UC Berkeley
Haas School of Business
Game Theory
(EMBA 296 & EWMBA 211)
Summer 2016

Preliminaries

Block 1
May 19-20, 2016

Game theory

- Game theory is about what happens when decision makers (spouses, workers, managers, presidents) interact.

- In the past fifty years, game theory has gradually became a standard language in economics.

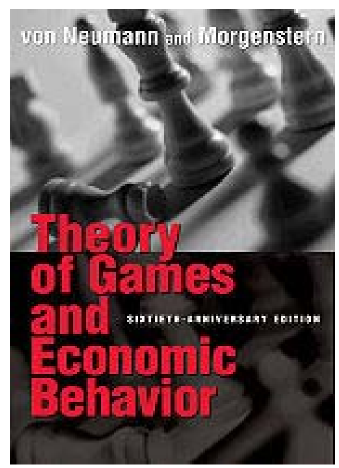
- The power of game theory is its <u>generality</u> and (mathematical) <u>precision</u>.

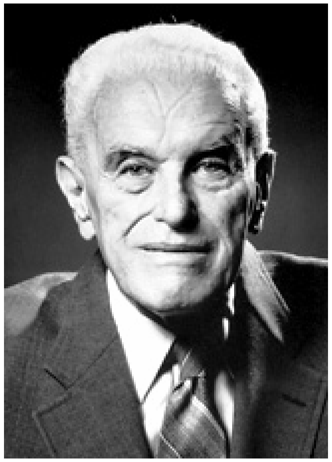
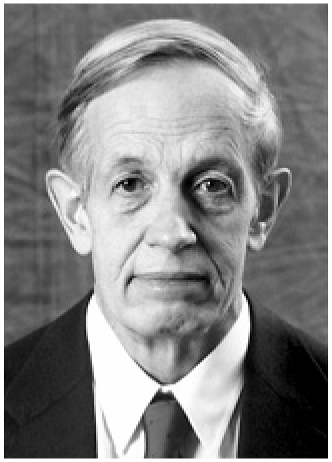
- Because game theory is <u>rich</u> and <u>crisp</u>, it could unify many parts of social science.

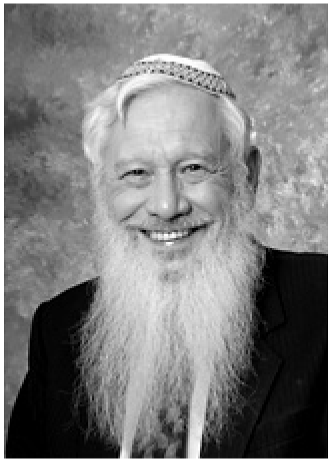
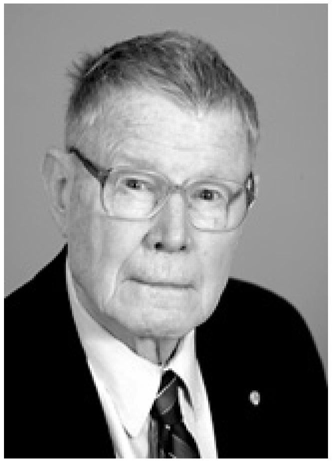
- The spread of game theory outside of economics has suffered because of the misconception that it requires a lot of fancy math.

- Game theory is also a natural tool for understanding complex social and economic phenomena in the real world.

The paternity of game theory



What is game theory good for?

Q Is game theory meant to predict what decision makers do, to give them advice, or what?

A The tools of analytical game theory are used to predict, postdict (explain), and prescribe.

Remember: even if game theory is not always accurate, descriptive failure is prescriptive opportunity!

As Milton Friedman said famously observed "theories do not have to be realistic to be useful." A theory can be *useful* in three ways:

A. descriptive (how people actually choose)

B. prescriptive (as a practical aid to choice)

C. normative (how people ought to choose)

Aumann (1987):

> “*Game theory is a sort of umbrella or ‘unified field’ theory for the rational side of social science, where ‘social’ is interpreted broadly, to include human as well as non-human players (computers, animals, plants).*”

Game theory in practice

Farhan Zaidi, the General Manager of the LA Dodgers (PHD in economics from UC Berkeley), and the person Billy Beane called “absolutely brilliant.”

