UC Berkeley
Haas School of Business
Game Theory
(EMBA 296 & EWMBA 211)
Summer 2015

Preliminaries

Block 1
May 22, 2015

Game theory

- Game theory is about what happens when decision makers (spouses, workers, managers, presidents) interact.

- In the past fifty years, game theory has gradually became a standard language in economics.

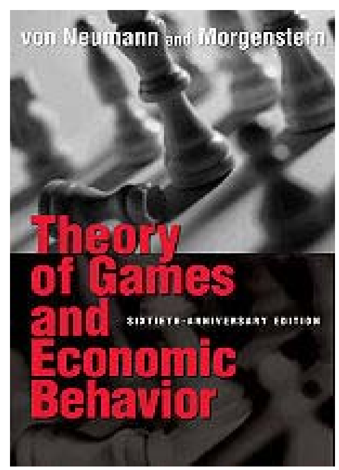
- The power of game theory is its <u>generality</u> and (mathematical) <u>precision</u>.

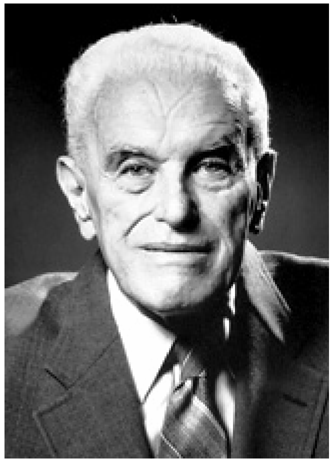
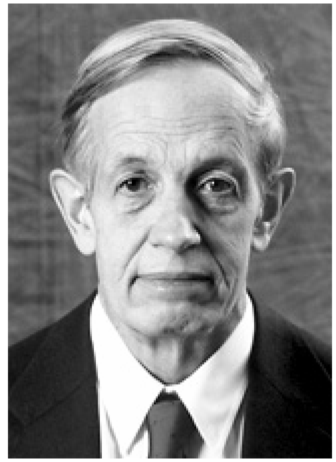
- Because game theory is rich and crisp, it could unify many parts of social science.

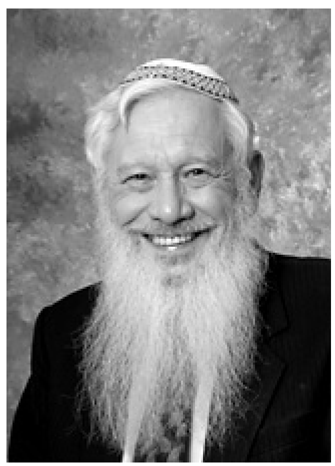
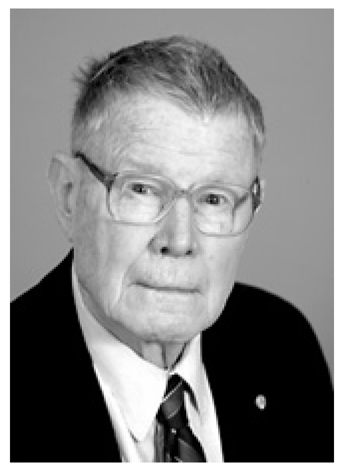
- The spread of game theory outside of economics has suffered because of the misconception that it requires a lot of fancy math.

- Game theory is also a natural tool for understanding complex social and economic phenomena in the real world.

The paternity of game theory



What is game theory good for?

Q Is game theory meant to predict what decision makers do, to give them advice, or what?

A The tools of analytical game theory are used to predict, postdict (explain), and prescribe.

Remember: even if game theory is not always accurate, descriptive failure is prescriptive opportunity!

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As Milton Friedman said famously observed "theories do not have to be realistic to be useful." A theory can be *useful* in three ways:

A. descriptive (how people actually choose)

B. prescriptive (as a practical aid to choice)

C. normative (how people ought to choose)

Aumann (1987):

> "*Game theory is a sort of umbrella or 'unified field' theory for the rational side of social science, where 'social' is interpreted broadly, to include human as well as non-human players (computers, animals, plants).*"

Game theory in practice

Farhan Zaidi, the General Manager of the LA Dodgers (PHD in economics from UC Berkeley), and the person Billy Beane called "absolutely brilliant."

