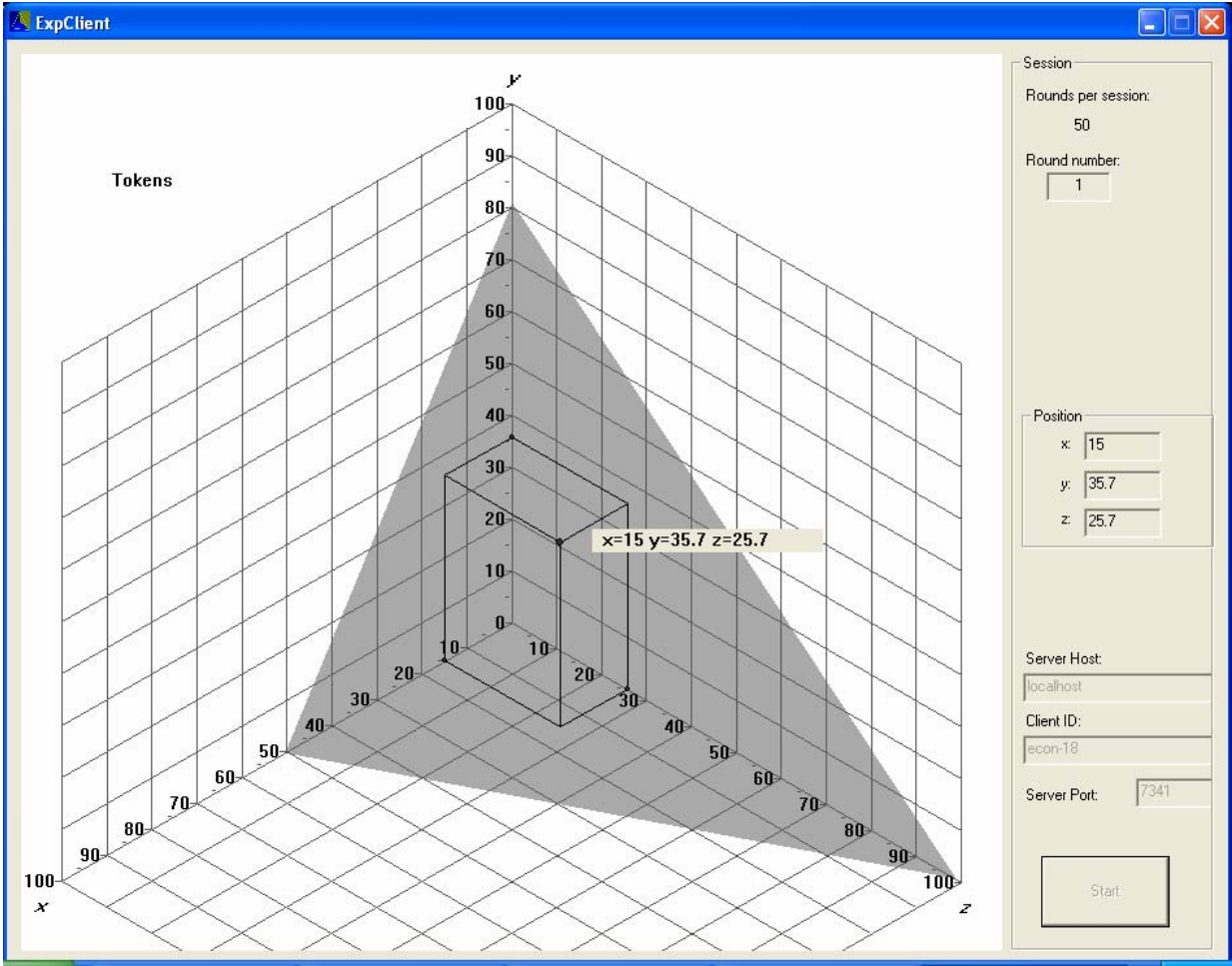
## **A new experimental design**

An experimental design that has a couple of fundamental innovations over previous work:

- A selection of a bundle of contingent commodities from a budget set (a portfolio choice problem).
- A graphical experimental interface that allows for the collection of a rich individual-level data set.

# The experimental computer program dialog windows





## Rationality

Let  $\{(p^i, x^i)\}_{i=1}^{50}$  be some observed individual data ( $p^i$  denotes the  $i$ -th observation of the price vector and  $x^i$  denotes the associated portfolio).

A utility function  $u(x)$  *rationalizes* the observed behavior if it achieves the maximum on the budget set at the chosen portfolio

$$u(x^i) \geq u(x) \text{ for all } x \text{ s.t. } p^i \cdot x^i \geq p^i \cdot x.$$



## Revealed preference

A portfolio  $x^i$  is *directly revealed preferred* to a portfolio  $x^j$  if  $p^i \cdot x^i \geq p^i \cdot x^j$ , and  $x^i$  is *strictly directly revealed preferred* to  $x^j$  if the inequality is strict.

The relation *indirectly revealed preferred* is the transitive closure of the directly revealed preferred relation.

**Generalized Axiom of Revealed Preference (GARP)** *If  $x^i$  is indirectly revealed preferred to  $x^j$ , then  $x^j$  is not strictly directly revealed preferred (i.e.  $p^j \cdot x^j \leq p^j \cdot x^i$ ) to  $x^i$ .*

GARP is tied to utility representation through a theorem, which was first proved by Afriat (1967).

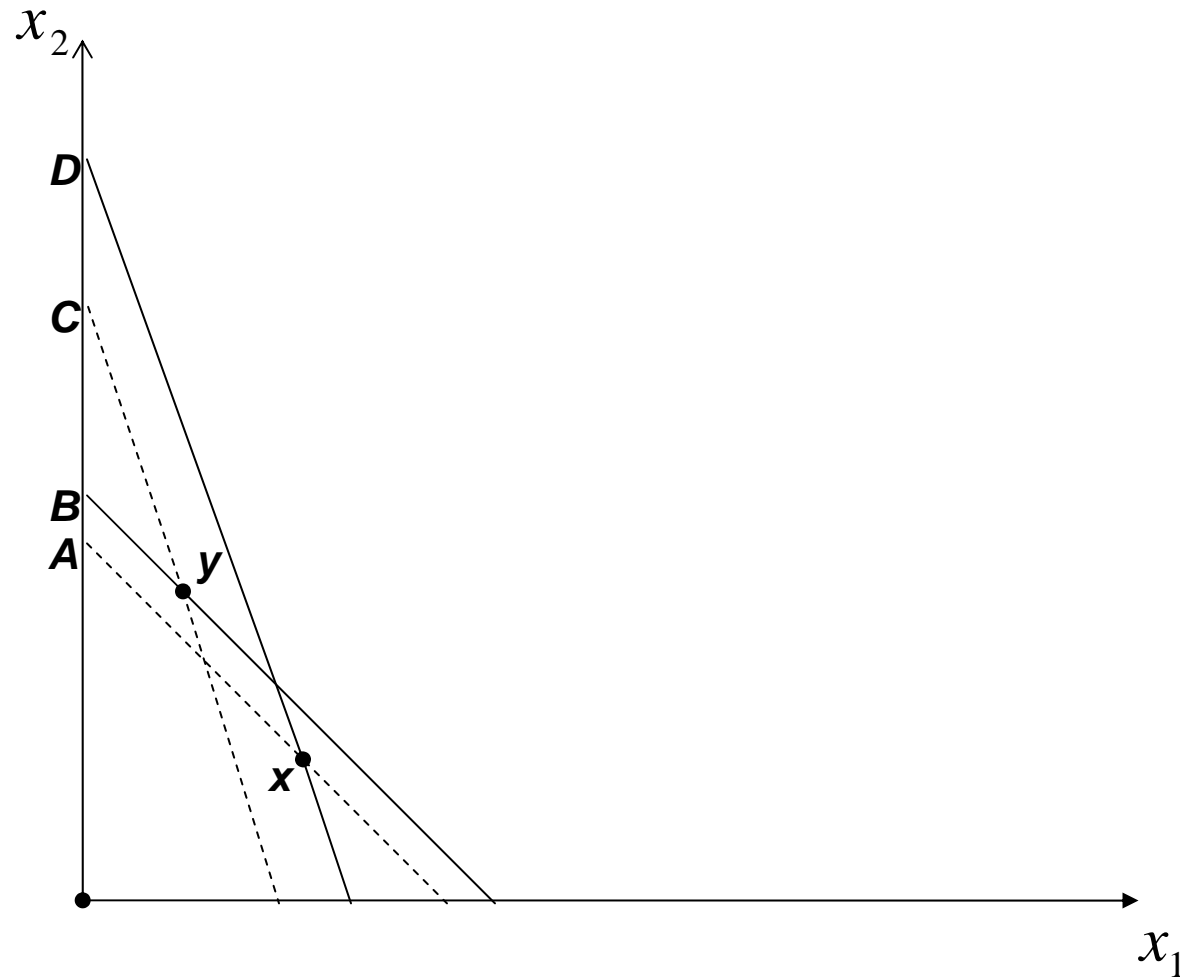
**Afriat's Theorem** *The following conditions are equivalent:*

- *The data satisfy GARP.*
- *There exists a non-satiated utility function that rationalizes the data.*
- *There exists a concave, monotonic, continuous, non-satiated utility function that rationalizes the data.*

**Afriat's critical cost efficiency index (CCEI)** *The amount by which each budget constraint must be relaxed in order to remove all violations of GARP.*

The CCEI is bounded between zero and one. The closer it is to one, the smaller the perturbation required to remove all violations and thus the closer the data are to satisfying GARP.

## The construction of the CCEI for a simple violation of GARP



The agent is 'wasting' as much as  $A/B < C/D$  of his income by making inefficient choices.

## A benchmark level of consistency

A random sample of hypothetical subjects who implement the power utility function

$$u(x) = \frac{x^{1-\rho}}{1-\rho},$$

commonly employed in the empirical analysis of choice under uncertainty, with error.

The likelihood of error is assumed to be a decreasing function of the utility cost of an error.

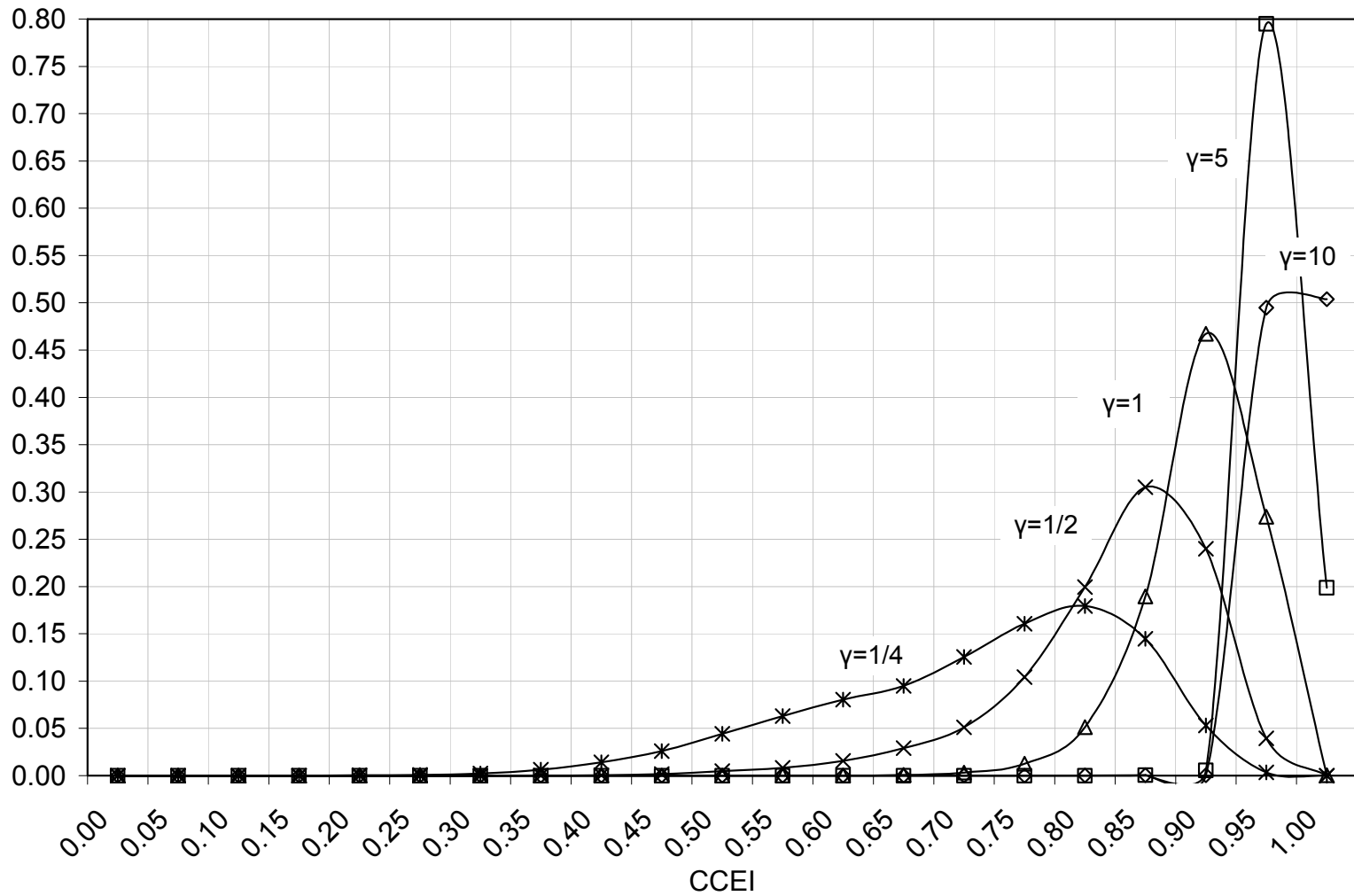
More precisely, we assume an idiosyncratic preference shock that has a logistic distribution

$$\Pr(x^*) = \frac{e^{\gamma \cdot u(x^*)}}{\int_{x:p \cdot x=1} e^{\gamma \cdot u(x)}},$$

where the precision parameter  $\gamma$  reflects sensitivity to differences in utility.

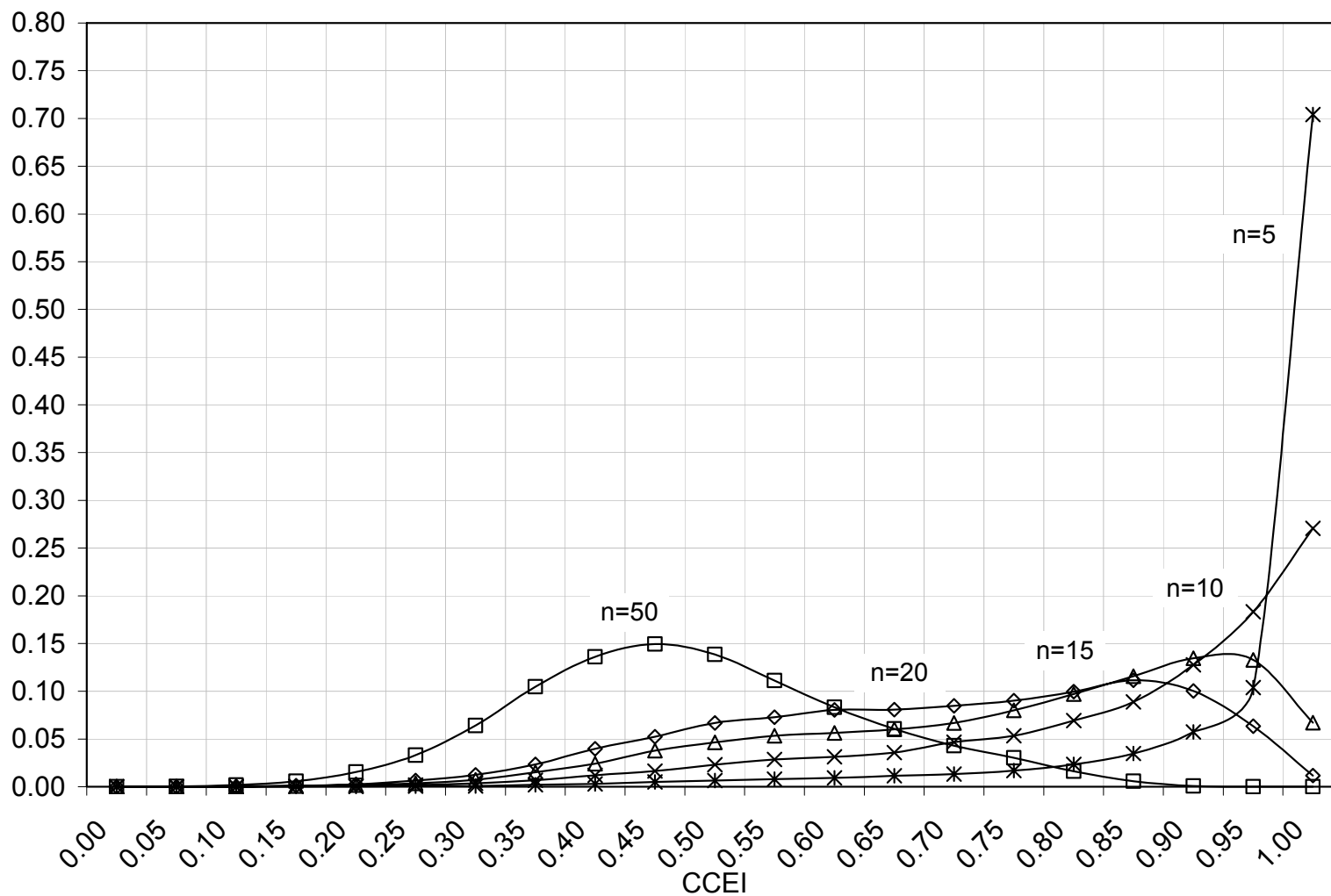
If utility maximization is not the correct model, is our experiment sufficiently powerful to detect it?