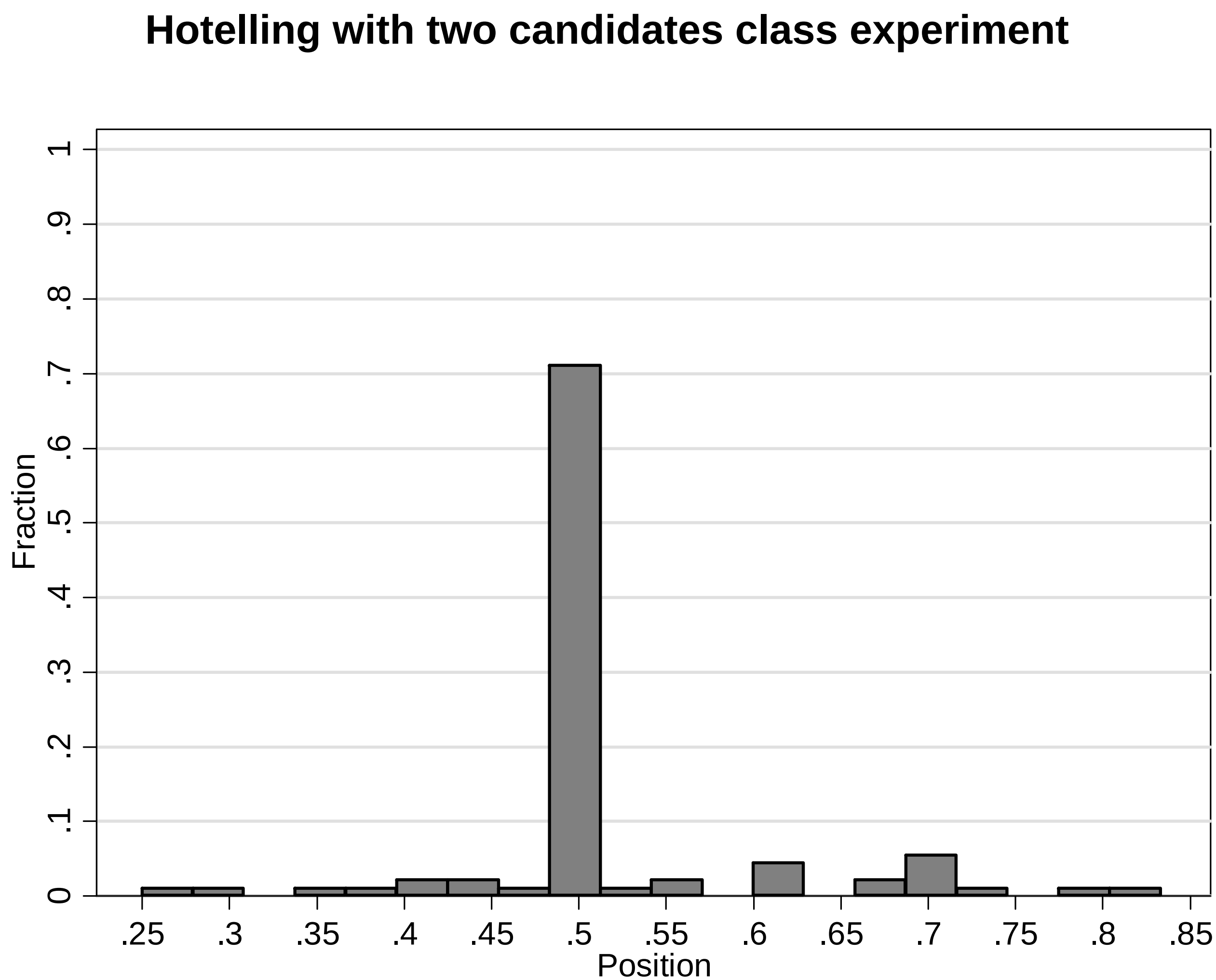
Four examples

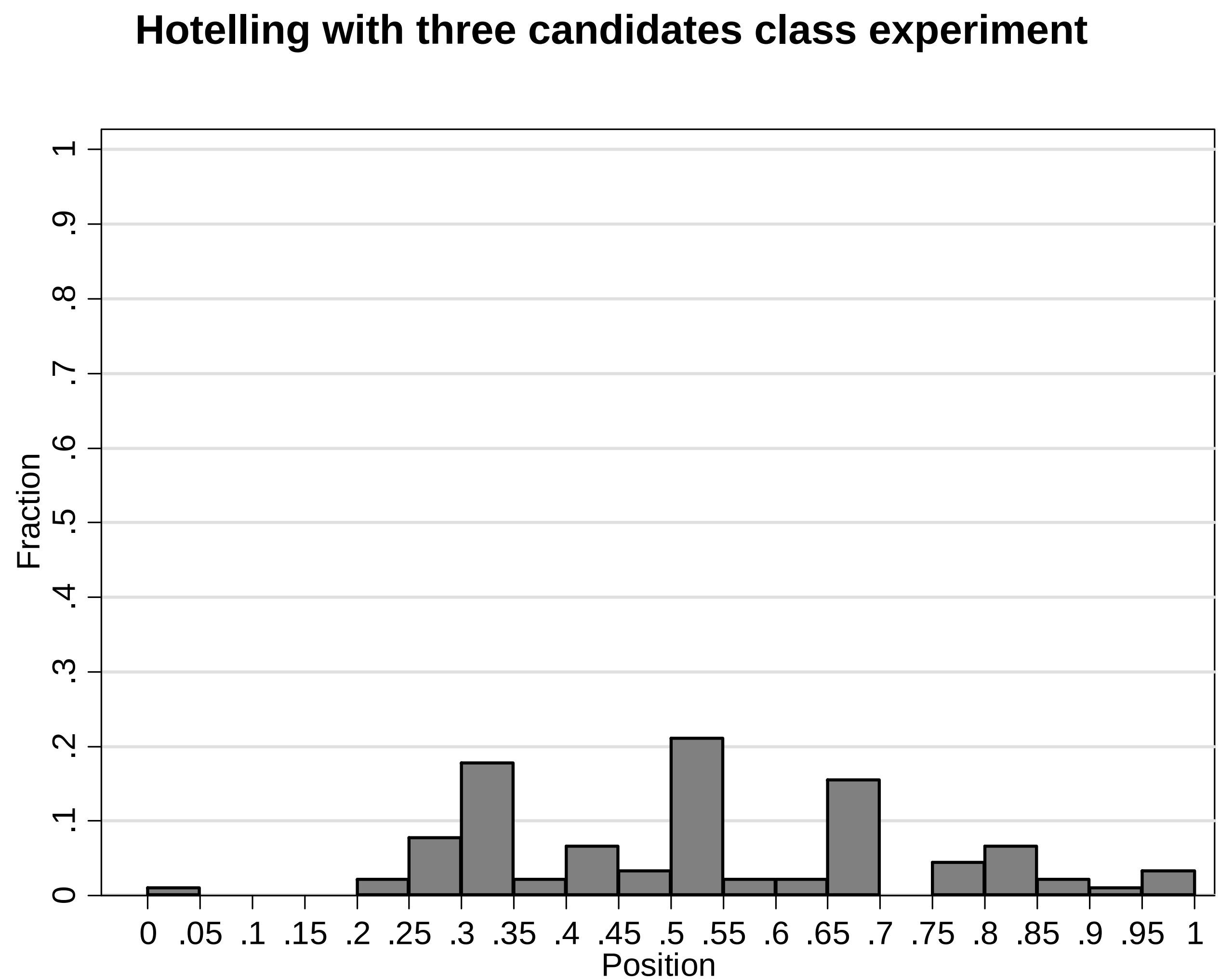
Example I: Hotelling's electoral competition game

– There are two candidates and a continuum of voters, each with a favorite position on the interval $[0, 1]$.

– Each voter's distaste for any position is given by the distance between the position and her favorite position.

– A candidate attracts the votes off all citizens whose favorite positions are closer to her position.


Hotelling with two candidates class experiment
Fraction
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
.25
.3
.35
.4
.45
.5
.55
.6
.65
.7
.75
.8
.85
Position


Hotelling with three candidates class experiment
Fraction
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
0
.05
.1
.15
.2
.25
.3
.35
.4
.45
.5
.55
.6
.65
.7
.75
.8
.85
.9
.95
1
Position

Example II: Keynes's beauty contest game

- Simultaneously, everyone choose a number (integer) in the interval $[0, 100]$.

- The person whose number is closest to $2/3$ of the average number wins a fixed prize.

John Maynard Keynes (1936):