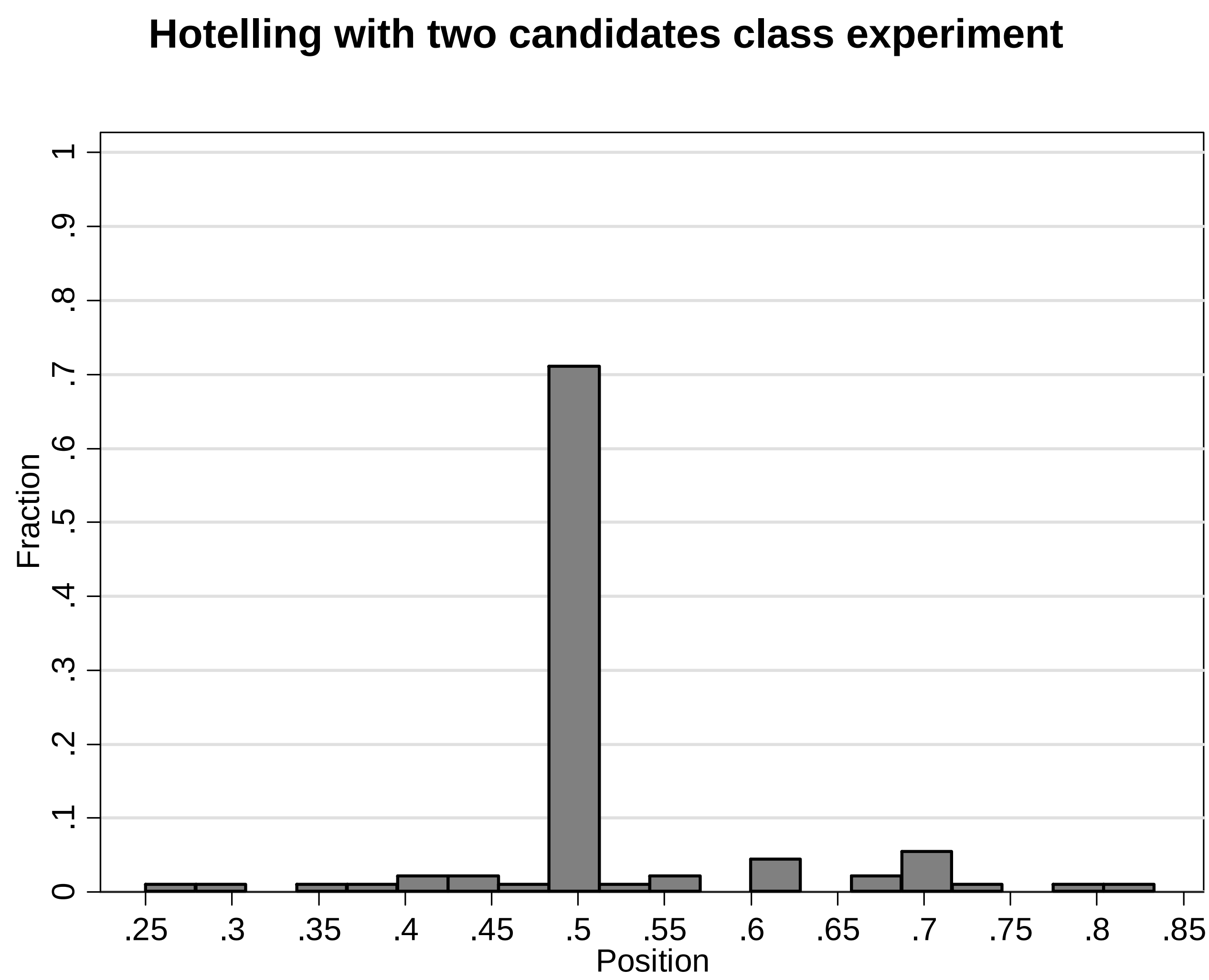
Three examples

Example I: Hotelling's electoral competition game

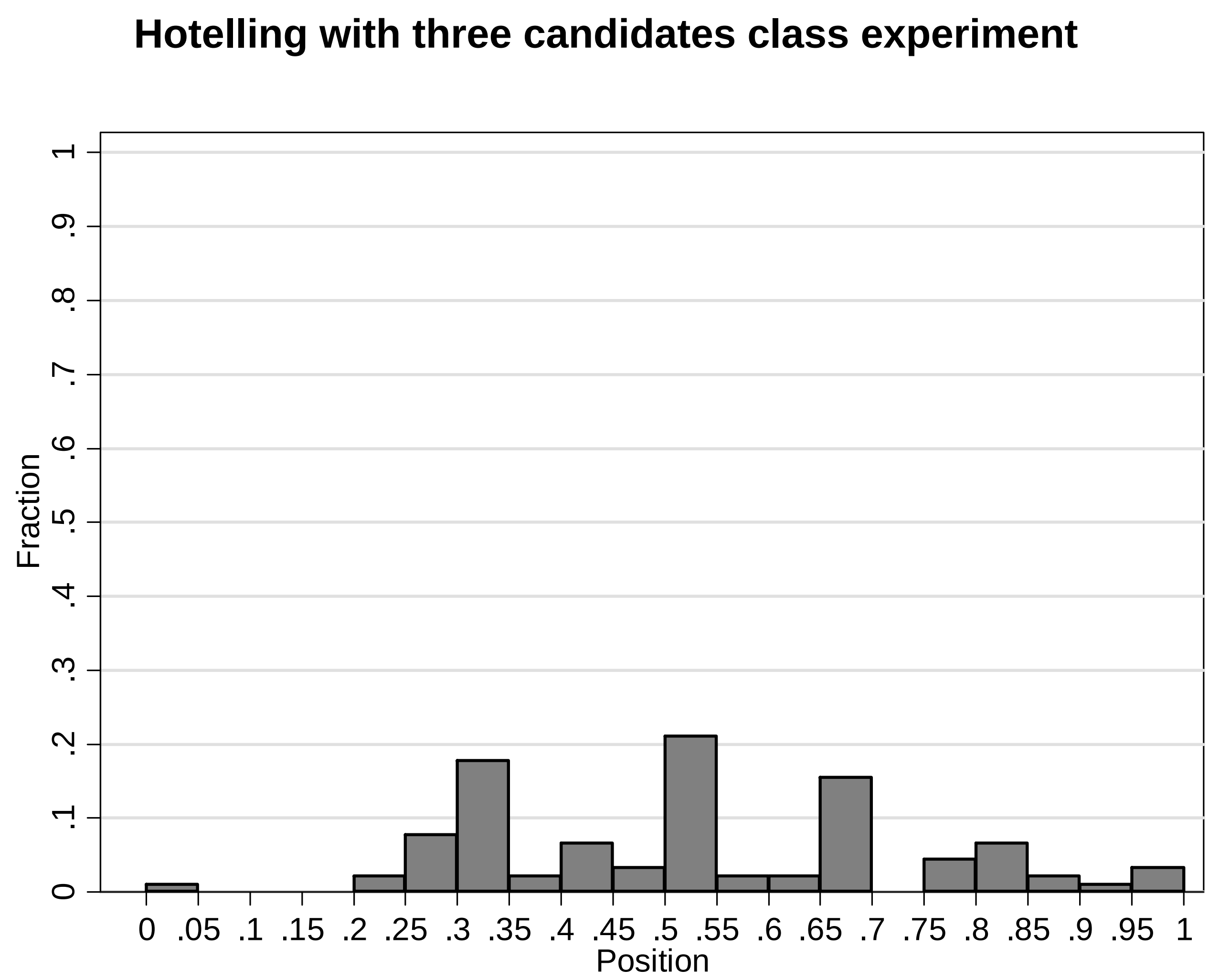
- There are two candidates and a continuum of voters, each with a favorite position on the interval $[0, 1]$.

- Each voter's distaste for any position is given by the distance between the position and her favorite position.

- A candidate attracts the votes off all citizens whose favorite positions are closer to her position.