The distributions of GARP violations –  $\rho=1/2$  and different  $\gamma$

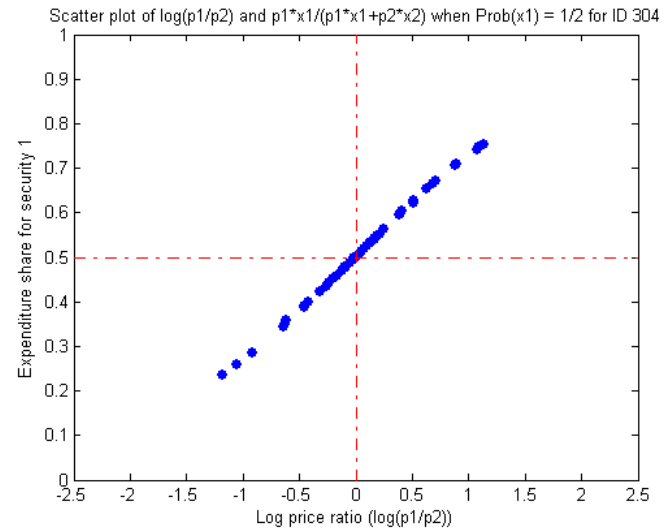
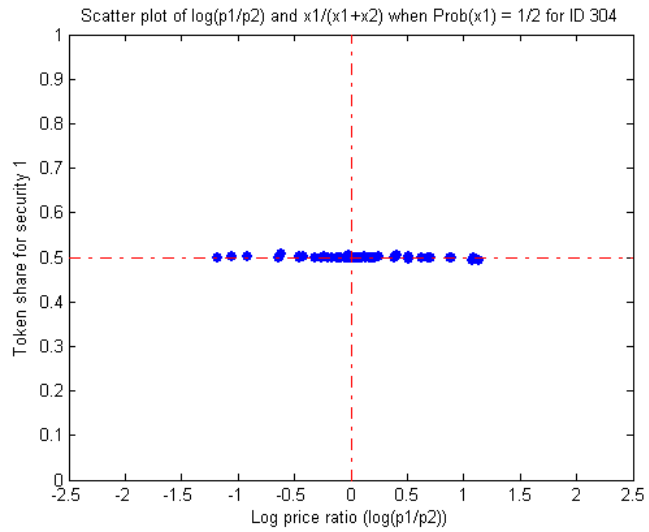
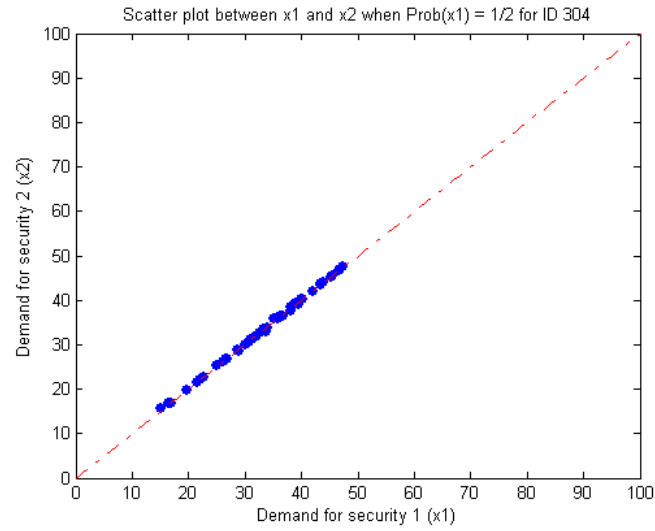


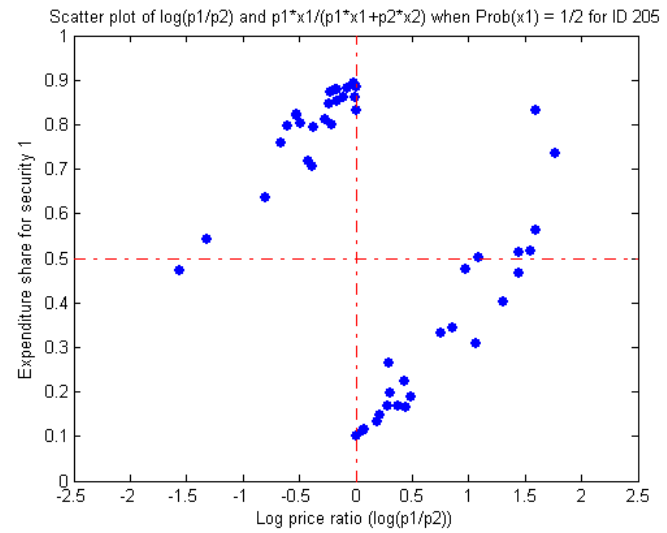
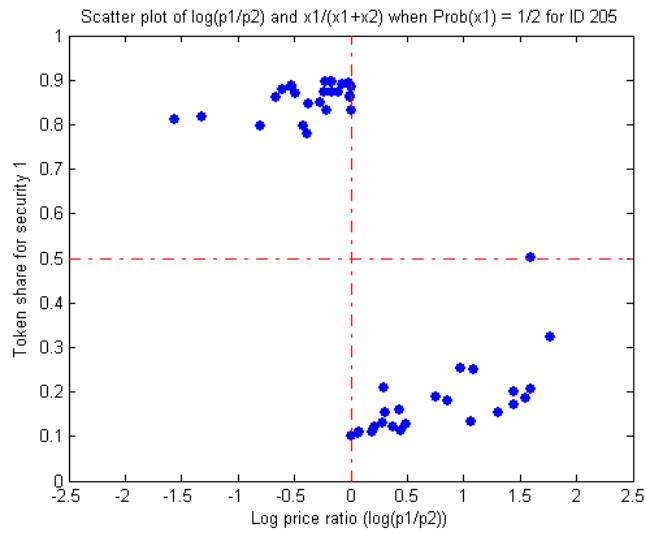
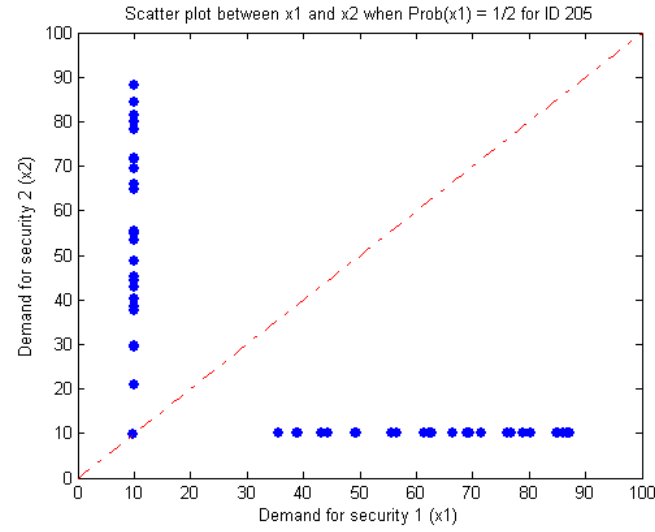


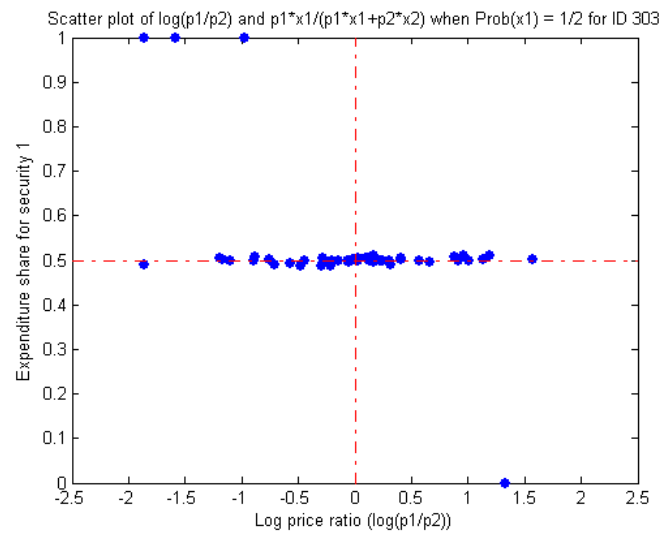
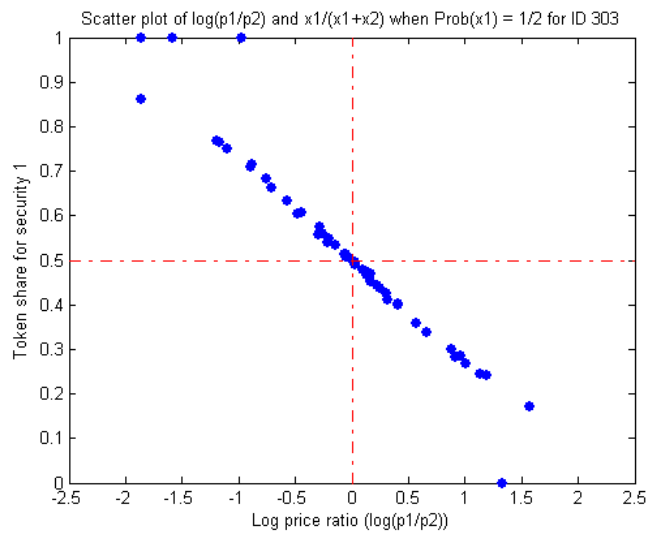
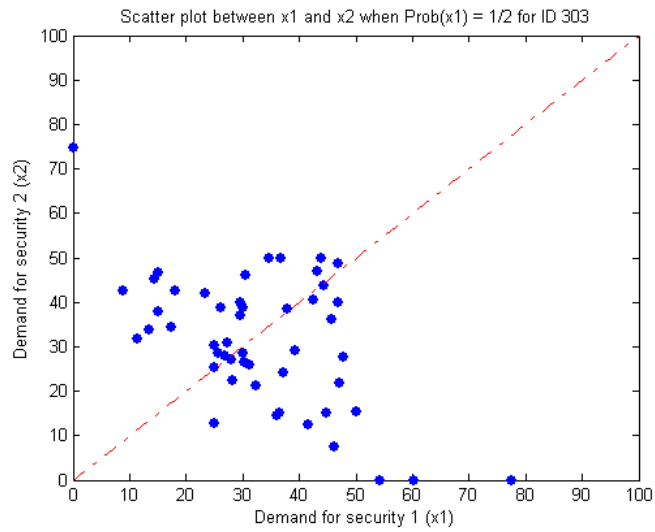
# Bronnars' (1987) test ( $\gamma=0$ )

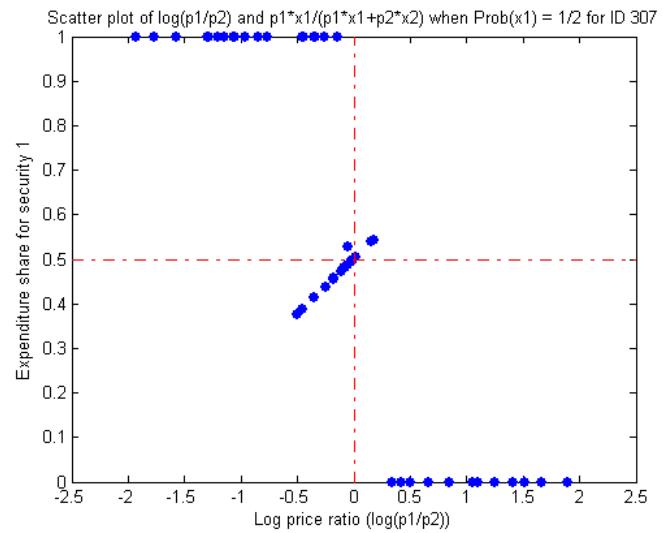
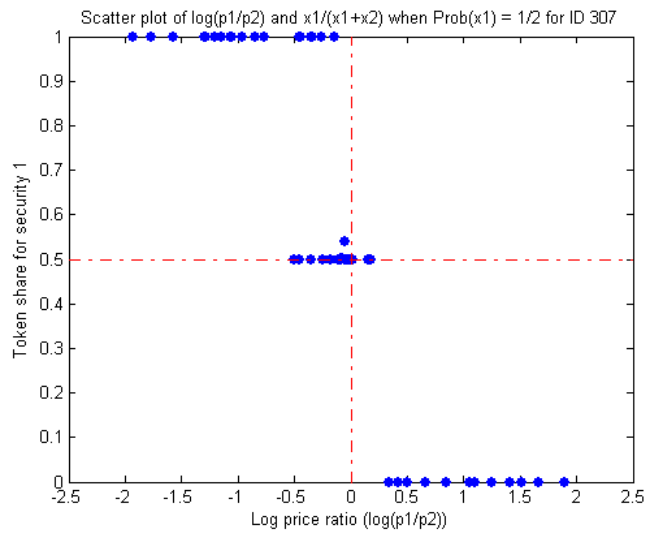
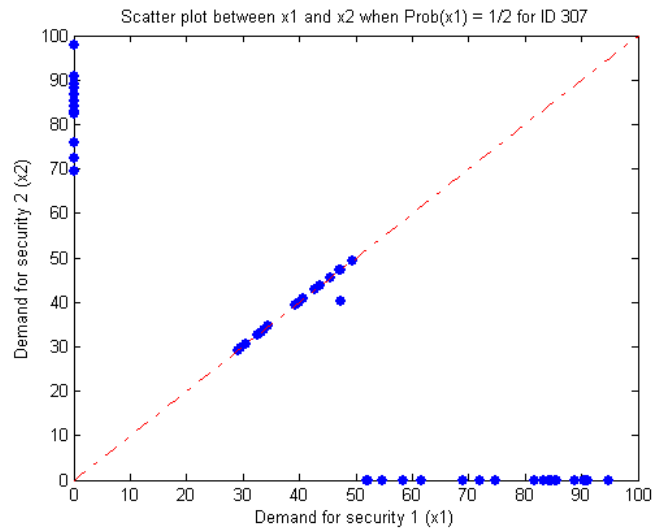


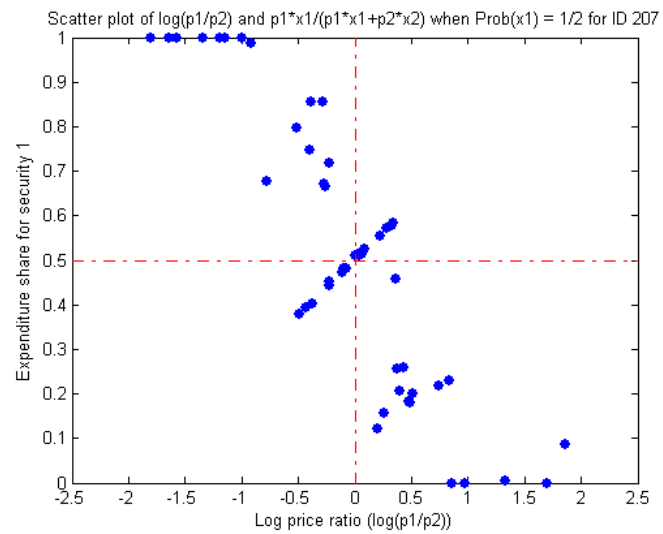
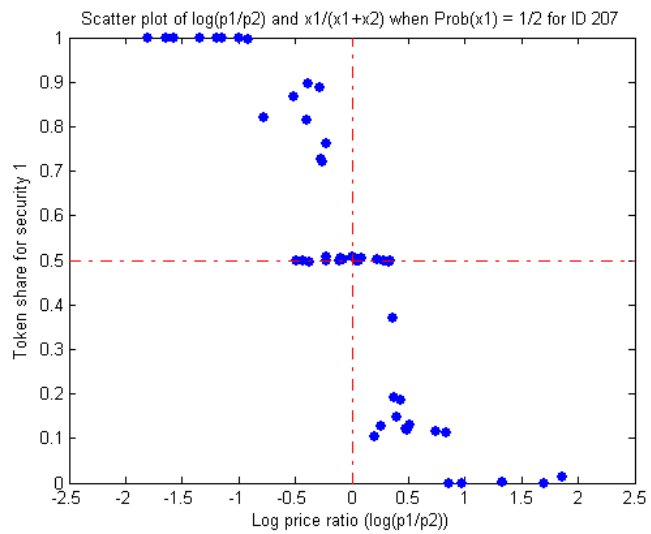
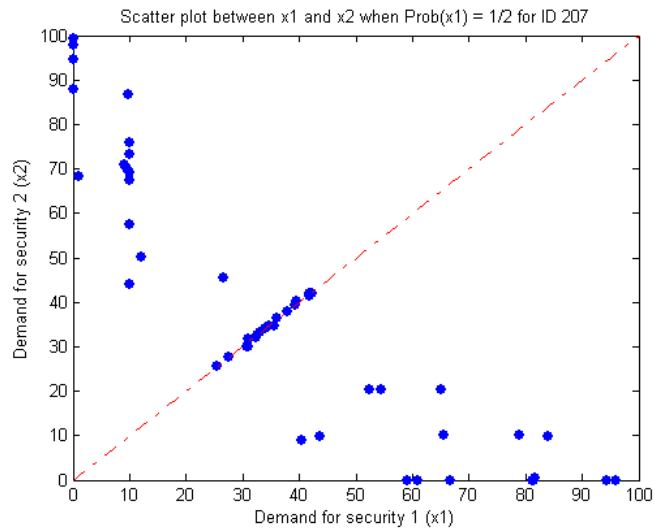
# Individual-level data

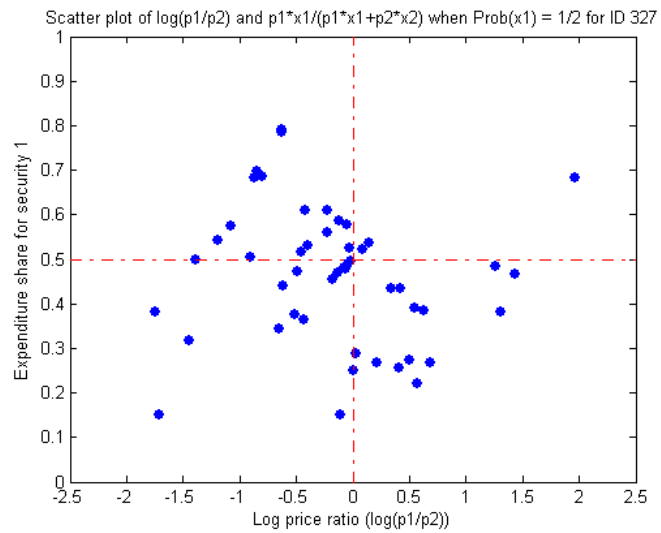
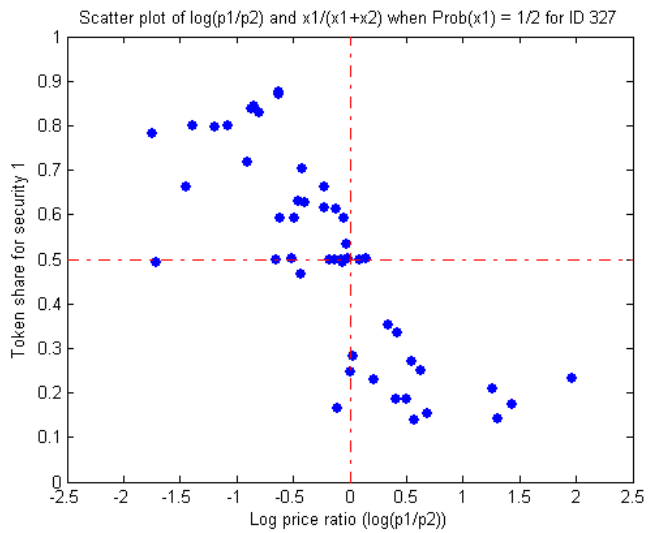
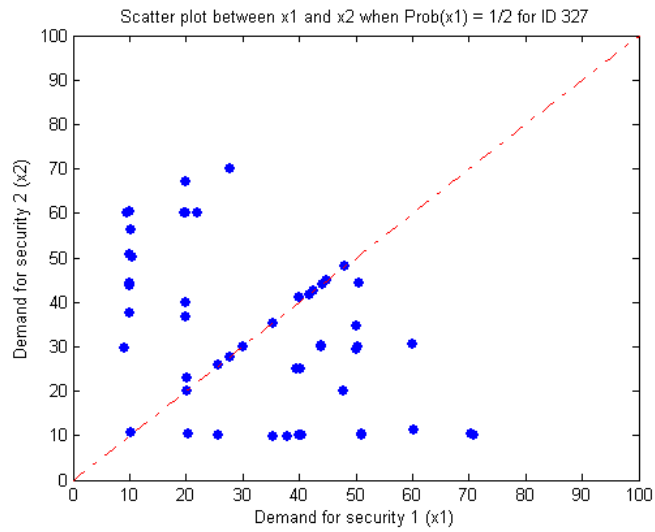


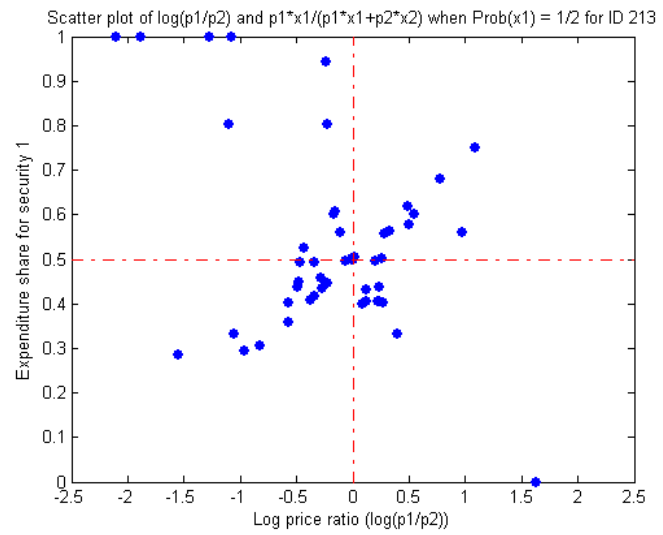
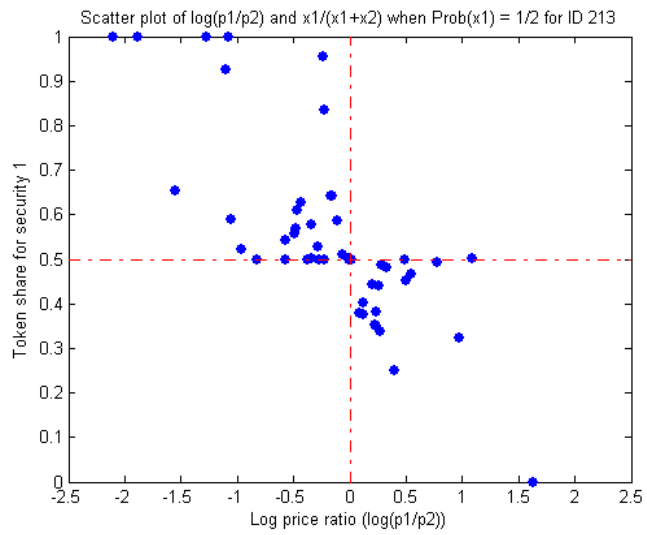
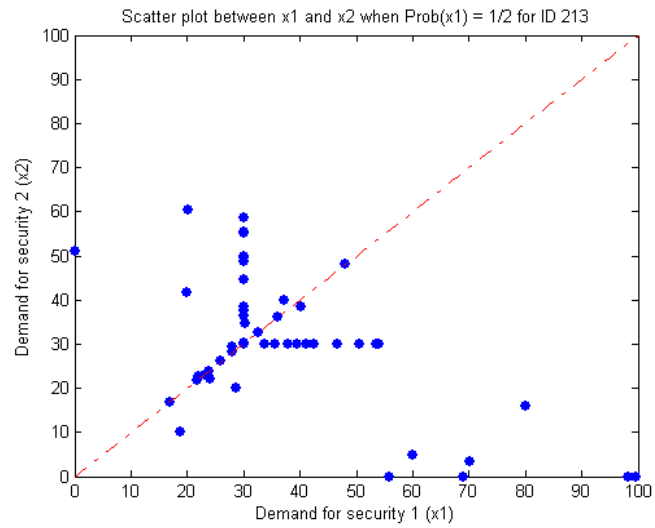