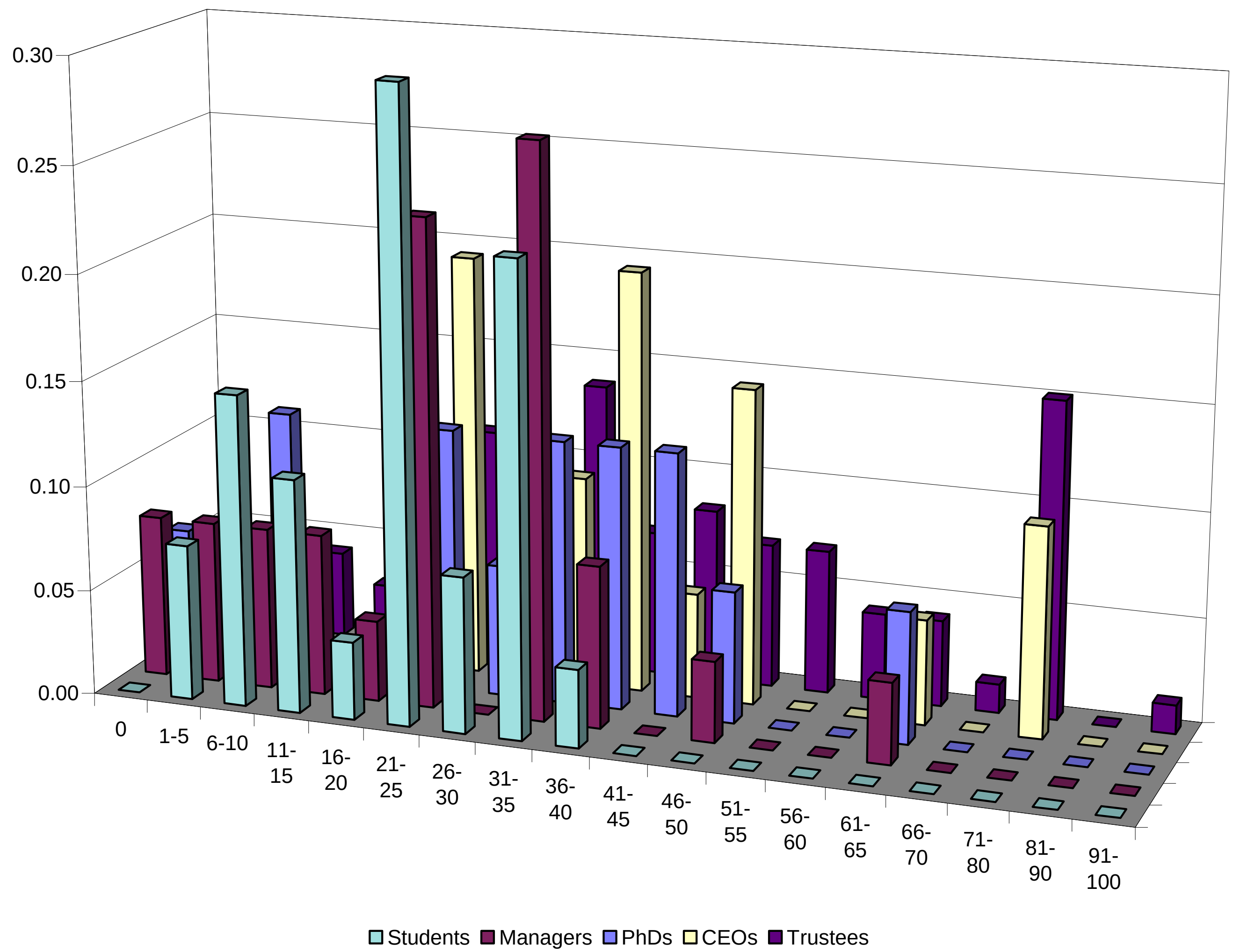
> "*It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.*"

$\Longrightarrow$ self-fulfilling price bubbles!

Beauty contest results

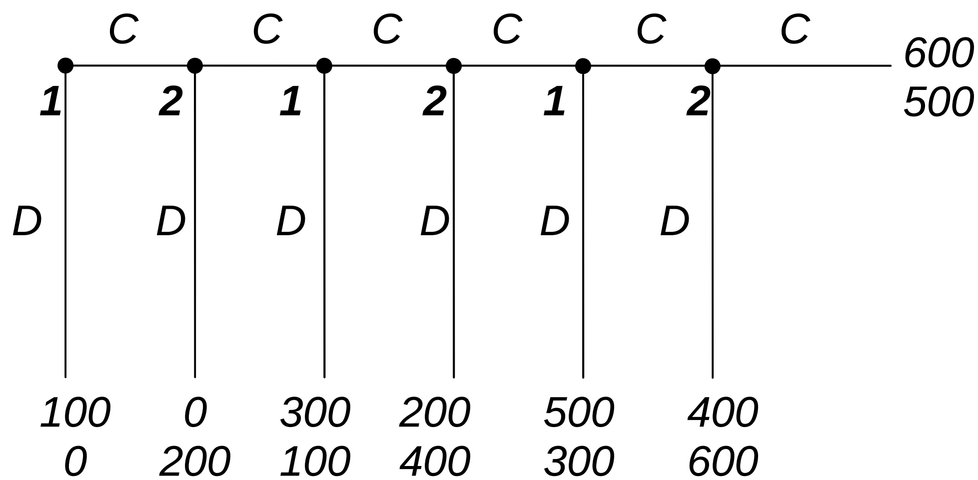
	Portfolio Managers	Economics PhDs	CEOs	Caltech students	Caltech trustees
Mean	24.3	27.4	37.8	21.9	42.6
Median	24.4	30.0	36.5	23.0	40.0
Fraction choosing zero	7.7%	12.5%	10.0%	7.4%	2.7%

	Germany	Singapore	UCLA	Wharton	High school (US)
Mean	36.7	46.1	42.3	37.9	32.4
Median	33.0	50.0	40.5	35.0	28.0
Fraction choosing zero	3.0%	2.0%	0.0%	0.0%	3.8%



0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1-5
6-10
11-15
16-20
21-25
26-30
31-35
36-40
41-45
46-50
51-55
56-60
61-65
66-70
71-80
81-90
91-100
Students
Managers
PhDs
CEOs
Trustees

Example III: the centipede game (graphically resembles a centipede insect)



The centipede game class experiment

Down	*0.311*
Continue, Down	*0.311*
Continue, Continue, Down	*0.267*
Continue, Continue, Continue	*0.111*

Eye movements can tell us a lot about how people play this game (and others).

Example IV: auctions

From Babylonia to eBay, auctioning has a very long history.

Babylon:

- women at marriageable age.