Hotelling with two candidates class experiment
Fraction
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
.25
.3
.35
.4
.45
.5
.55
.6
.65
.7
.75
.8
.85
Position


Hotelling with three candidates class experiment
Fraction
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
0 .05 .1 .15 .2 .25 .3 .35 .4 .45 .5 .55 .6 .65 .7 .75 .8 .85 .9 .95 1
Position

Example II: Keynes's beauty contest game

– Simultaneously, everyone choose a number (integer) in the interval $[0, 100]$.

– The person whose number is closest to $2/3$ of the average number wins a fixed prize.

John Maynard Keynes (1936):

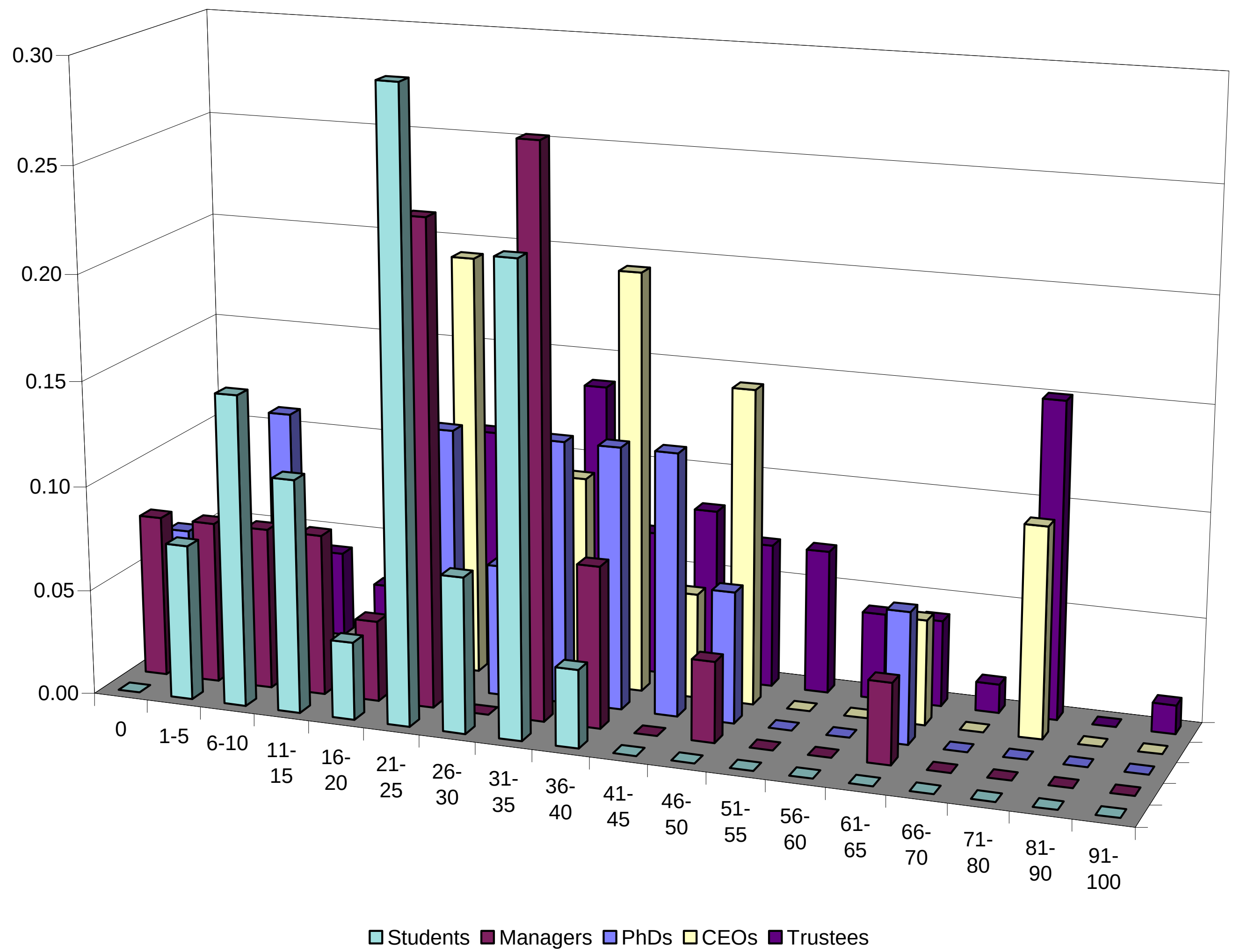
> "*It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.*"

$\Longrightarrow$ self-fulfilling price bubbles!