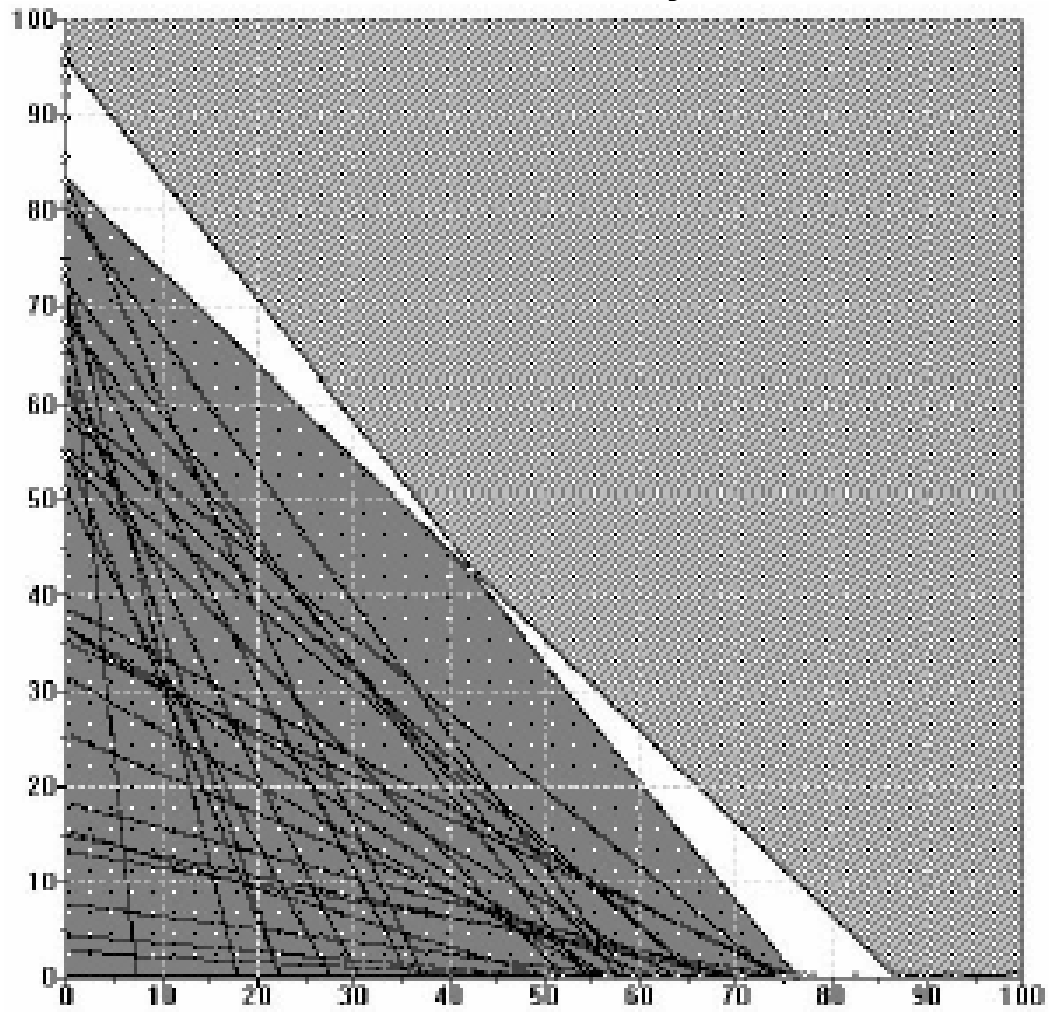




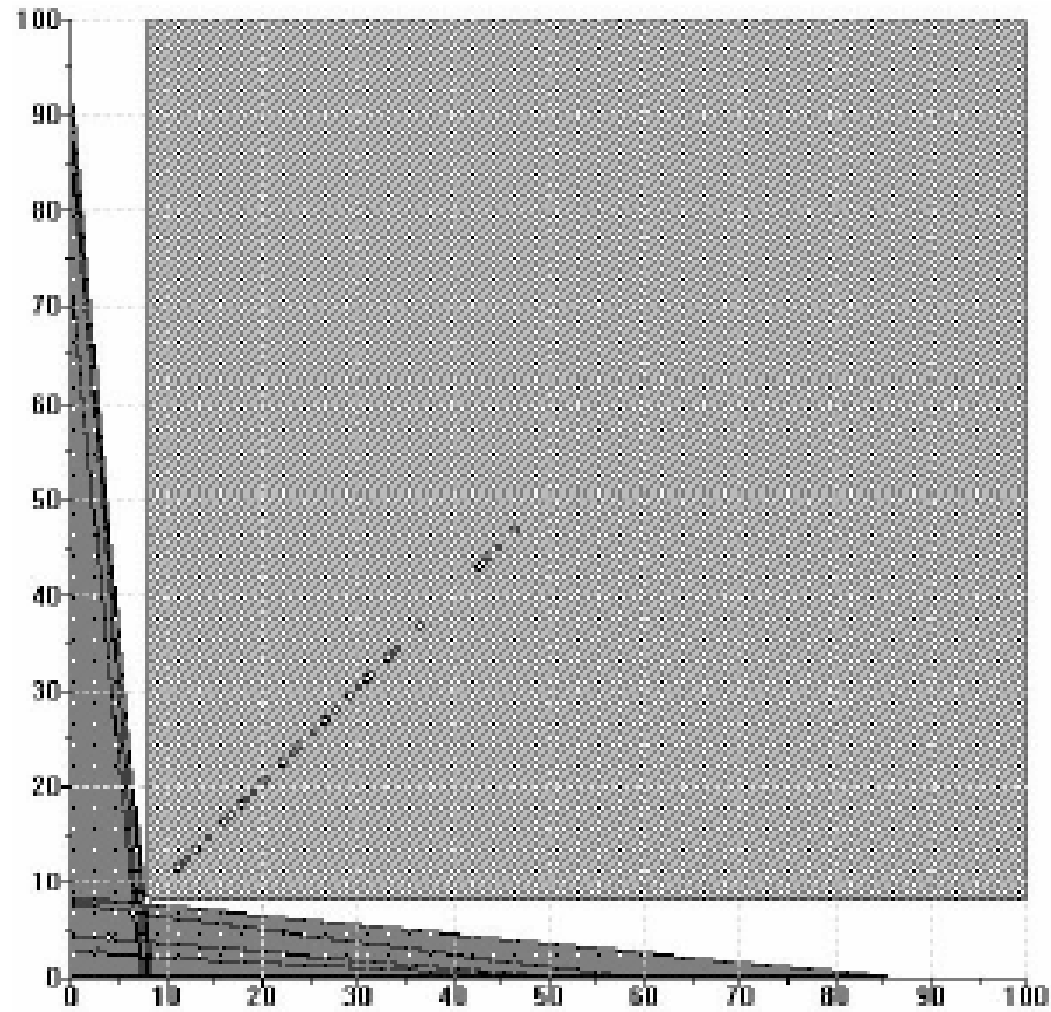
## Recoverability

- Revealed preference relations in the data contain the information that is necessary for recovering preferences.
- Varian (*ECMA*, 1982) uses GARP to generate an algorithm that can recover preferences from choices.
- This approach is purely nonparametric making no assumptions about the parametric form of the underlying utility function.

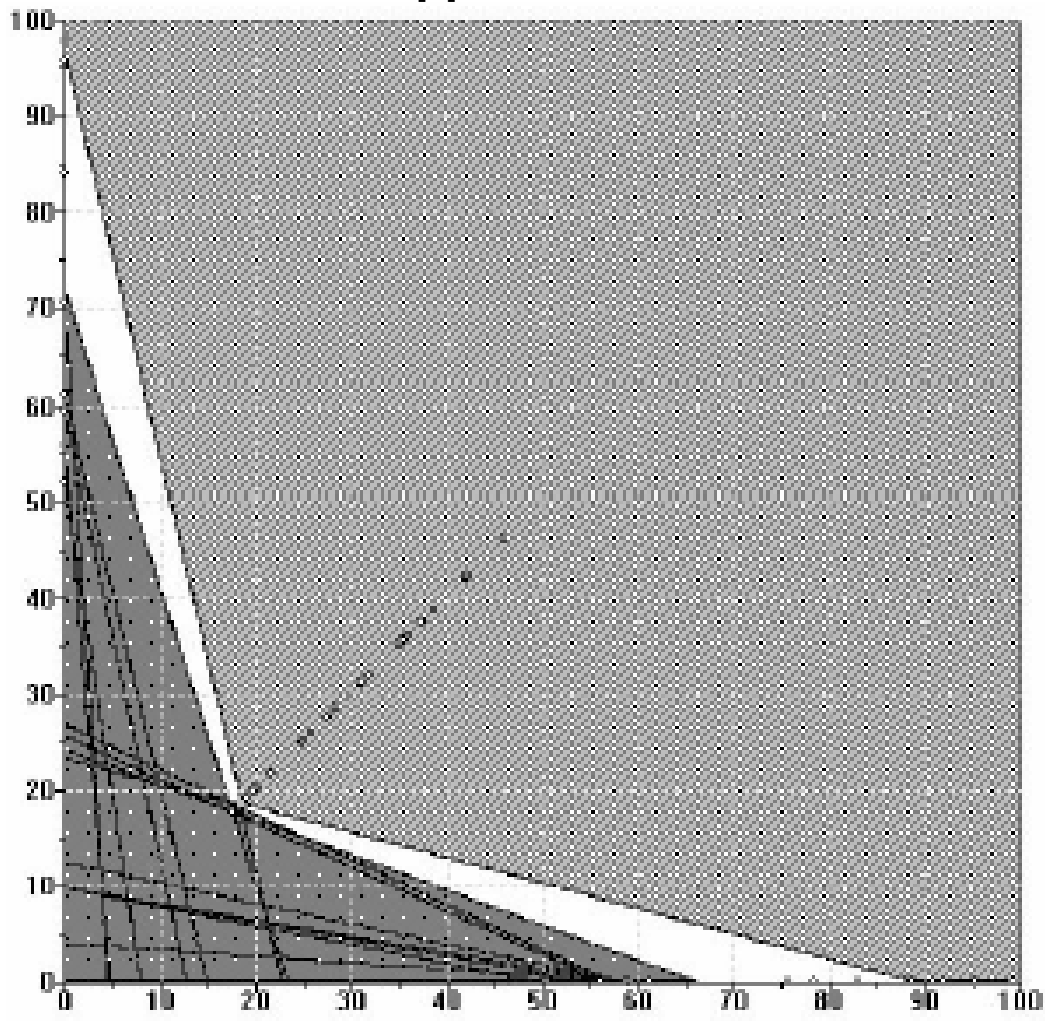
# Risk neutrality



# Infinite risk aversion



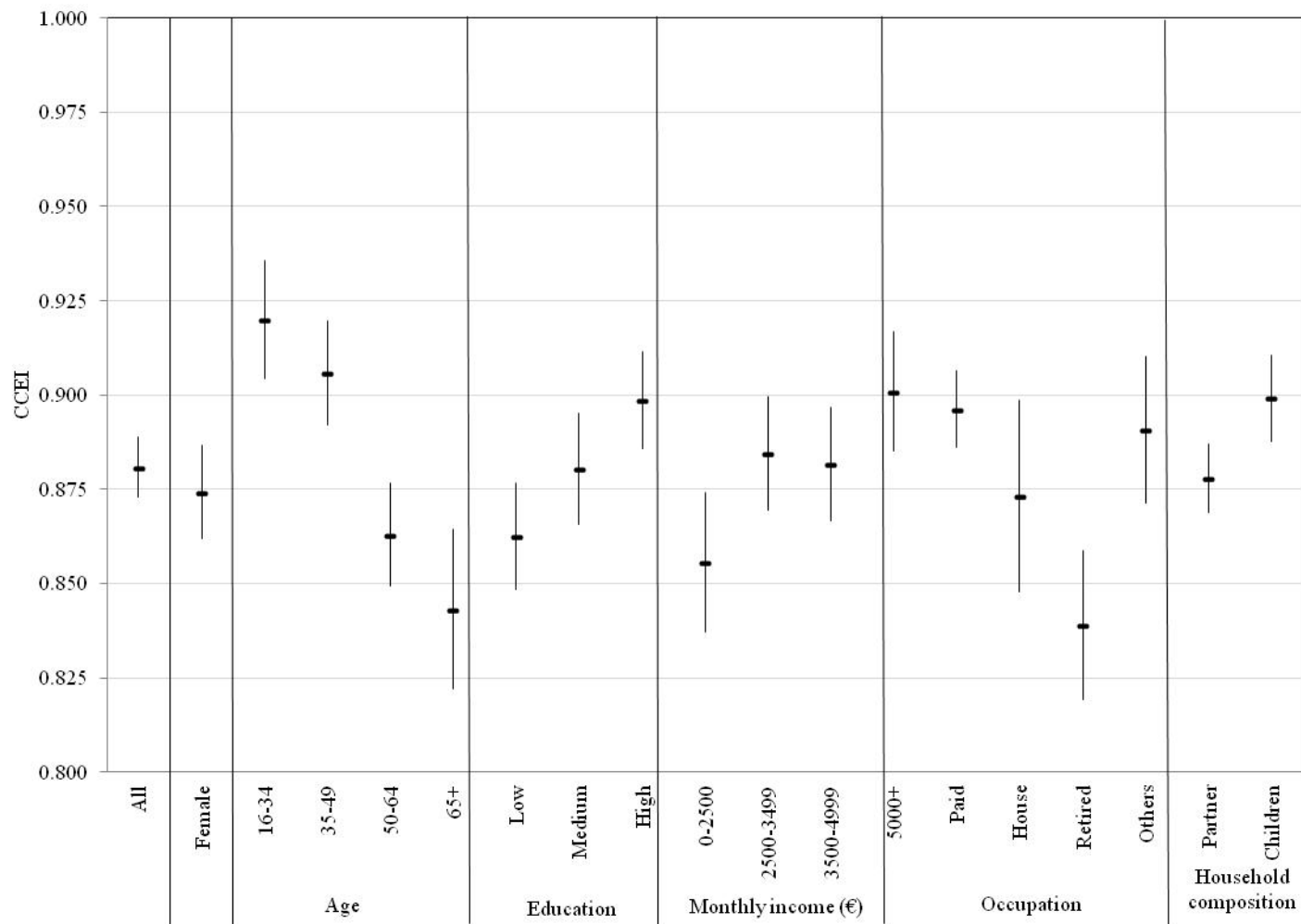
# Loss / disappointment aversion



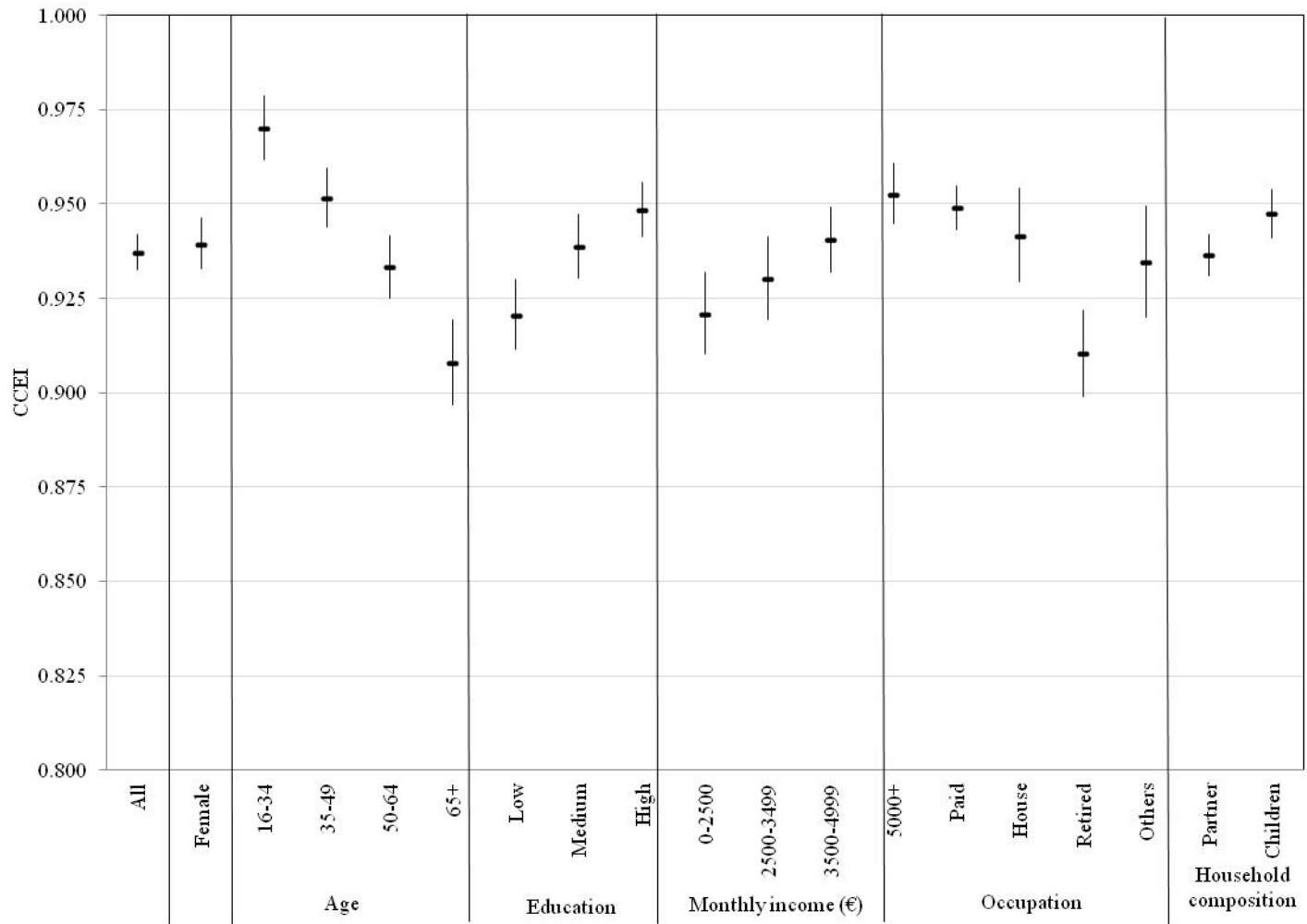
## The CentERpanel

- A representative sample of over 2,000 Dutch-speaking households (5,000 individual members) in the Netherlands.
- A wide range of individual socio-demographic and economic information for the panel members.
- The subjects in the experiment were randomly recruited from the entire CentERpanel body.