Athens, Rome, and medieval Europe:

- rights to collect taxes, dispose of confiscated property, lease of land and mines,

and many more...

The word “auction” comes from the Latin *augere*, meaning “to increase.”

The earliest use of the English word “auction” given by the *Oxford English Dictionary* dates from 1595 and concerns an auction “when will be sold Slaves, household goods, etc.”

In this era, the auctioneer lit a short candle and bids were valid only if made before the flame went out – Samuel Pepys (1633-1703) –

- Auctions, broadly defined, are used to allocate significant economics resources.

  Examples: works of art, government bonds, offshore tracts for oil exploration, radio spectrum, and more.

- Auctions take many forms. A game-theoretic framework enables to understand the consequences of various auction designs.

- Game theory can suggest the design likely to be most effective, and the one likely to raise the most revenues.

Types of auctions

Sequential / simultaneous

Bids may be called out sequentially or may be submitted simultaneously in sealed envelopes:

- English (or oral) – the seller actively solicits progressively higher bids and the item is soled to the highest bidder.

- Dutch – the seller begins by offering units at a "high" price and reduces it until all units are soled.

- Sealed-bid – all bids are made simultaneously, and the item is sold to the highest bidder.

First-price / second-price

The price paid may be the highest bid or some other price:

- First-price – the bidder who submits the highest bid wins and pay a price equal to her bid.

- Second-prices – the bidder who submits the highest bid wins and pay a price equal to the second highest bid.

Variants: all-pay (lobbying), discriminatory, uniform, Vickrey (William Vickrey, Nobel Laureate 1996), and more.

Private-value / common-value

Bidders can be certain or uncertain about each other's valuation:

- In private-value auctions, valuations differ among bidders, and each bidder is certain of her own valuation and can be certain or uncertain of every other bidder's valuation.

- In common-value auctions, all bidders have the same valuation, but bidders do not know this value precisely and their estimates of it vary.

Types of games

We study four groups of game theoretic models:

I strategic games

II extensive games (with perfect and imperfect information)

III repeated games

IV coalitional games

Adam Brandenburger:

> *There is nothing so practical as a good [game] theory. A good theory confirms the conventional wisdom that "less is more." A good theory does less because it does not give answers. At the same time, it does a lot more because it helps people organize what they know and uncover what they do not know. A good theory gives people the tools to discover what is best for them.*

Side note I: individual preferences

Consider some (finite) set of alternatives $(x, y, z, ...)$.

- Formally, we represent the decision-maker's preferences by a binary relation $\succsim$ defined on the set of consumption bundles.

- For any pair of bundles x and y, if the decision-maker says that x is at least as good as y, we write

$$x \succsim y$$

  and say that x is *weakly preferred* to y.

Bear in mind: economic theory often seeks to convince you with simple examples and then gets you to extrapolate. This simple construction works in wider (and wilder circumstances).

From the weak preference relation $\succsim$ we derive two other relations on the set of alternatives:

– Strict performance relation

$$x \succ y \text{ if and only if } x \succsim y \text{ and not } y \succsim x.$$

The phrase $x \succ y$ is read x is *strictly preferred* to y.

– Indifference relation

$$x \sim y \text{ if and only if } x \succsim y \text{ and } y \succsim x.$$

The phrase $x \sim y$ is read x is *indifferent* to y.

Side note II: individual rationality

Economic theory begins with two assumptions about preferences. These assumptions are so fundamental that we can refer to them as "axioms" of decision theory.

[1] Completeness

$$x \succsim y \text{ or } y \succsim x$$

for any pair of bundles x and y.

[2] Transitivity

$$\text{if } x \succsim y \text{ and } y \succsim z \text{ then } x \succsim z$$

for any three bundles x, y and z.

Together, completeness and transitivity constitute the formal definition of *rationality* as the term is used in economics. Rational economic agents are ones who

> have the ability to make choices [1], and whose choices display a logical consistency [2].

(Only) the preferences of a rational agent can be represented, or summarized, by a *utility function*.

Strategic games

A strategic game consists of

- a set of players (decision makers)
- for each player, a set of possible actions
- for each player, preferences over the set of action profiles (outcomes).

In strategic games, players move simultaneously. A wide range of situations may be modeled as strategic games.

A two-player (finite) strategic game can be described conveniently in a so-called bi-matrix.

For example, a generic 2×2 (two players and two possible actions for each player) game

$$\begin{array}{c|c|c|} & L & R \\ \hline T & A_1, A_2 & B_1, B_2 \\ \hline B & C_1, C_2 & D_1, D_2 \\ \hline \end{array}$$

where the two rows (resp. columns) correspond to the possible actions of player 1 (resp. 2).

Applying the definition of a strategic game to the 2×2 game above yields:

- Players: $\{1, 2\}$
- Action sets: $A_1 = \{T, B\}$ and $A_2 = \{L, R\}$
- Action profiles (outcomes):

$$A = A_1 \times A_2 = \{(T, L), (T, R), (B, L), (B, R)\}$$

- Preferences: $\succsim_1$ and $\succsim_2$ are given by the bi-matrix.