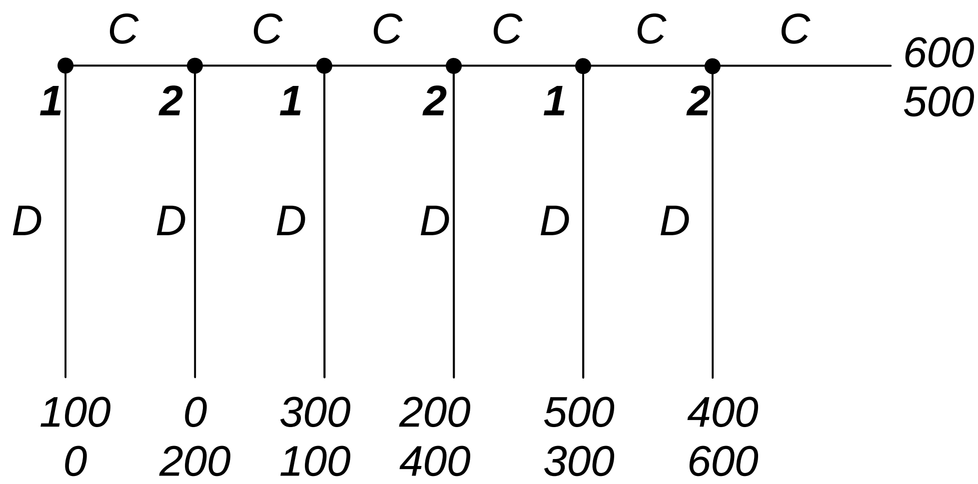
Beauty contest results

	Portfolio Managers	Economics PhDs	CEOs	Caltech students	Caltech trustees
Mean	24.3	27.4	37.8	21.9	42.6
Median	24.4	30.0	36.5	23.0	40.0
Fraction choosing zero	7.7%	12.5%	10.0%	7.4%	2.7%

	Germany	Singapore	UCLA	Wharton	High school (US)
Mean	36.7	46.1	42.3	37.9	32.4
Median	33.0	50.0	40.5	35.0	28.0
Fraction choosing zero	3.0%	2.0%	0.0%	0.0%	3.8%


0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1-5
6-10
11-15
16-20
21-25
26-30
31-35
36-40
41-45
46-50
51-55
56-60
61-65
66-70
71-80
81-90
91-100
Students
Managers
PhDs
CEOs
Trustees

Example III: the centipede game (graphically resembles a centipede insect)



The centipede game class experiment

Down	*0.311*
Continue, Down	*0.311*
Continue, Continue, Down	*0.267*
Continue, Continue, Continue	*0.111*

Eye movements can tell us a lot about how people play this game (and others).

Types of games

We study four groups of game theoretic models:

I strategic games

II extensive games (with perfect and imperfect information)

III repeated games

IV coalitional games

Side note I: individual preferences

Consider some (finite) set of alternatives $(x, y, z, ...)$.

– Formally, we represent the decision-maker's preferences by a binary relation $\succsim$ defined on the set of consumption bundles.

– For any pair of bundles x and y, if the decision-maker says that x is at least as good as y, we write

$$x \succsim y$$

and say that x is *weakly preferred* to y.

Bear in mind: economic theory often seeks to convince you with simple examples and then gets you to extrapolate. This simple construction works in wider (and wilder circumstances).

From the weak preference relation $\succsim$ we derive two other relations on the set of alternatives:

– Strict performance relation

$$x \succ y \text{ if and only if } x \succsim y \text{ and not } y \succsim x.$$

The phrase $x \succ y$ is read x is *strictly preferred* to y.

– Indifference relation

$$x \sim y \text{ if and only if } x \succsim y \text{ and } y \succsim x.$$

The phrase $x \sim y$ is read x is *indifferent* to y.

Side note II: individual rationality

Economic theory begins with two assumptions about preferences. These assumptions are so fundamental that we can refer to them as "axioms" of decision theory.