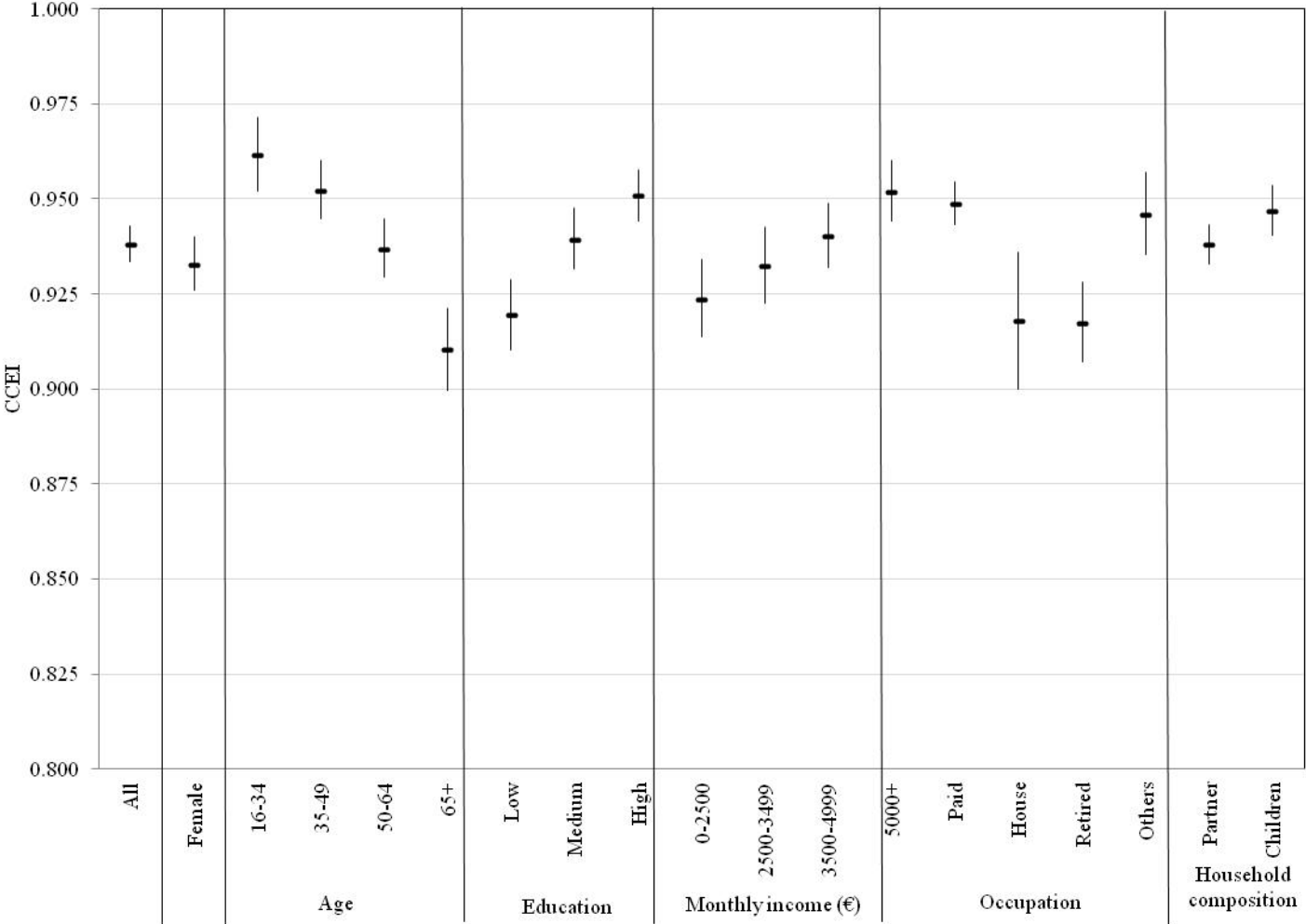
## Mean CCEI scores: equiprobable lotteries



# Mean CCEI scores: income in a few days and income 60 days after that



# Mean CCEI scores: income in 60 days and income another 60 days after that

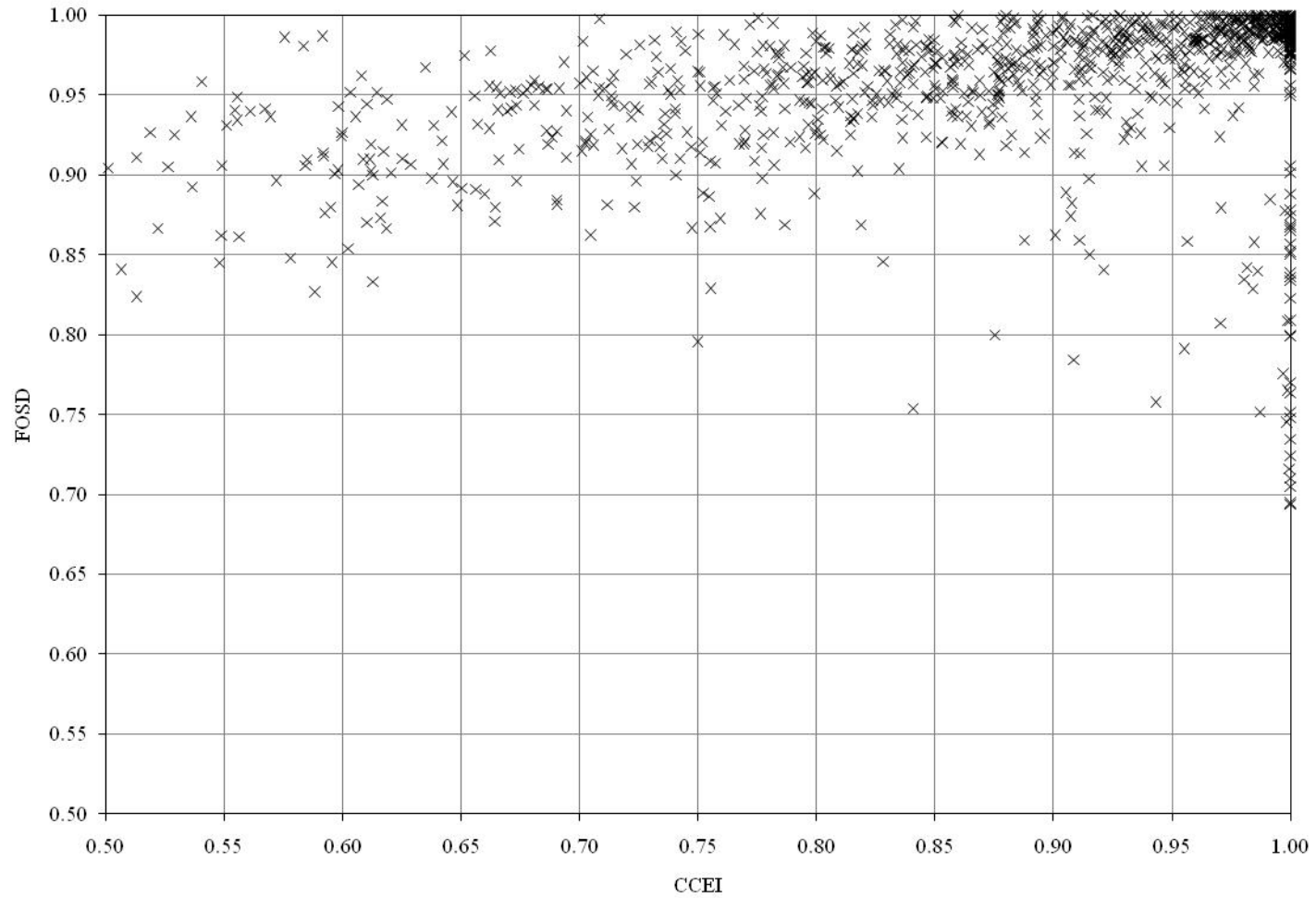




## Dominance

- Violations of monotonicity with respect to first-order stochastic dominance (FOSD) are errors, regardless of risk attitudes.
- A decision to allocate *less* tokens to the *cheaper* account violates dominance but need not involve a violation of GARP.
- We use expected payoff calculations (largest upward probabilistic shift) to assess how nearly choice behavior complies with dominance.

## A scatterplot of CCEI and FOSD scores



## Wealth differentials

- ⇒ The heterogeneity in wealth is not well-explained either by standard observables (income, education, family structure) or by standard unobservables (intertemporal substitution, risk tolerance).
- ⇒ If consistency with utility maximization in the experiment were a good proxy for (financial) decision-making quality then the degree to which consistency differ across subjects should help explain wealth differentials.

The relationship between CCEI scores and wealth

	(1)	(2)	(3)
CCEI	1.351** (0.566)	1.109** (0.534)	101888.0* (52691.9)
Log 2008 household income	0.584*** (0.132)	0.606*** (0.126)	
2008 household income			1.776*** (0.4)
Female	-0.313* (0.177)	-0.356** (0.164)	-32484.3* (17523.9)
Partnered	0.652*** (0.181)	0.595*** (0.171)	46201.9*** (17173.7)
# of children	0.090 (0.093)	0.109 (0.086)	14078.6* (8351.5)
Age	Y	Y	Y
Education	Y	Y	Y
Occupation	Y	Y	Y
Constant	6.292 (6.419)	0.469 (3.598)	76214.4 (559677.5)
$R^2$	0.179	0.217	0.188
# of obs.	517	566	568

The robustness of the correlation -- controls for constraints

	(1)	(2)	(3)	(4)	(5)
CCEI	1.322** (0.570)	1.318** (0.574)	1.925*** (0.672)	1.888*** (0.652)	1.441** (0.578)
Log household income					
2008	19.770 (14.629)	1.000 .	0.544*** (0.137)	0.285* (0.165)	0.616*** (0.128)
2008 <sup>2</sup>	-2.194 (1.533)				
2008 <sup>3</sup>	0.082 (0.053)				
2006				0.232 (0.231)	
2004				0.215 (0.174)	
Female	-0.291 (0.181)	-0.201 (0.173)	-0.337* (0.185)	-0.296 (0.186)	-0.321* (0.176)
Partnered	0.598*** (0.181)	0.561*** (0.178)	0.734*** (0.192)	0.707*** (0.193)	0.641*** (0.179)
# of children	0.091 (0.092)	0.101 (0.096)	0.018 (0.099)	0.031 (0.095)	0.088 (0.093)
Age	Y	Y	Y	Y	Y
Education	Y	Y	Y	Y	N
Occupation	Y	Y	Y	Y	Y
Constant	-47.059 (46.275)	0.864 (6.545)	5.354 (6.93)	3.016 (7.109)	6.398 (6.484)
$R^2$	0.187		0.205	0.217	0.177
# of obs.	517	517	449	449	517

The robustness of the correlation -- controls for preferences and beliefs

	(1)	(2)	(3)	(4)	(5)
CCEI	1.379** (0.568)	1.396** (0.568)	1.404** (0.569)	1.214* (0.625)	1.237** (0.623)
Risk tolerance					
Quantitative (experiment)	-0.768 (0.714)	-0.808 (0.711)	-0.766 (0.718)		
Qualitative (survey)		0.017 (0.074)	0.023 (0.076)		
Qualitative (survey) missing		-0.190 (0.335)	-0.162 (0.482)		
Conscientiousness			0.089 (0.072)		
Conscientiousness missing			-0.040 (0.668)		
Longevity expectations					-0.034 (0.040)
Log 2008 household income	0.589*** (0.132)	0.578*** (0.131)	0.572*** (0.133)	0.443*** (0.123)	0.434*** (0.123)
Female	-0.316* (0.177)	-0.310* (0.181)	-0.323* (0.181)	-0.415** (0.186)	-0.417** (0.186)
Partnered	0.655*** (0.181)	0.658*** (0.181)	0.642*** (0.182)	0.686*** (0.204)	0.687*** (0.205)
# of children	0.086 (0.093)	0.087 (0.093)	0.083 (0.093)	0.075 (0.102)	0.083 (0.102)
Age	Y	Y	Y	Y	Y
Education	Y	Y	Y	Y	Y
Occupation	Y	Y	Y	Y	Y
Constant	6.840 (6.361)	6.883 (6.357)	6.496 (6.395)	3.777 (15.258)	4.411 (15.256)
$R^2$	0.179	0.176	0.176	0.163	0.163
# of obs.	517	517	517	414	414

Evaluating alternative measures of *DMQ*

	(1)	(2)	(3)	(4)
CCEI	1.253* (0.712)	1.401* (0.729)	1.269* (0.729)	1.177** (0.583)
CCEI (combined dataset)	0.099 -0.38			
von Gaudecker et al. (2011)			0.927* (0.485)	
Cognitive Reflection Test (CRT)				0.120* (0.071)
CRT missing				-0.203 (0.237)
Log 2008 household income	0.586*** (0.132)	0.388* (0.155)	0.383* (0.154)	0.577*** (0.132)
Female	-0.314* (0.177)	-0.218 (0.212)	-0.207 (0.211)	-0.292* (0.176)
Partnered	0.653*** (0.181)	0.907*** (0.230)	0.926*** (0.228)	0.690*** (0.181)
# of children	0.089 (0.093)	0.105 (0.114)	0.096 (0.113)	0.091 (0.092)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	6.237 (6.424)	10.056 (6.976)	8.355 (6.990)	6.855 (6.464)
$R^2$	0.177	0.225	0.232	0.181
# of obs.	517	326	326	517

The sources of the relationship

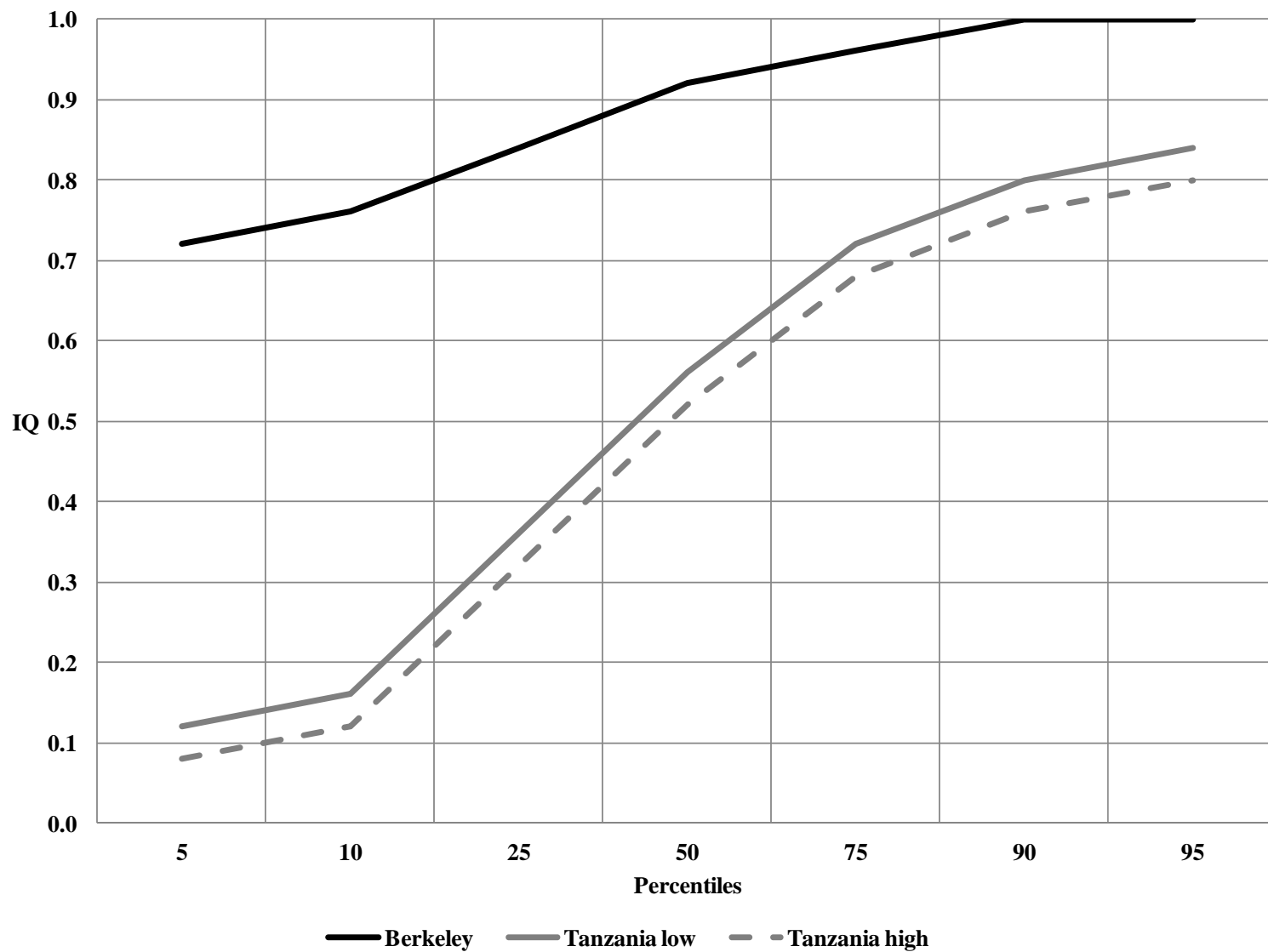
	(1)	(2)	(3)	(4)
	Have checking	Fraction in checking	Have saving	Fraction in saving
CCEI	0.03 (0.032)	-0.098* (0.057)	-0.047 (0.053)	-0.162* (0.097)
Log 2008 household income	0.001 (0.002)	-0.029** (0.013)	0.003 (0.010)	-0.068*** (0.021)
Female	0.007 (0.005)	0.023 (0.020)	0.014 (0.019)	0.038 (0.033)
Partnered	-0.005 (0.004)	-0.031 (0.020)	0.017 (0.022)	-0.054 (0.033)
# of children	0.000 (0.001)	-0.004 (0.010)	-0.025* (0.014)	-0.043*** (0.013)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	0.998*** (0.172)	0.106 (0.822)	1.126 (0.848)	1.448 (1.288)
$R^2$	-0.007	0.021	-0.011	0.083
# of obs.	512	512	502	502



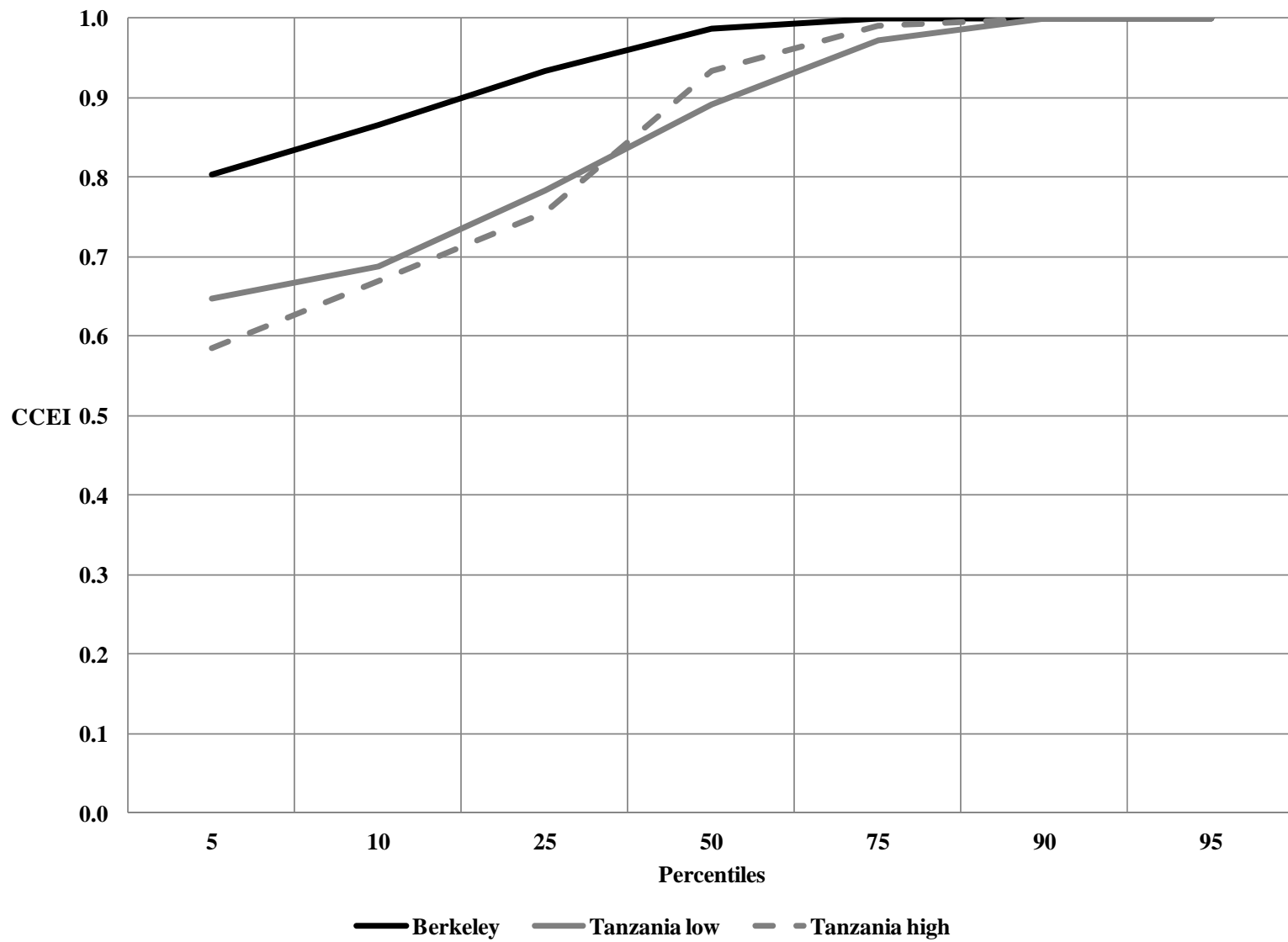
The sources of the relationship (cont.)