Rock-Paper-Scissors

(over a dollar)

	R	P	S
R	$0, 0$	$-1, 1$	$1, -1$
P	$1, -1$	$0, 0$	$-1, 1$
S	$-1, 1$	$1, -1$	$0, 0$

Each player's set of actions is $\{Rock, Papar, Scissors\}$ and the set of action profiles is

$$\{RR, RP, RS, PR, PP, PS, SR.SP, SS\}.$$

In rock-paper-scissors

$$PR \sim_1 SP \sim_1 RS \succ_1 PP \sim_1 RR \sim_1 SS \succ_1 PS \sim_1 SR \sim_1 PS$$

and

$$PR \sim_2 SP \sim_2 RS \prec_2 PP \sim_2 RR \sim_2 SS \prec_2 PS \sim_2 SR \sim_2 PS$$

This is a zero-sum or a strictly competitive game.

Classical 2×2 **games**

- The following simple 2×2 games represent a variety of strategic situations.

- Despite their simplicity, each game captures the essence of a type of strategic interaction that is present in more complex situations.

- These classical games "span" the set of almost *all* games (strategic equivalence).

Game I: Prisoner's Dilemma

	$Work$	$Goof$
$Work$	$3, 3$	$0, 4$
$Goof$	$4, 0$	$1, 1$

A situation where there are gains from cooperation but each player has an incentive to "free ride."

Examples: team work, duopoly, arm/advertisement/R&D race, public goods, and more.

Game II: Battle of the Sexes (BoS)

	$Ball$	$Show$
$Ball$	$2, 1$	$0, 0$
$Show$	$0, 0$	$1, 2$

Like the Prisoner's Dilemma, Battle of the Sexes models a wide variety of situations.

Examples: political stands, mergers, among others.

Game III-V: Coordination, Hawk-Dove, and Matching Pennies

	Ball	*Show*
Ball	2, 2	0, 0
Show	0, 0	1, 1

	Dove	*Hawk*
Dove	3, 3	1, 4
Hawk	4, 1	0, 0

	Head	*Tail*
Head	1, −1	−1, 1
Tail	−1, 1	1, −1

Best response and dominated actions

Action T is player 1's *best response* to action L player 2 if T is the optimal choice when 1 *conjectures* that 2 will play L.

Player 1's action T is *strictly* dominated if it is never a best response (inferior to B no matter what the other players do).

In the Prisoner's Dilemma, for example, action $Work$ is strictly dominated by action $Goof$. As we will see, a strictly dominated action is not used in any Nash equilibrium.

Nash equilibrium

Nash equilibrium (NE) is a steady state of the play of a strategic game – no player has a profitable deviation given the actions of the other players.

Put differently, a NE is a set of actions such that all players are doing their best given the actions of the other players.

Mixed strategy Nash equilibrium in the BoS

Suppose that, each player can randomize among all her strategies so choices are not deterministic:

		q L	$1-q$ R
p	T	pq	$p(1-q)$
$1-p$	B	$(1-p)q$	$(1-p)(1-q)$

Let p and q be the probabilities that player 1 and 2 respectively assign to the strategy *Ball*.

Player 2 will be indifferent between using her strategy B and S when player 1 assigns a probability p such that her expected payoffs from playing B and S are the same. That is,

$$1p + 0(1 - p) = 0p + 2(1 - p)$$
$$p = 2 - 2p$$
$$p^* = 2/3$$

Hence, when player 1 assigns probability $p^* = 2/3$ to her strategy B and probability $1 - p^* = 1/3$ to her strategy S, player 2 is indifferent between playing B or S any mixture of them.

Similarly, player 1 will be indifferent between using her strategy B and S when player 2 assigns a probability q such that her expected payoffs from playing B and S are the same. That is,

$$\begin{gathered} 2q + 0(1-q) = 0q + 1(1-q) \\ 2q = 1 - q \\ q^* = 1/3 \end{gathered}$$

Hence, when player 2 assigns probability $q^* = 1/3$ to her strategy B and probability $1 - q^* = 2/3$ to her strategy S, player 2 is indifferent between playing B or S any mixture of them.

In terms of best responses:

$$B_1(q) = \begin{cases} p = 1 & if \quad p > 1/3 \\ p \in [0,1] & if \quad p = 1/3 \\ p = 0 & if \quad p < 1/3 \end{cases}$$

$$B_2(p) = \begin{cases} q = 1 & if \quad p > 2/3 \\ q \in [0,1] & if \quad p = 2/3 \\ q = 0 & if \quad p < 2/3 \end{cases}$$

The BoS has two Nash equilibria in pure strategies $\{(B,B),(S,S)\}$ and one in mixed strategies $\{(2/3,1/3)\}$. In fact, any game with a finite number of players and a finite number of strategies for each player has Nash equilibrium (Nash, 1950).