[1] Completeness

$$x \succsim y \text{ or } y \succsim x$$

for any pair of bundles x and y.

[2] Transitivity

$$\text{if } x \succsim y \text{ and } y \succsim z \text{ then } x \succsim z$$

for any three bundles x, y and z.

Together, completeness and transitivity constitute the formal definition of *rationality* as the term is used in economics. Rational economic agents are ones who

> have the ability to make choices [1], and whose choices display a logical consistency [2].

(Only) the preferences of a rational agent can be represented, or summarized, by a *utility function*.

Strategic games

A strategic game consists of

- a set of players (decision makers)
- for each player, a set of possible actions
- for each player, preferences over the set of action profiles (outcomes).

In strategic games, players move simultaneously. A wide range of situations may be modeled as strategic games.

A two-player (finite) strategic game can be described conveniently in a so-called bi-matrix.

For example, a generic 2×2 (two players and two possible actions for each player) game

	L	R
T	A_1, A_2	B_1, B_2
B	C_1, C_2	D_1, D_2

where the two rows (resp. columns) correspond to the possible actions of player 1 (resp. 2).

For example, rock-paper-scissors (over a dollar):

	R	P	S
R	$0, 0$	$-1, 1$	$1, -1$
P	$1, -1$	$0, 0$	$-1, 1$
S	$-1, 1$	$1, -1$	$0, 0$

Each player's set of actions is $\{Rock, Papar, Scissors\}$ and the set of action profiles is

$$\{RR, RP, RS, PR, PP, PS, SR.SP, SS\}.$$

In rock-paper-scissors

$$PR \sim_1 SP \sim_1 RS \succ_1 PP \sim_1 RR \sim_1 SS \succ_1 PS \sim_1 SR \sim_1 PS$$

and

$$PR \sim_2 SP \sim_2 RS \prec_2 PP \sim_2 RR \sim_2 SS \prec_2 PS \sim_2 SR \sim_2 PS$$

This is a zero-sum or a strictly competitive game.

Classical 2×2 games

- The following simple 2×2 games represent a variety of strategic situations.

- Despite their simplicity, each game captures the essence of a type of strategic interaction that is present in more complex situations.

- These classical games "span" the set of almost *all* games (strategic equivalence).

Game I: Prisoner's Dilemma

	$Work$	$Goof$
$Work$	$3,3$	$0,4$
$Goof$	$4,0$	$1,1$

A situation where there are gains from cooperation but each player has an incentive to "free ride."

Examples: team work, duopoly, arm/advertisement/R&D race, public goods, and more.

Game II: Battle of the Sexes (BoS)

	Ball	*Show*
Ball	$2, 1$	$0, 0$
Show	$0, 0$	$1, 2$

Like the Prisoner's Dilemma, Battle of the Sexes models a wide variety of situations.

Examples: political stands, mergers, among others.

Game III-V: Coordination, Hawk-Dove, and Matching Pennies

	Ball	*Show*
Ball	2, 2	0, 0
Show	0, 0	1, 1

	Dove	*Hawk*
Dove	3, 3	1, 4
Hawk	4, 1	0, 0

	Head	*Tail*
Head	1, −1	−1, 1
Tail	−1, 1	1, −1

Best response and dominated actions

Action a is player 1's *best response* to action b player 1 if it is the optimal choice when 1 *conjectures* that 2 will play b.

In any game, player 1's action a' is *strictly* dominated if it is never a best response (inferior no matter what the other players do).

In the Prisoner's Dilemma, for example, action $Work$ is strictly dominated by action $Goof$. As we will see, a strictly dominated action is not used in any Nash equilibrium.

Nash equilibrium

Nash equilibrium (NE) is a steady state of the play of a strategic game – no player has a profitable deviation given the actions of the other players.

Put differently, a NE is a set of actions such that all players are doing their best given the actions of the other players.

Conclusions

Adam Brandenburger:

> *There is nothing so practical as a good [game] theory. A good theory confirms the conventional wisdom that "less is more." A good theory does less because it does not give answers. At the same time, it does a lot more because it helps people organize what they know and uncover what they do not know. A good theory gives people the tools to discover what is best for them.*