	(5)	(6)	(7)	(8)
	Have stocks	Fraction in stocks	Have a house	Fraction in house
CCEI	0.167 (0.163)	0.001 (0.050)	0.352** (0.152)	0.324** (0.129)
Log 2008 household income	0.148*** (0.031)	0.013 (0.009)	0.134*** (0.029)	0.096*** (0.024)
Female	0.007 (0.050)	0.009 (0.013)	-0.038 (0.050)	-0.066 (0.043)
Partnered	0.005 (0.049)	-0.007 (0.014)	0.207*** (0.051)	0.127*** (0.044)
# of children	0.003 (0.026)	0.000 (0.007)	0.048** (0.020)	0.063*** (0.019)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	-3.152* (1.856)	-0.317 (0.398)	-1.047 (1.760)	-1.151 (1.419)
$R^2$	0.079	0.002	0.148	0.123
# of obs.	514	514	479	479

# Is there a development gap in rationality (IQ)?



# Is there a development gap in rationality (CCEI)?



**Substantive rationality**

## Substantive rationality

One approach is to posit the “kinky” preference ordering implied by Afriat’s theorem and go no further in attempting to rationalize the data.

Q. What kind of preferences could give rise to these choices?

A. The best candidate would be preferences generated by a *rank-dependent utility function* (Quiggin, 1993).

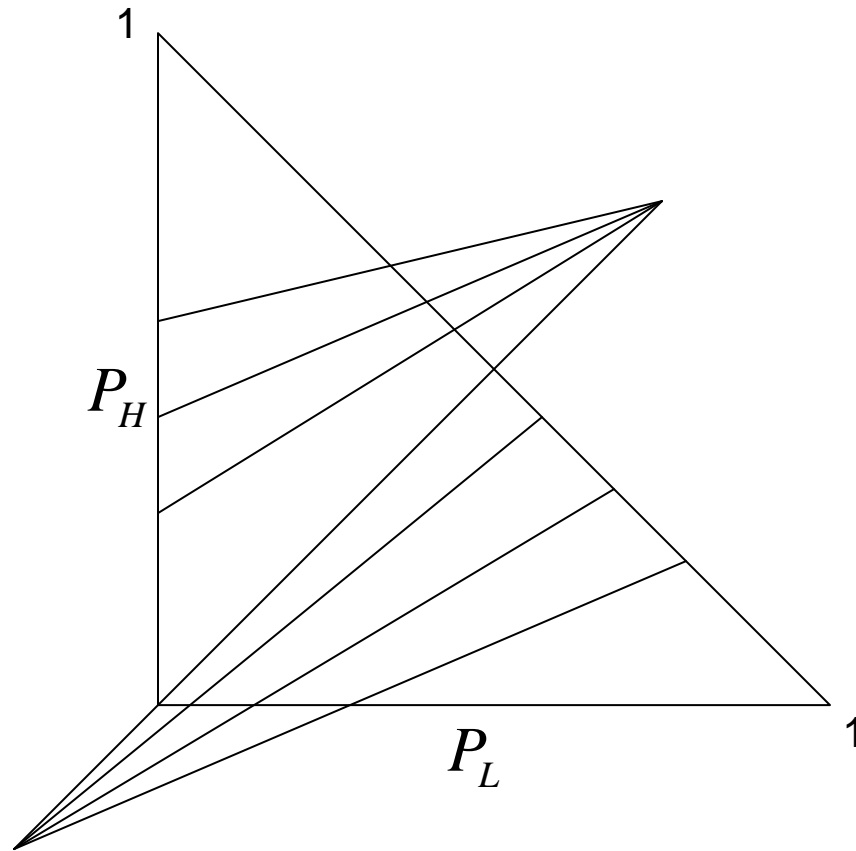
The theory of Gul (1991) implies that the utility function over portfolios takes the form

$$\min \{ \alpha u(x_1) + u(x_2), u(x_1) + \alpha u(x_2) \},$$

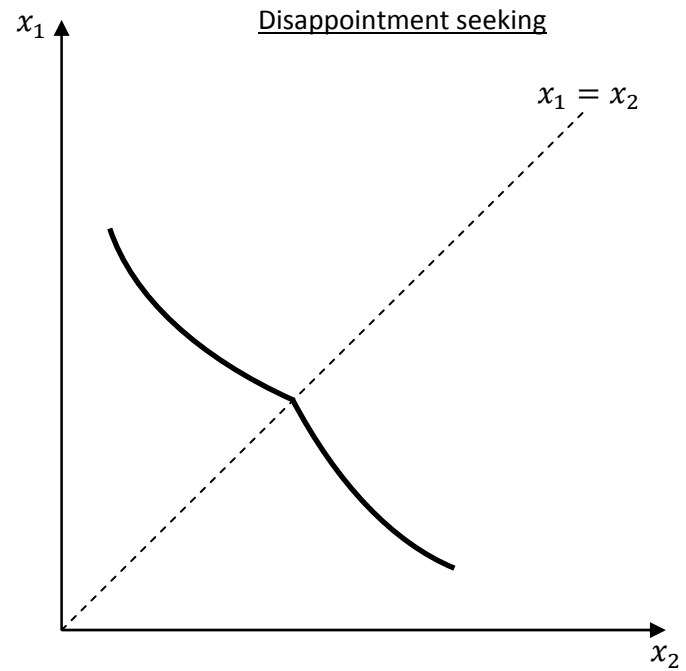
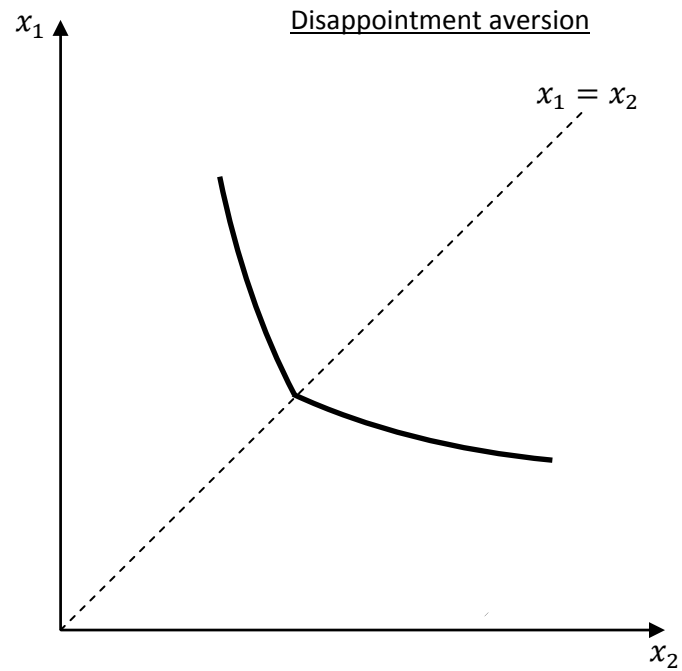
where  $\alpha \geq 1$  measures loss/disappointment aversion and  $u(\cdot)$  is the utility of consumption in each state.

If  $\alpha > 1$  there is a kink at the point where  $x_1 = x_2$  and if  $\alpha = 1$  we have the standard EUT representation.

The indifference map of Gul (1991) in the Marschak-Machina triangle

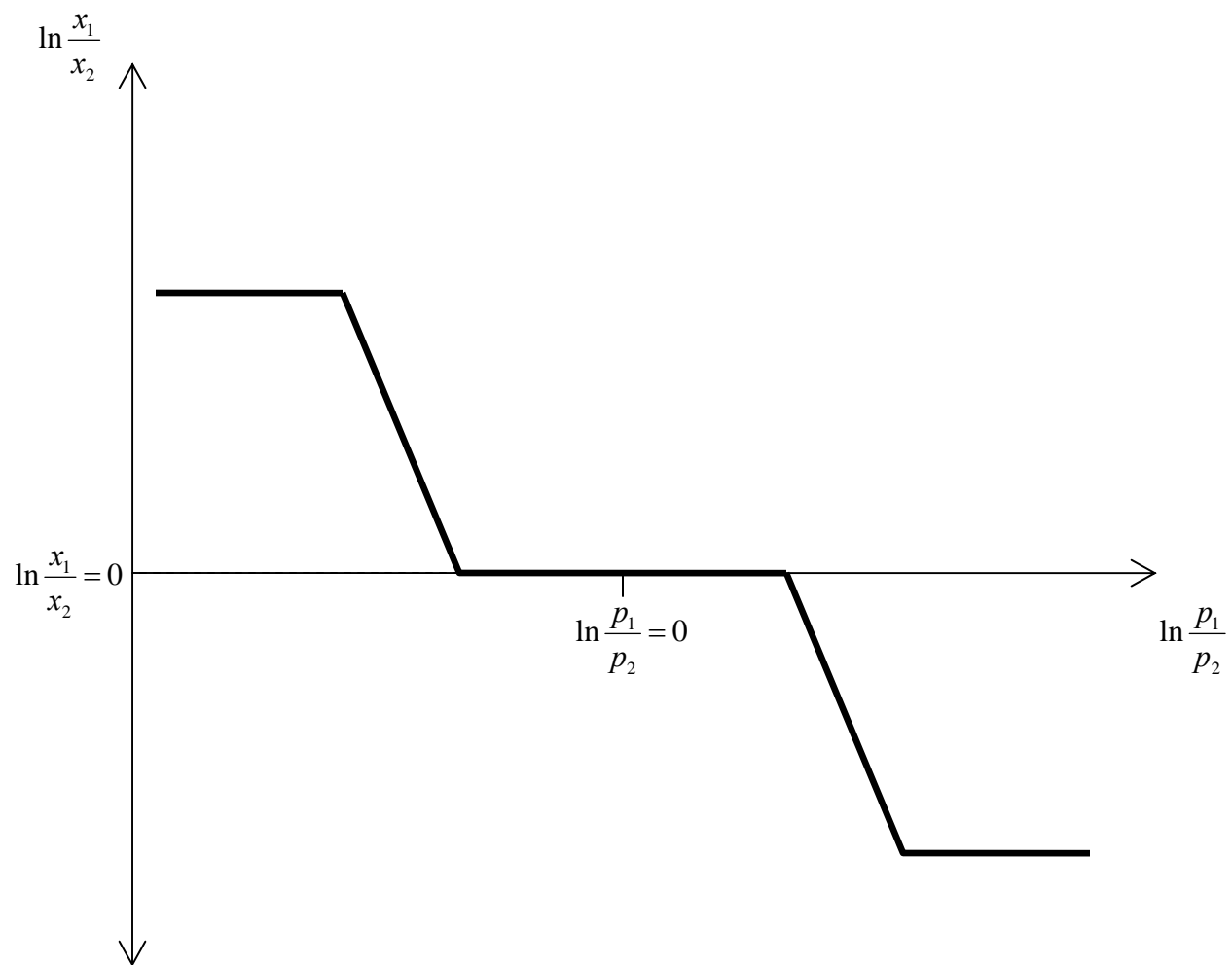


## The indifference map of Gul (1991)

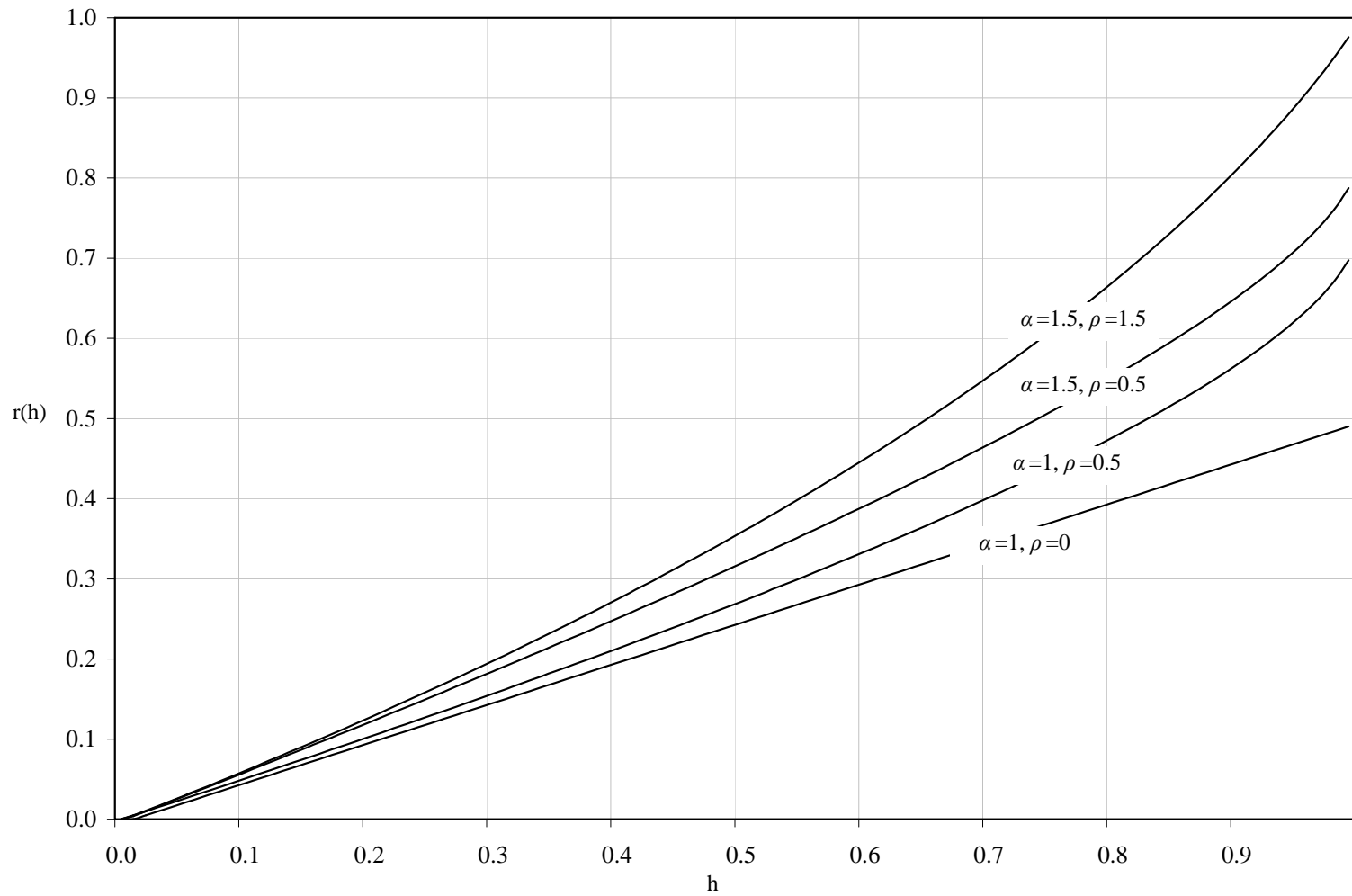




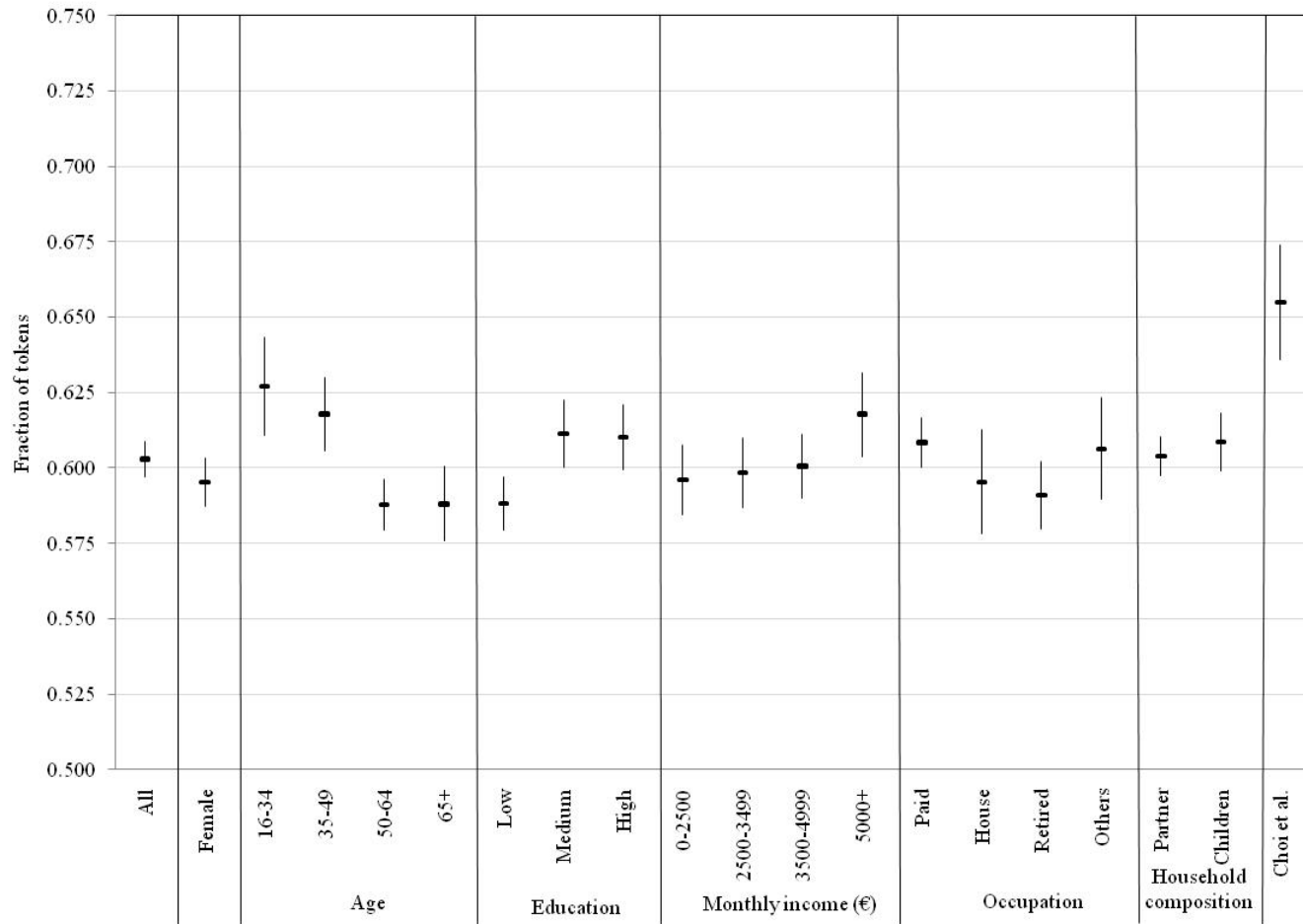
## An illustration of the derived demand



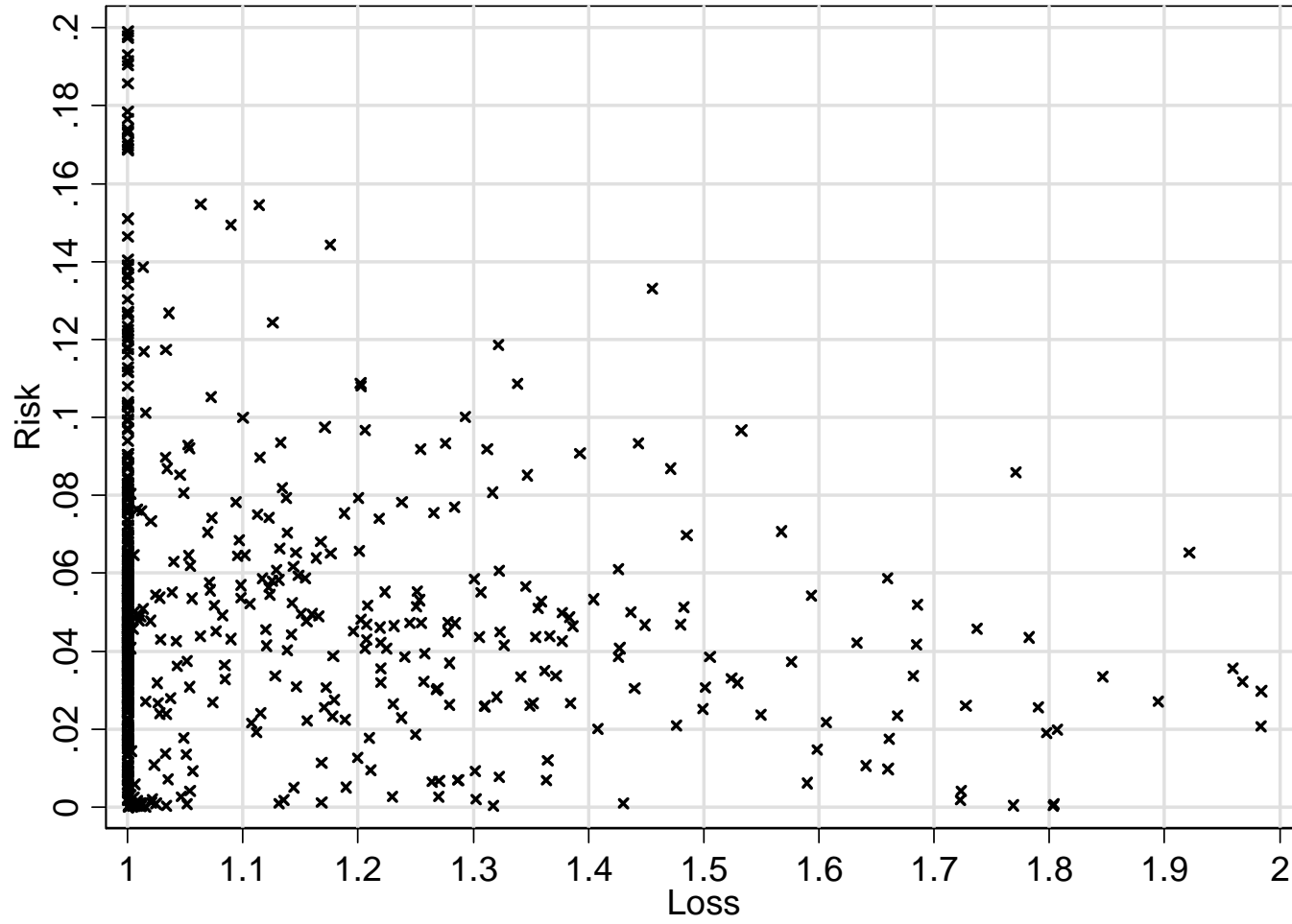
# The risk premium for different levels of risk and disappointment aversion



# Risk aversion – the fraction of tokens allocated to the cheaper asset



# Scatterplot of the CARA NLLS estimates



## **Ambiguity aversion**

## **Ambiguity aversion**

- The distinction between settings with risk and ambiguity dates back to at least the work of Knight (1921).
- Ellsberg (1961) countered the reduction of subjective uncertainty to risk with several thought experiments.
- A large theoretical literature (axioms over preferences) has developed models to accommodate this behavior.