Oligopoly

(only if time permits)

- Oligopoly is a market in which only a few firms compete with one another, and entry of new firms is impeded.

- The situation is known as the Cournot model after Antoine Augustin Cournot, a French economist, philosopher and mathematician (1801-1877).

- In the basic example, a single good is produced by two firms (the industry is a "duopoly").

Cournot's oligopoly model (1838) (Antoine Augustin Cournot, an economist, philosopher and mathematician, 1801-1877).

- A single good is produced by two firms (the industry is a "duopoly").

- The cost for firm $i = 1, 2$ for producing q_i units of the good is given by $c_i q_i$ ("unit cost" is constant equal to $c_i > 0$).

- If the firms' total output is $Q = q_1 + q_2$ then the market price is

$$P = A - Q$$

  if $A \geq Q$ and zero otherwise (linear inverse demand function). We also assume that $A > c$.

To find the Nash equilibria of the Cournot's game, we can use the procedures based on the firms' best response functions.

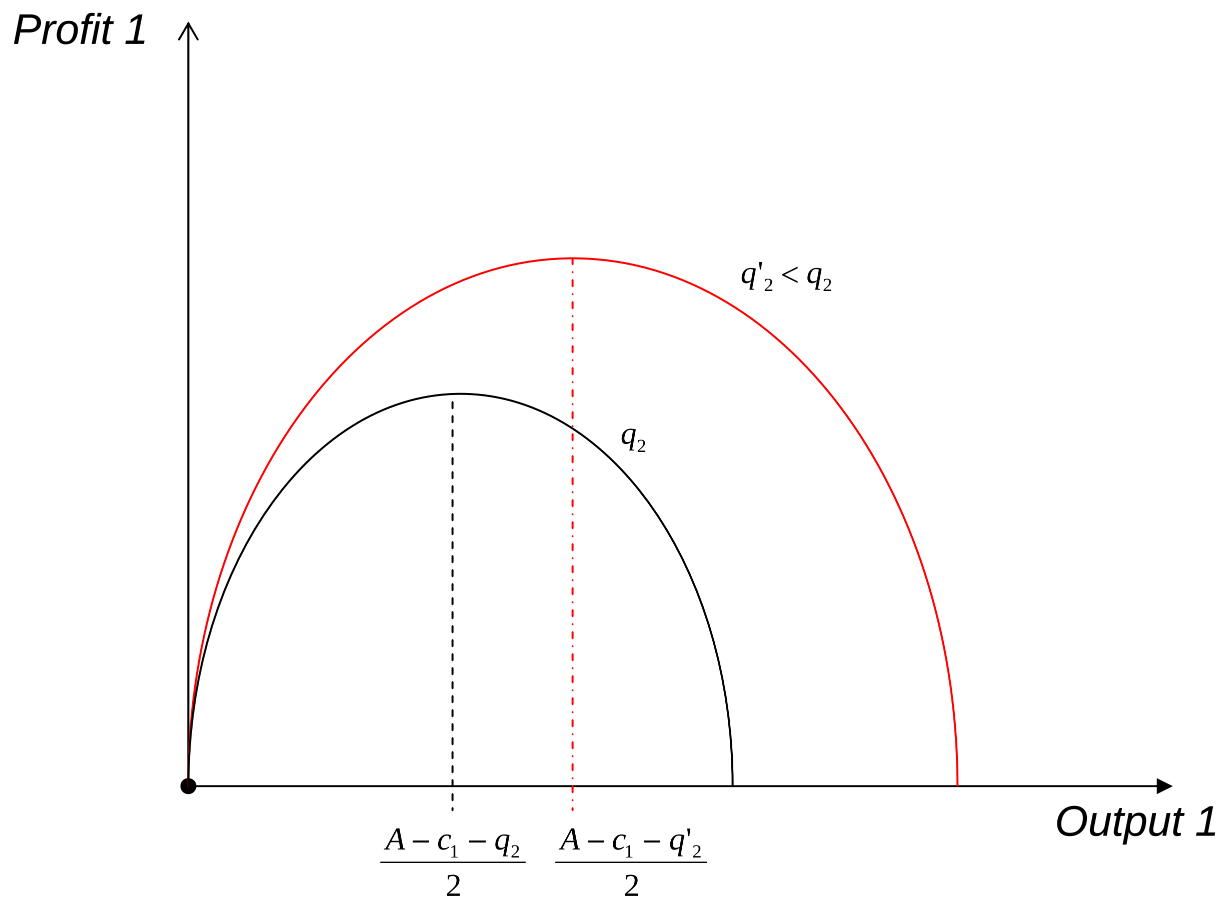
But first we need the firms payoffs (profits):

$$\begin{aligned}\pi_1 &= Pq_1 - c_1q_1 \\ &= (A-Q)q_1 - c_1q_1 \\ &= (A-q_1-q_2)q_1 - c_1q_1 \\ &= (A-q_1-q_2-c_1)q_1\end{aligned}$$

and similarly,

$$\pi_2 = (A-q_1-q_2-c_2)q_2$$

Firm 1's profit as a function of its output (given firm 2's output)



To find firm 1's best response to any given output q_2 of firm 2, we need to study firm 1's profit as a function of its output q_1 for given values of q_2.