## Experiments à la Ellsberg

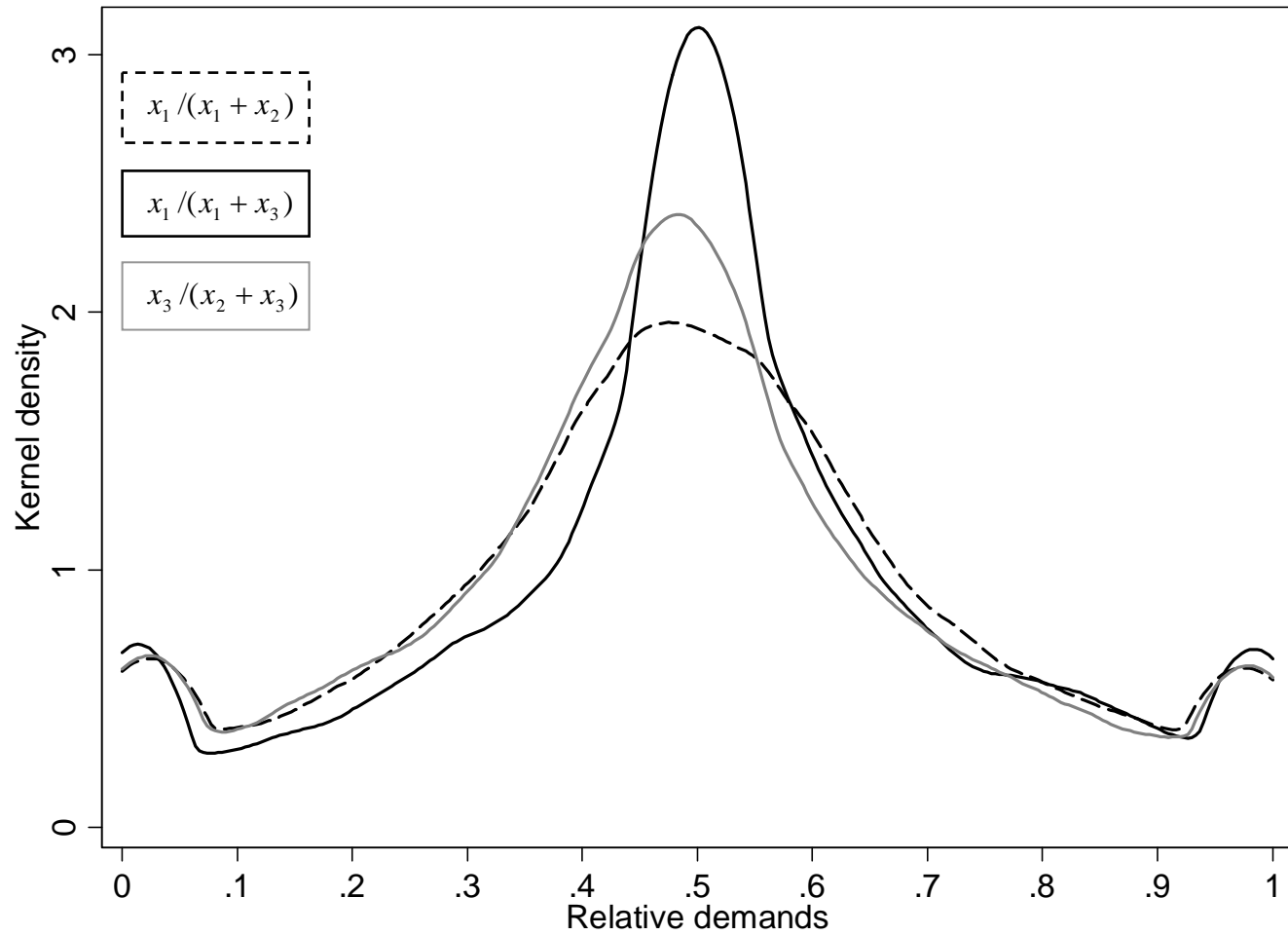
Consider the following four two-color Ellsberg-type urns (Halevy, 2007):

- I. 5 red balls and 5 black balls
- II. an unknown number of red and black balls
- III. a bag containing 11 tickets with the numbers 0-10; the number written on the drawn ticket determines the number of red balls
- IV. a bag containing 2 tickets with the numbers 0 and 10; the number written on the drawn ticket determines the number of red balls

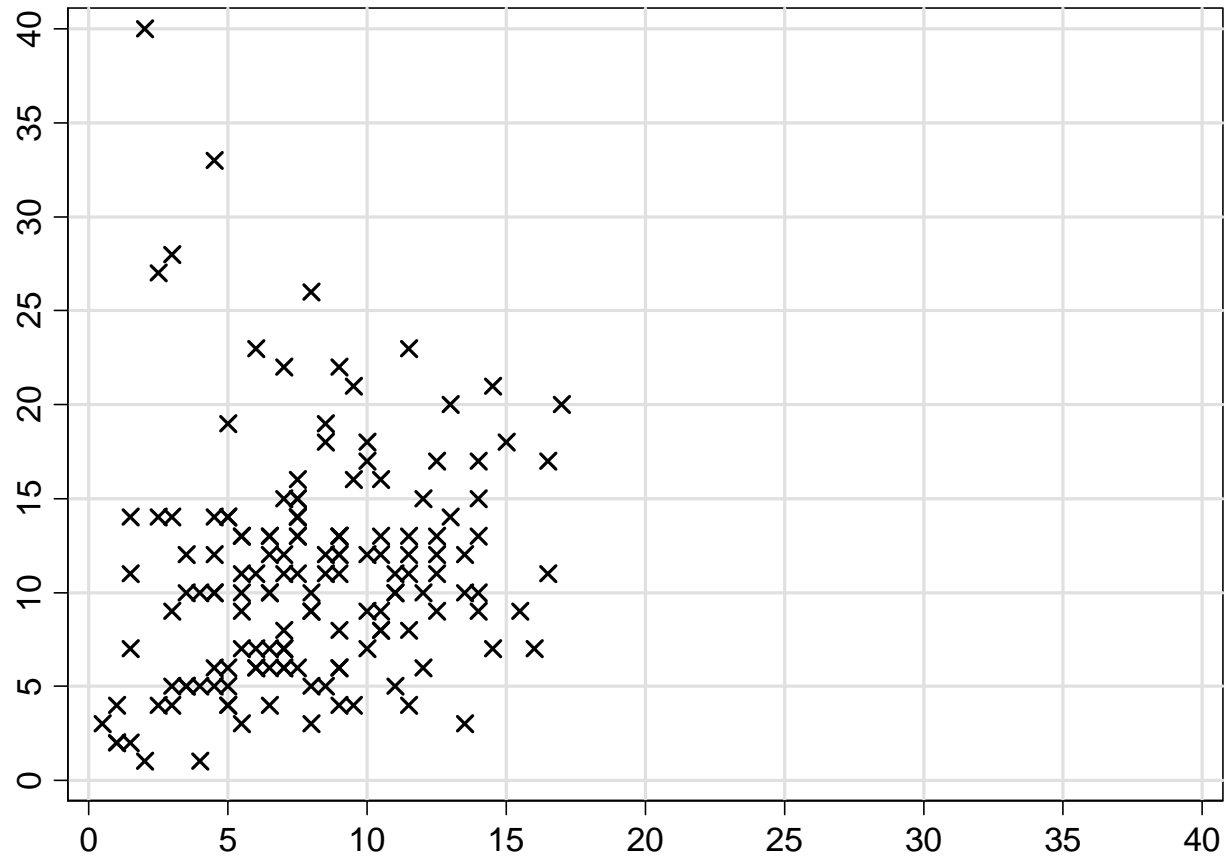
- A cleverly designed experiment that allows distinguishing between four models of ambiguity aversion — SEU, MEU, REU and RNEU.
- The different models of ambiguity aversion generate different predictions about how the urns will be ordered.
- For each subject, there will be a unique model that predicts (is consistent with) the subject's reservation values.
- No single model predicts all the observed behaviors, and all models are represented in the pool of subjects.



## The distribution of relative demands



**The number of diagonal portfolios by subject  
(ambiguity - vertical axis / loss - horizontal axis)**



## Parametric analysis

There is a variety of theoretical models of attitudes toward risk and ambiguity, but they all give rise to one of two main specifications:

- [1] kinked specification rationalized by  $\alpha$ -Maxmin Expected Utility ( $\alpha$ -MEU), Choquet Expected Utility, or Contraction Expected Utility.
- [2] smooth specification, based on the class of Recursive Expected Utility (REU) models.

The kinked specification has the form

$$\alpha \left[ \frac{2}{3}u(\min\{x_1, x_3\}) + \frac{1}{3}u(x_2) \right] \\ + (1 - \alpha) \left[ \frac{2}{3}u(\max\{x_1, x_3\}) + \frac{1}{3}u(x_2) \right],$$

where  $\alpha$  is the ambiguity parameter.

The indifference curves have a “kink” at all unambiguous portfolios where  $x_1 = x_3$ .

The general form of the smooth specification is

$$\int_{\Delta S} \varphi \left( \int_S u(x_s) d\pi(s) \right) d\mu(\pi),$$

where  $\mu \in \Delta(\Delta(S))$  is a (second-order) distribution over possible priors  $\pi$  on  $S$  and  $\varphi : u(\mathbf{R}_+) \rightarrow \mathbf{R}$  is a possibly nonlinear transformation over expected utility levels.

The REU models are based a cardinal utility indicator – the preferences generated are not independent of a change in the scale of utility.

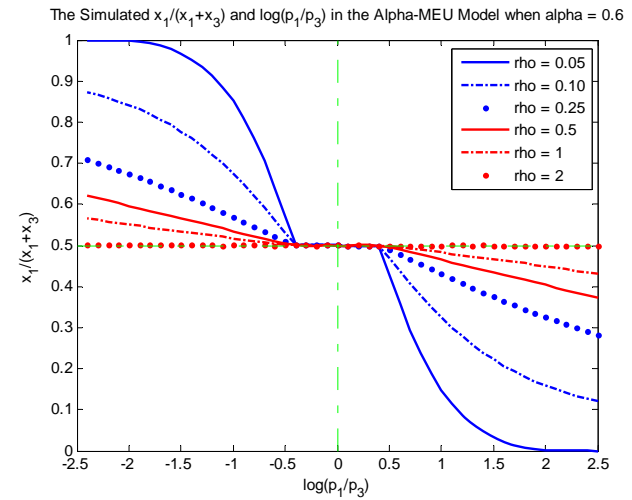
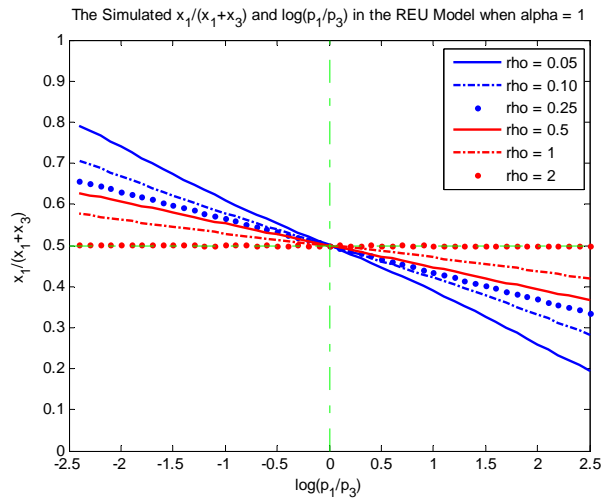
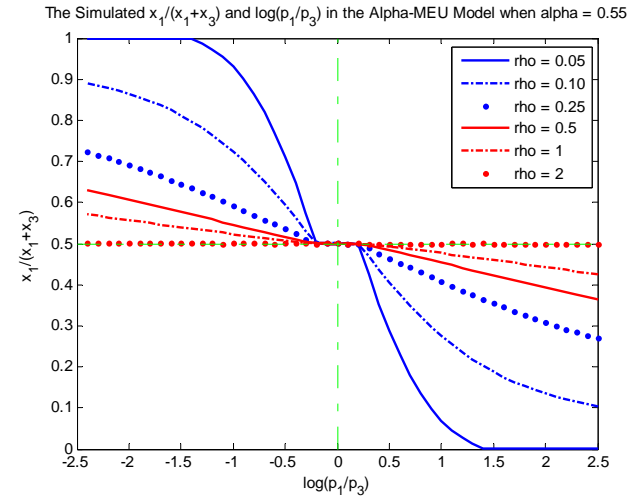
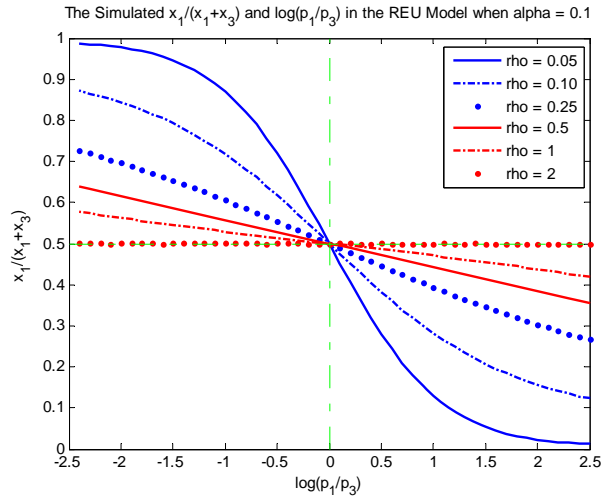
To clarify, suppose risk preferences are represented by a von Neumann-Morgenstern utility function  $u(x)$  with constant absolute risk aversion (CARA)

$$u(x) = -Ae^{-\rho x},$$

where  $x$  is the number of tokens,  $A$  a scale parameter, and  $\rho$  is the coefficient of absolute risk aversion.

The concavity of the transformation  $\varphi$  implies that the ranking of uncertain prospects will not be invariant to changes in  $A$  (cannot identify  $\varphi$  and  $A$  separately).

# Smooth (left) and kinked (right) specifications



## Estimation

→ Our first parametric assumption is that risk preferences are represented by a von Neumann-Morgenstern utility function with CARA,

$$u(x) = -e^{-\rho x}.$$

→ We restrict, WLOG, the parameters so that preferences are always risk and ambiguity averse –  $\rho \geq 0$  in both specifications,  $1/2 \leq \alpha \leq 1$  in the kinked specification and  $0 \leq \alpha$  in the smooth specification.

→ For each subject  $n$  and for each specification, we generate estimates of the ambiguity and risk aversion parameters,  $\hat{\alpha}_n$  and  $\hat{\rho}_n$ , using nonlinear least squares (NLLS).



## Econometric results

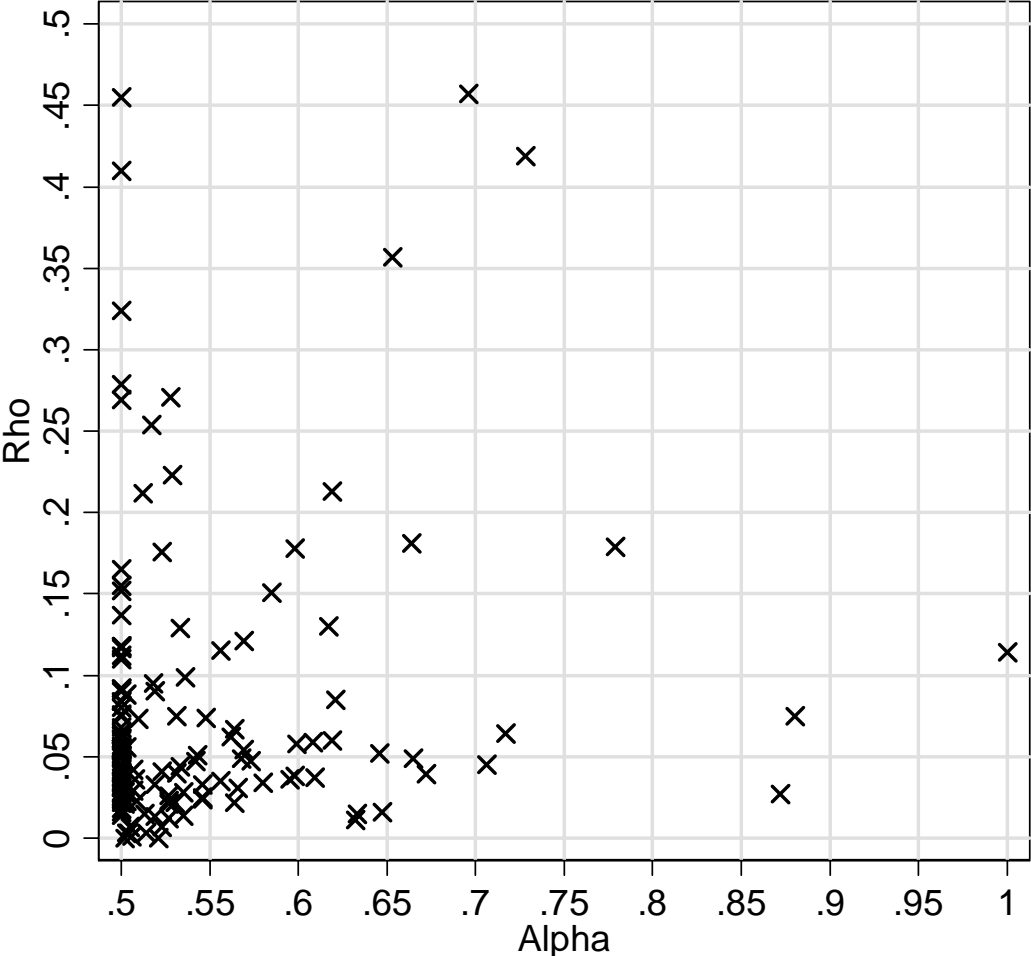
### Kinked specification

56 subjects (38.9%) have non-kinky preferences  $\hat{\alpha}_n \approx 1/2$ . We cannot reject the hypothesis that  $\alpha_n = 1/2$  for a total of 109 subjects (75.7%) at the 5% significance level.

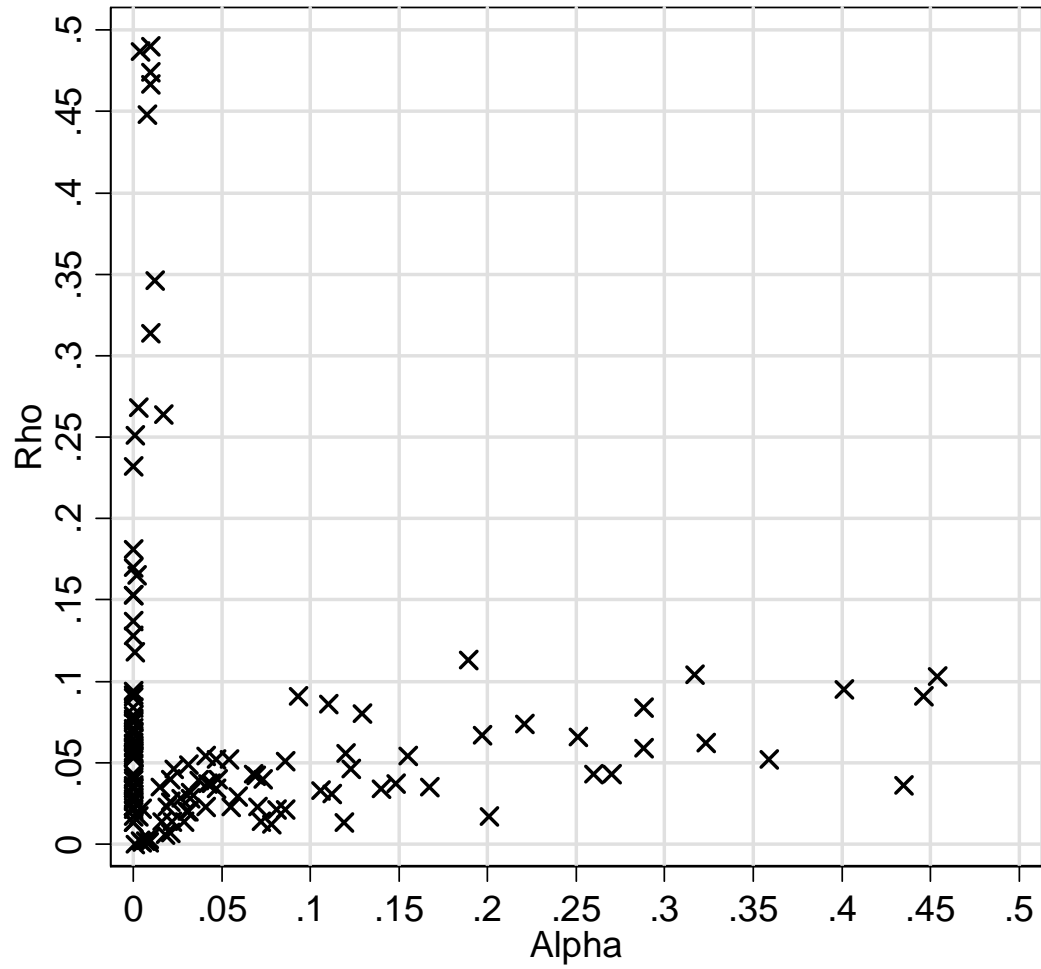
### Smooth specification

44 subjects (30.6%) have  $\hat{\alpha}_n \approx 0$ , indicating ambiguity neutrality. We cannot reject the hypothesis that  $\alpha_n = 0$  for a total of 97 subjects (67.4%) at the 5% significance level .

# Scatterplot of the estimated parameters – kinked specification



# Scatterplot of the estimated parameters – smooth specification



- There is the strong tendency for subjects to equate the demand for the securities that pay off in the ambiguous states,  $x_1$  and  $x_3$ .
- This feature of the data is consistent with the kinked specification but not with the smooth specification.
- But the tendency to equate  $x_1$  and  $x_3$  cannot be attributed solely to ambiguity aversion.
- In the first place, there is also strong tendency to equate the demand for the securities that pay off in any pair of states.

## A model of ambiguity aversion and loss/disappointment aversion

- If both loss aversion and ambiguity aversion are present in the data, we need a structural model in order to disentangle the two effects.
- In order to allow for kinks at portfolios where  $x_s = x_{s'}$  for any  $s \neq s'$ , we make use of the rank-dependent utility (RDU) model of Quiggin (1982).
- This is a generalization of the SEU model that replaces probabilities with *decision weights* when calculating the value of expected utility.
- In Quiggin (1982), the decision weight of each payout depends only on its (known) probability and its ranking position.

Following  $\alpha$ -MEU, we assume that the unknown probabilities  $\pi_1$  and  $\pi_3$  are skewed using the weights  $\alpha$  and  $1 - \alpha$  :

$$x_{\min} = \min\{x_1, x_3\}$$

is given a probability weight  $\frac{2}{3}\alpha$  and

$$x_{\max} = \max\{x_1, x_3\}$$

is given probability weight  $\frac{2}{3}(1 - \alpha)$  where the parameter  $\frac{1}{2} \leq \alpha \leq 1$  measures the degree of ambiguity aversion.

The utility of a portfolio  $\mathbf{x} = (x_1, x_2, x_3)$  takes the form

I.  $x_2 \leq x_{\min}$

$$\beta_1 u(x_2) + \beta_2 u(x_{\min}) + (1 - \beta_1 - \beta_2) u(x_{\max})$$

II.  $x_{\min} \leq x_2 \leq x_{\max}$

$$\beta_3 u(x_{\min}) + (\beta_1 + \beta_2 - \beta_3) u(x_2) + (1 - \beta_1 - \beta_2) u(x_{\max})$$

III.  $x_{\max} \leq x_2$

$$\beta_3 u(x_{\min}) + \beta_4 u(x_{\max}) + (1 - \beta_3 - \beta_4) u(x_2)$$

where

$$\begin{aligned}\beta_1 &= w\left(\frac{1}{3}\right), \\ \beta_2 &= w\left(\frac{2}{3}\alpha + \frac{1}{3}\right) - w\left(\frac{1}{3}\right), \\ \beta_3 &= w\left(\frac{2}{3}\alpha\right), \\ \beta_4 &= w\left(\frac{2}{3}\right) - w\left(\frac{2}{3}\alpha\right),\end{aligned}$$

and the mapping from the four parameters  $\beta_1, \dots, \beta_4$  to two parameters  $\delta$  and  $\gamma$  is as follows:

$$\begin{aligned}\beta_1 &= \frac{1}{3} + \gamma, \\ \beta_2 &= \frac{1}{3} + \delta, \\ \beta_3 &= \frac{1}{3} + \gamma + \delta, \\ \beta_4 &= \frac{1}{3} - \delta.\end{aligned}$$



The parameter  $\delta$  measures the degree of ambiguity aversion and the parameter  $\gamma$  measures the degree of loss aversion:

- $\delta \geq 0$  and  $\gamma = 0$  – kinked specification
- $\delta = 0$  and  $\gamma \geq 0$  – loss/disappointment aversion (Gul, 1991)
- $\delta = 0$  and  $\gamma = 0$  – standard SEU representation.

The indifference curves will have kinks where  $x_s = x_{s'}$  and agents will choose portfolios that satisfy  $x_s = x_{s'}$  for a non-negligible set of prices.

## Econometric results

The vast majority of the subjects are well described by the loss- and ambiguity-neutral SEU model. The remainder appear to have a significant degree of loss and/or ambiguity aversion

		Ambiguity		Total
		Neutral	Averse	
Loss	Neutral	60.4	16.7	77.1
	Averse	18.1	4.9	22.9
Total		78.5	21.5	

There is considerable heterogeneity in both  $\hat{\delta}_n$  or  $\hat{\gamma}_n$  and that their values are not correlated ( $r^2 = 0.029$ ).

**Procedural rationality**

## Procedural rationality

- How subjects come to make decisions that are consistent with an underlying preference ordering?
- Boundedly rational individuals use *heuristics* in their attempt to maximize an underlying preference ordering.
  - There is a distinction between the “revealed” preference ordering and the “true” underlying preference ordering.
  - Preferences have an EU representation, even though revealed preferences appear to be non-EU.

Suppose there are  $\ell$  states of nature and  $\ell$  associated Arrow securities and that the agent's behavior is represented by the decision problem

$$\begin{aligned} \max \quad & u(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} \in \mathbf{B}(\mathbf{p}) \cap \mathcal{A} \end{aligned}$$

where  $\mathbf{B}(\mathbf{p})$  is the budget set and  $\mathcal{A}$  is the set of portfolios corresponding to the various archetypes the agent uses to simplify his choice problem.

The only restriction we have to impose is that  $\mathcal{A}$  is a pointed cone (closed under multiplication by positive scalars), which is satisfied if  $\mathcal{A}$  is composed of *elimination* and *equalization* (Rubinstein, 1987).

We can derive the following properties of the agent's demand:

1. Let  $\mathbf{p}^k$  denotes the  $k$ -th observation of the price vector and

$$\mathbf{x}^k \in \arg \max \{u(\mathbf{x}) : \mathbf{x} \in \mathbf{B}(\mathbf{p}^k) \cap \mathcal{A}\}$$

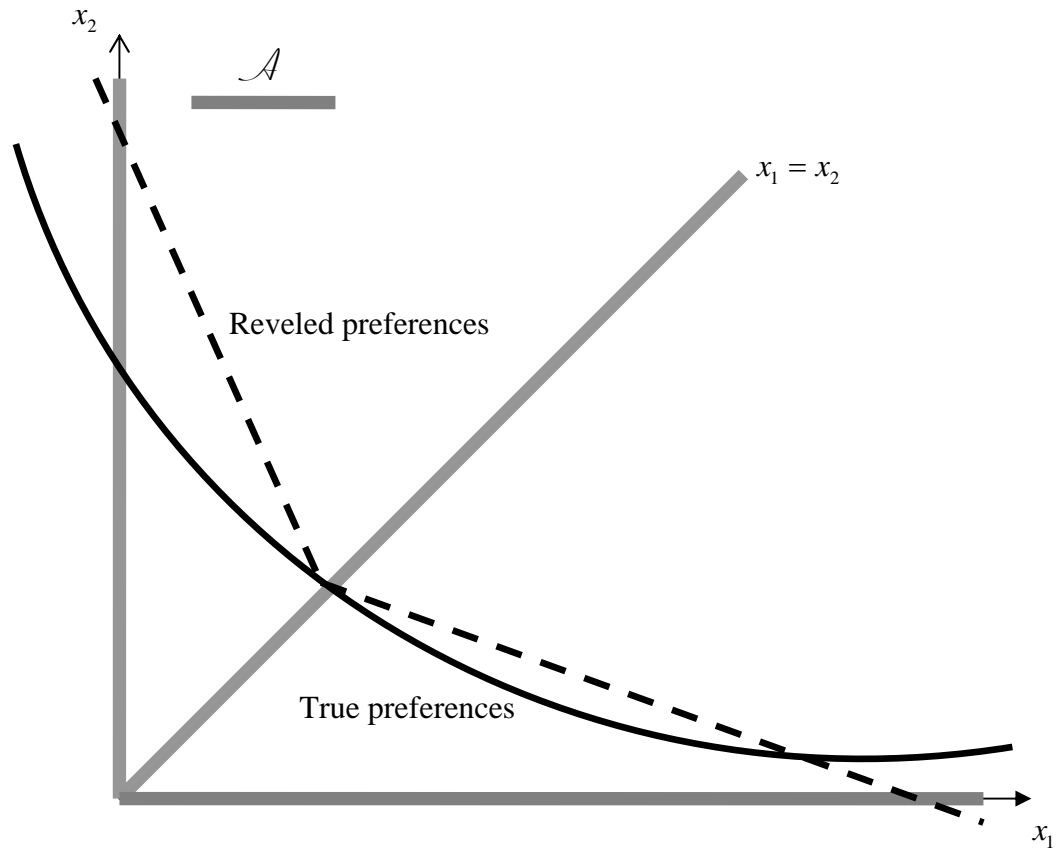
denotes the associated portfolio. Then the data  $\{\mathbf{p}^k, \mathbf{x}^k\}$  satisfy GARP.

2. There exists a utility function  $u^*(\mathbf{x})$  such that for any price vector  $\mathbf{p}$ ,

$$\mathbf{x}^* \in \arg \max \{u(\mathbf{x}) : \mathbf{x} \in \mathbf{B}(\mathbf{p}) \cap \mathcal{A}\}$$

$\Leftrightarrow$

$$\mathbf{x}^* \in \arg \max \{u^*(\mathbf{x}) : \mathbf{x} \in \mathbf{B}(\mathbf{p})\}.$$



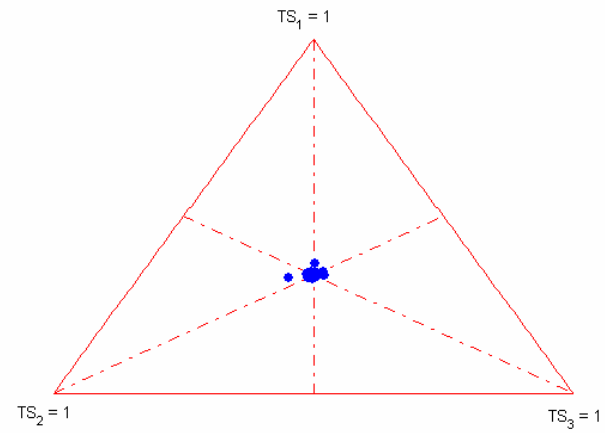
## Archetypes and polytypes

- We identify a finite number of stylized behaviors, which collectively pose a challenge to decision theory.
- We call these basic behaviors *archetypes*. We also find mixtures of archetypal behaviors, which we call *polytypes*.
- The archetypes account for a large proportion of the data set and play a role in the behavior of most subjects.
- The combinations of types defy any of the standard models of risk aversion.



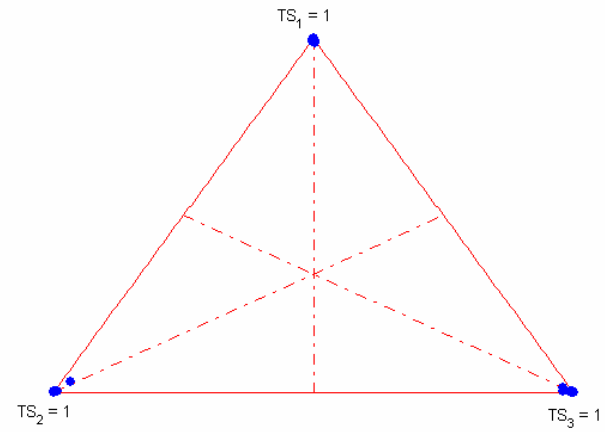
# Center

The Token Shares in the Risk Treatment for Subject ID 922



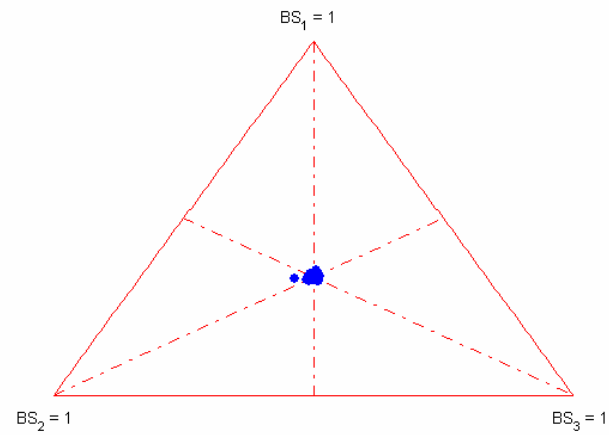
# Vertex

The Token Shares in the Risk Treatment for Subject ID 913



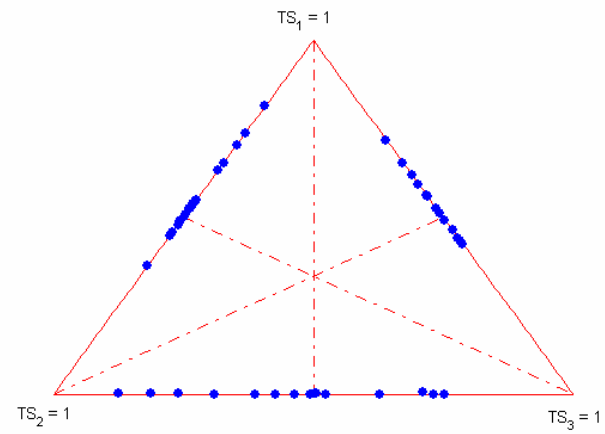
# Centroid (budget shares)

The Budget Shares in the Risk Treatment for Subject ID 1001



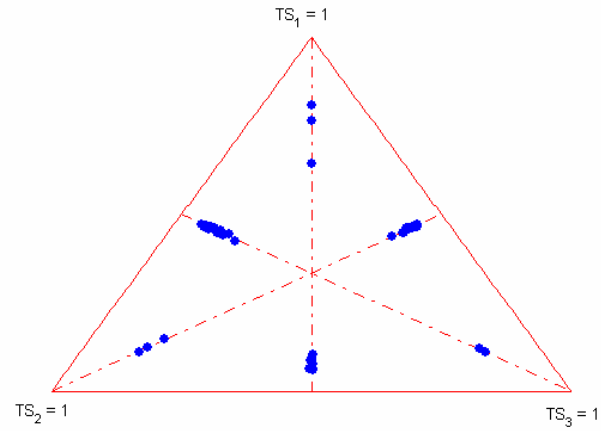
# Edge

The Token Shares in the Risk Treatment for Subject ID 905



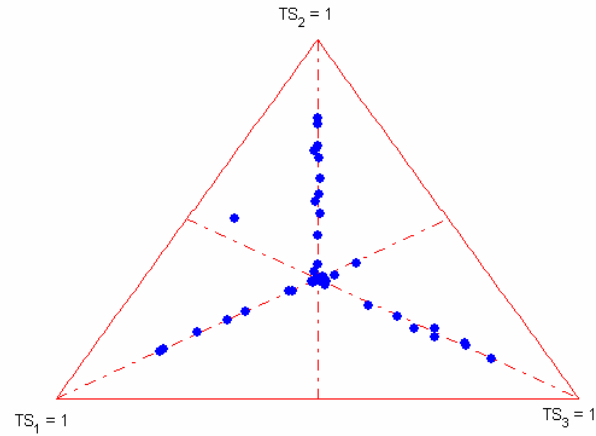
# Bisector

The Token Shares in the Risk Treatment for Subject ID 1003



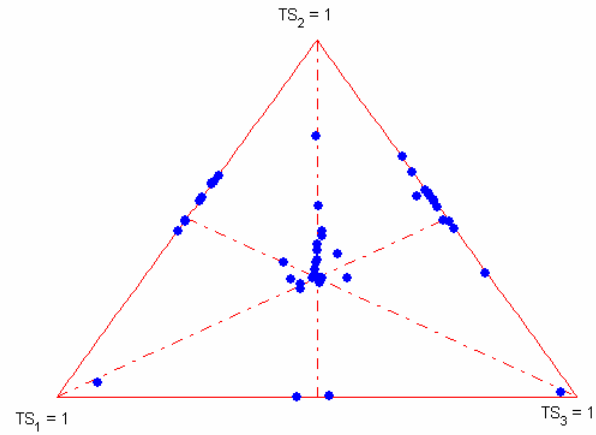
# Center and bisector

The Token Shares in the Risk Treatment for Subject ID 1007



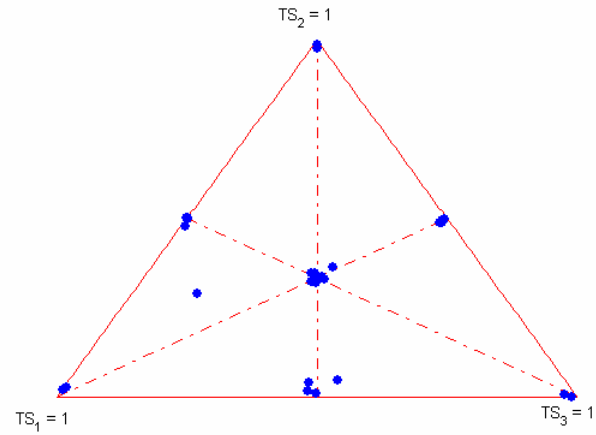
# Edge and bisector

The Token Shares in the Risk Treatment for Subject ID 1018



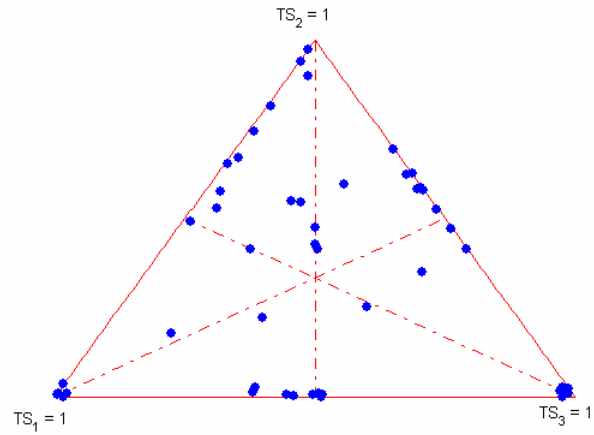
# Center, vertex, and edge

The Token Shares in the Risk Treatment for Subject ID 1023



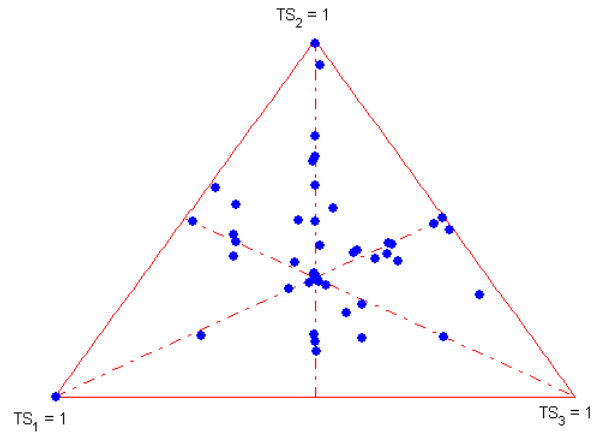
# Vertex and edge

The Token Shares in the Risk Treatment for Subject ID 914



# Center and bisector

The Token Shares in the Risk Treatment for Subject ID 1008

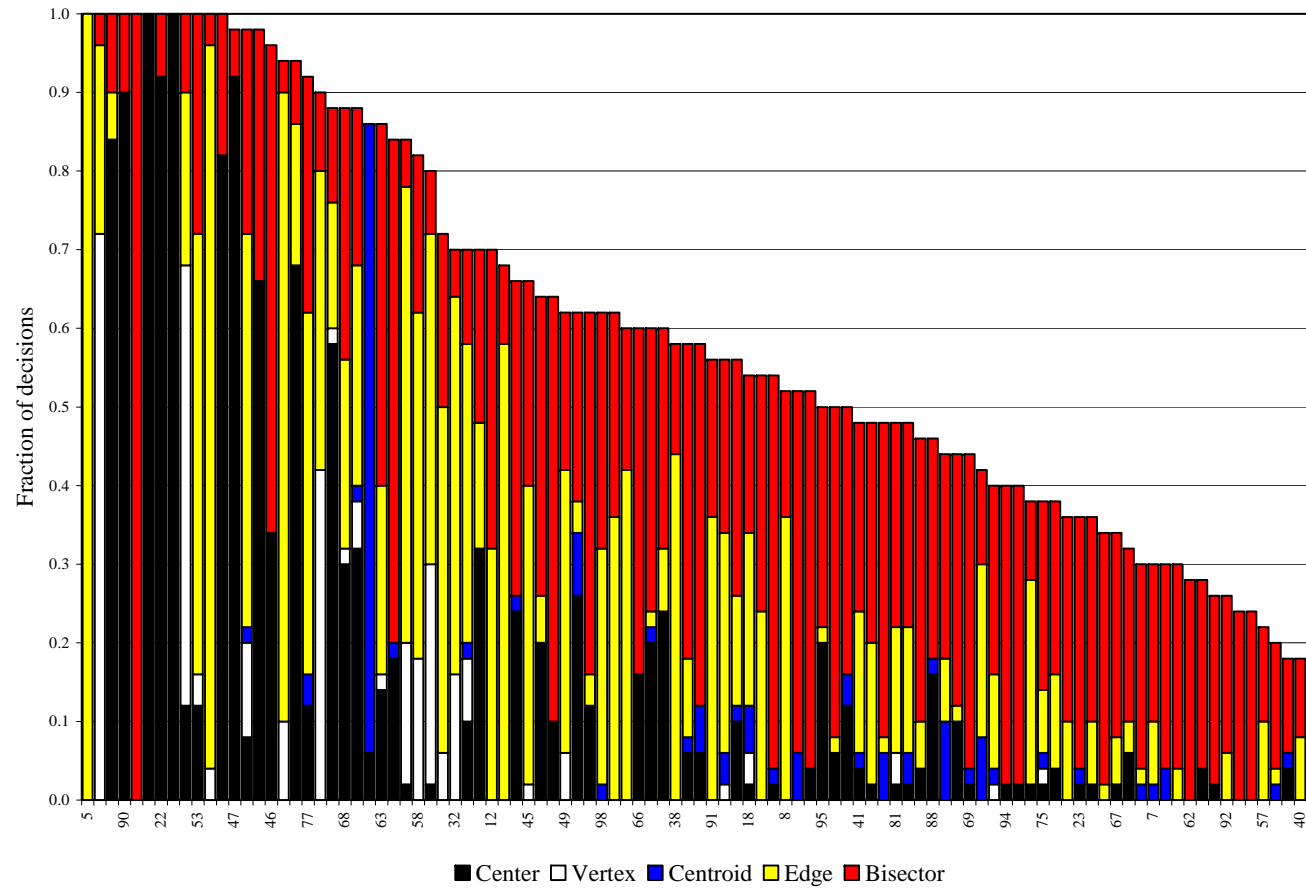


**The aggregate distribution of archetypes  
for different token confidence intervals**

	<b>Center</b>	<b>Vertex</b>	<b>Centroid</b>	<b>Edge</b>	<b>Bisector</b>	<b>All</b>
<b>0.1</b>	0.005	0.003	0.000	0.019	0.083	0.110
<b>0.25</b>	0.061	0.004	0.002	0.083	0.187	0.337
<b>0.5</b>	0.093	0.011	0.007	0.139	0.215	0.466
<b>1</b>	0.134	0.032	0.019	0.165	0.252	0.602
<b>2.5</b>	0.185	0.064	0.049	0.186	0.285	0.769



# The distribution of archetypes, by subject (one token confidence interval)





## Substantive rationality

What kind of preferences could give rise to these choices? The best candidate would be preferences generated by a *rank-dependent utility function* (Quiggin, 1993):

$$U(x_1, x_2, x_3) = \beta_L u(x_L) + \beta_M u(x_M) + \beta_H u(x_H)$$

where  $u(\cdot)$  is a von Neumann-Morgenstern utility function.

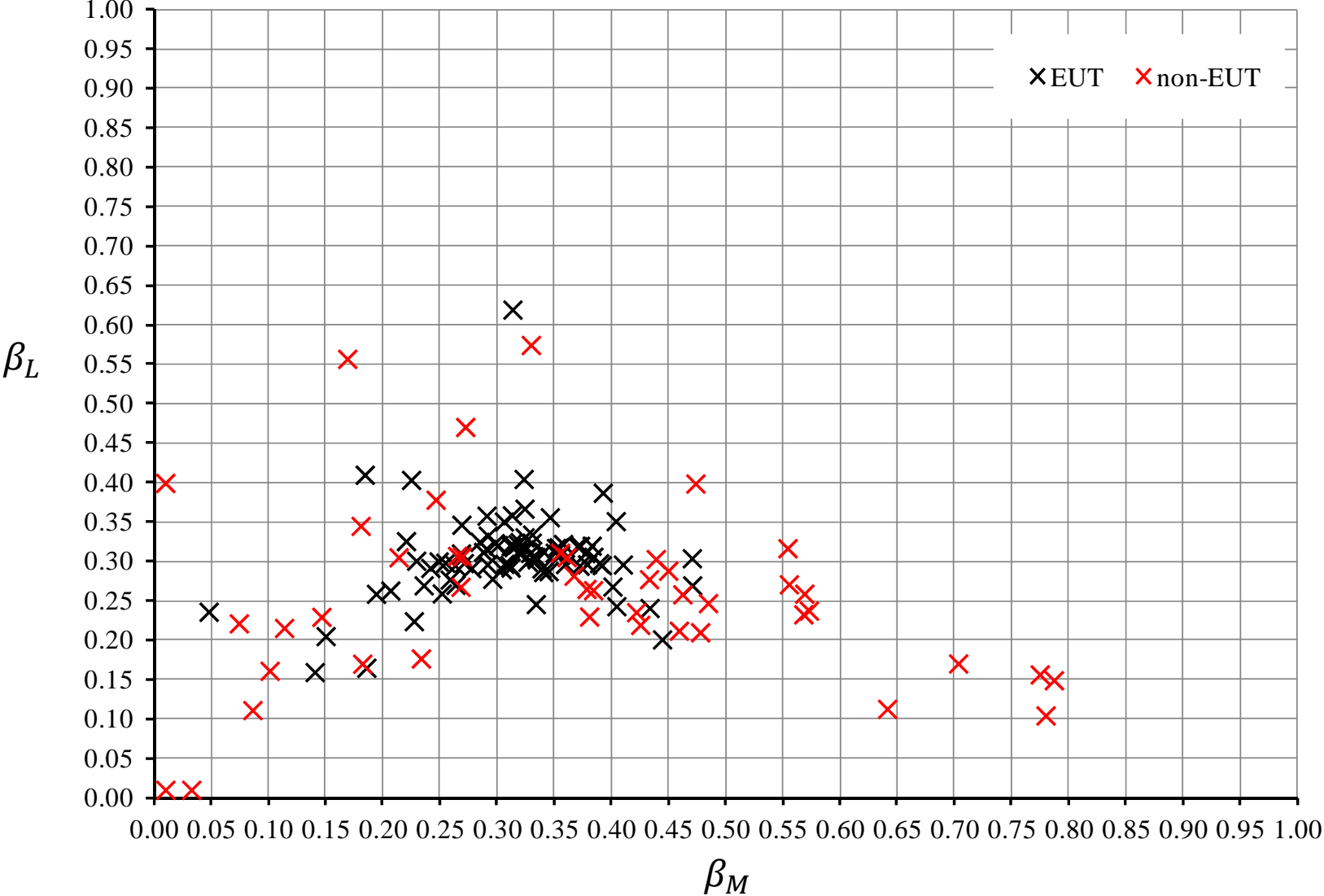
This formulation encompasses standard EU representation as a special case ( $\beta_L = \beta_M = \beta_H$ ).

## RDU estimation results

We assume that risk preferences are represented by a von Neumann-Morgenstern utility function  $u(x) = -e^{-\rho x}$  with constant absolute risk aversion (CARA).

$\beta_L = \beta_M = \beta_H$	66.2%
$\beta_L > \beta_M > \beta_H$	18.8%
$\beta_L < \beta_M < \beta_H$	3.0%
$\beta_L > \beta_M < \beta_H$	4.5%
$\beta_L < \beta_M > \beta_H$	7.5%

Scatterplot of the estimated RDU parameters



## **Time preferences**

- The long-standing interest: time discount rates decline as tradeoffs are pushed into the temporal distance.
  - Subjects often choose the larger and later of two rewards when both are distant in time, but prefer the smaller and earlier one as both rewards draw nearer to the present.
- Interpreted as non-constant time discounting, these preference reversals have important implications.
  - Under standard assumptions, non-constant time discounting implies time-inconsistency – self-control problems and a demand for commitment thus emerge.

## Experimental design

Different from the prototype experiment regarding risk:

- Choice problems (dataset  $E$ ) involve tradeoffs between income in a few days and income 60 days after that.
- Choice problems (dataset  $L$ ) involve tradeoffs between income in 60 days and income another 60 days after that.

The exact same budget sets – randomly ordered.

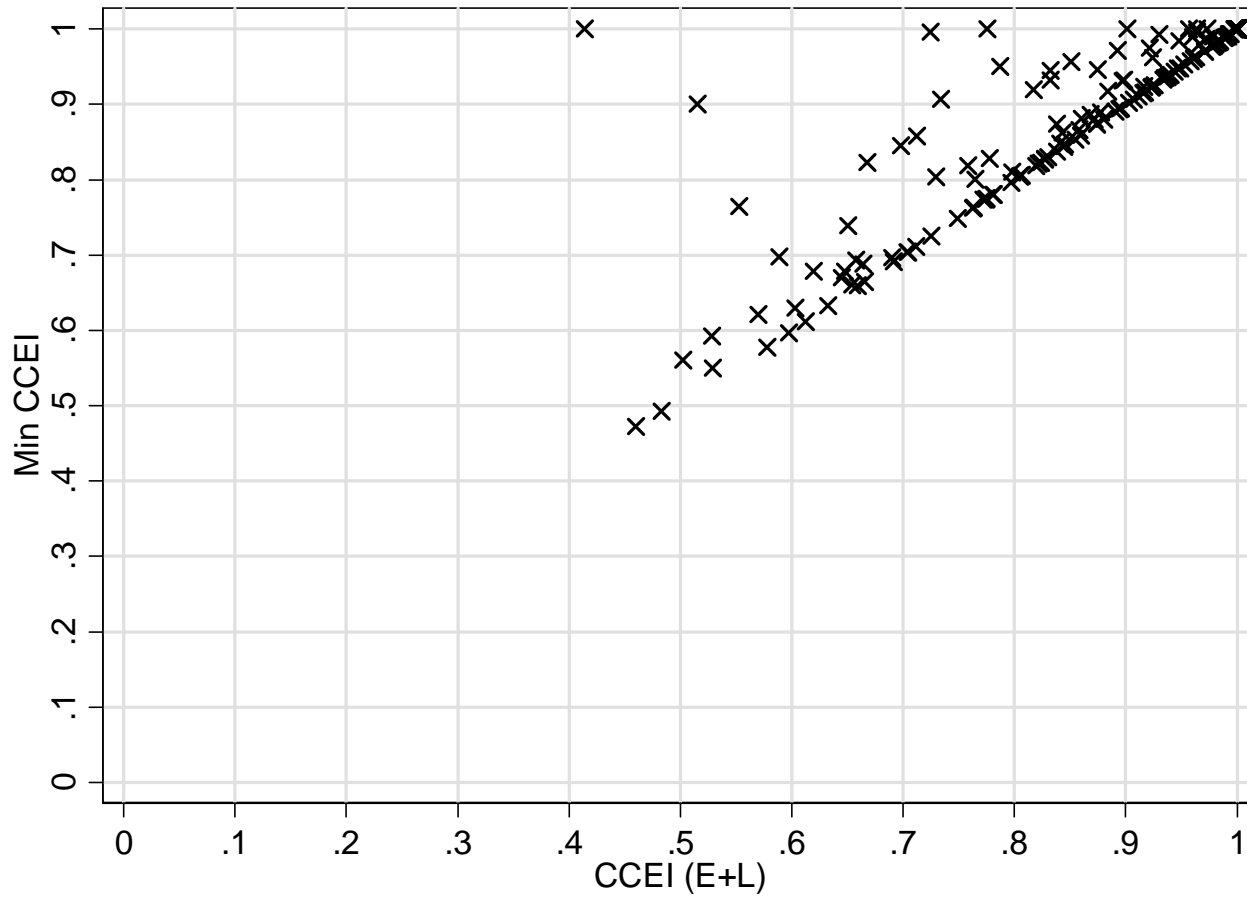
## A GARP test of dynamic preference reversals

A non-parametric econometric approach for testing whether preferences in the two treatments are the same (for a given subject):

- Combine dataset  $E$  with dataset  $L$ .
- Compute the CCEI for this combined dataset.
- Compare that number to the min CCEI in each of the separate treatments.

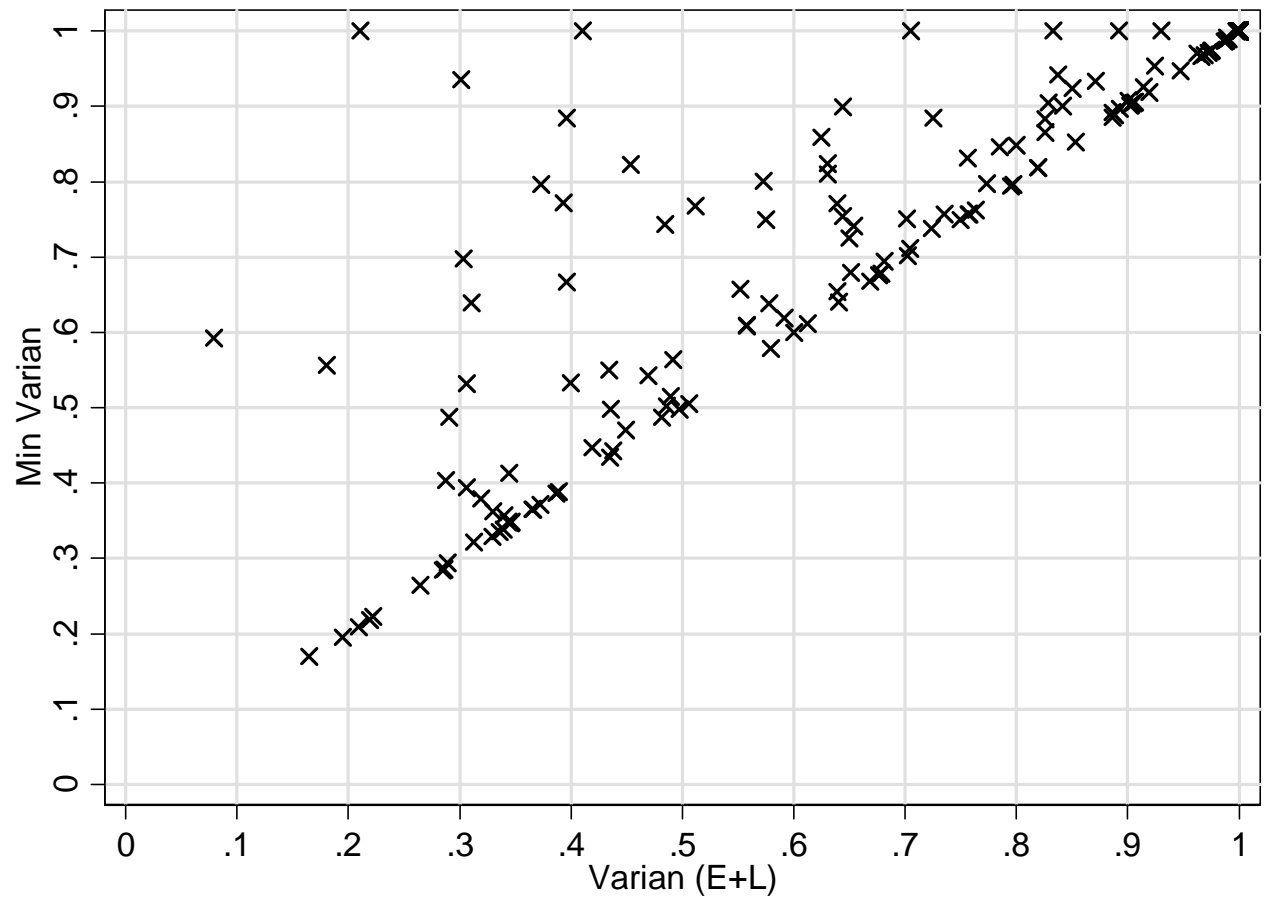
The CCEI for the combined dataset can be no bigger than the minimum of the CCEIs for the separate datasets  $E$  and  $L$ .

The minimum (vertical axis) and combined (horizontal axis) CCEI scores





The minimum (vertical axis) and combined (horizontal axis) Varian (1990, 1991) scores



## Concluding remarks

- The clarity that is achieved by putting behavior under the microscope is well worth the necessary simplification.
- Many experiments “censor” the data to avoid the ugly truth that life is more complicated than our theories.
- Otherwise, no theory fits perfectly and the analysis inevitably involves modeling and econometric compromises.

- A very rich data set well-suited to studying behavior at the level of the individual subject.
- Although individual behavior is quite heterogeneous, there is a high level of consistency in the individual-level decisions.
  - What kind of preferences are consistent with the observed choices (substantive rationality)?
  - How individuals come to make decisions that are consistent with an underlying preference ordering (procedural rationality)?