Using calculus, we set the derivative of firm 1's profit with respect to q_1 equal to zero and solve for q_1:

$$q_1 = \frac{1}{2}(A - q_2 - c_1).$$

We conclude that the best response of firm 1 to the output q_2 of firm 2 depends on the values of q_2 and c_1.

Because firm 2's cost function is $c_2 \neq c_1$, its best response function is given by

$$q_2 = \frac{1}{2}(A - q_1 - c_2).$$

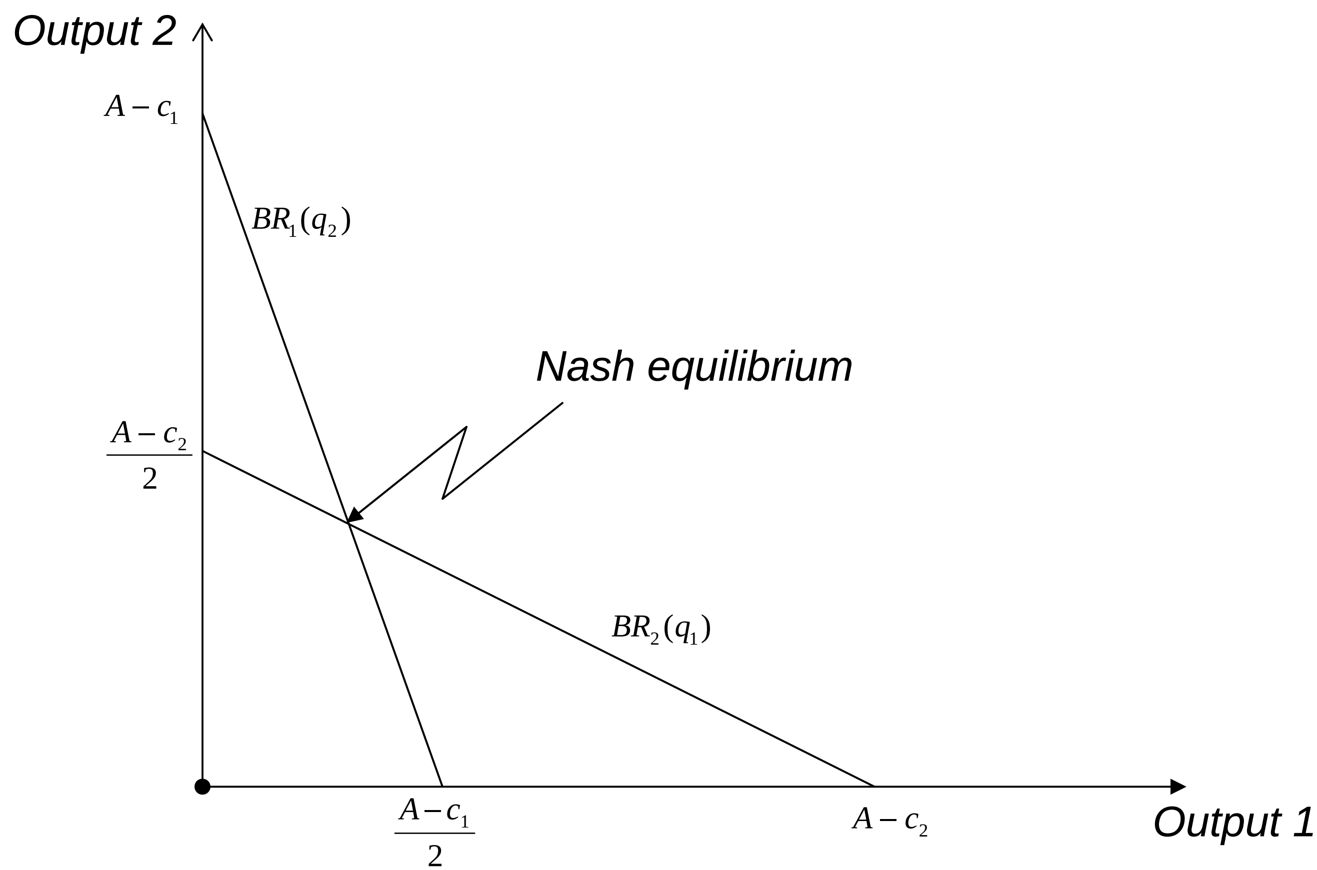
A Nash equilibrium of the Cournot's game is a pair (q_1^*, q_2^*) of outputs such that q_1^* is a best response to q_2^* and q_2^* is a best response to q_1^*.

From the figure below, we see that there is exactly one such pair of outputs

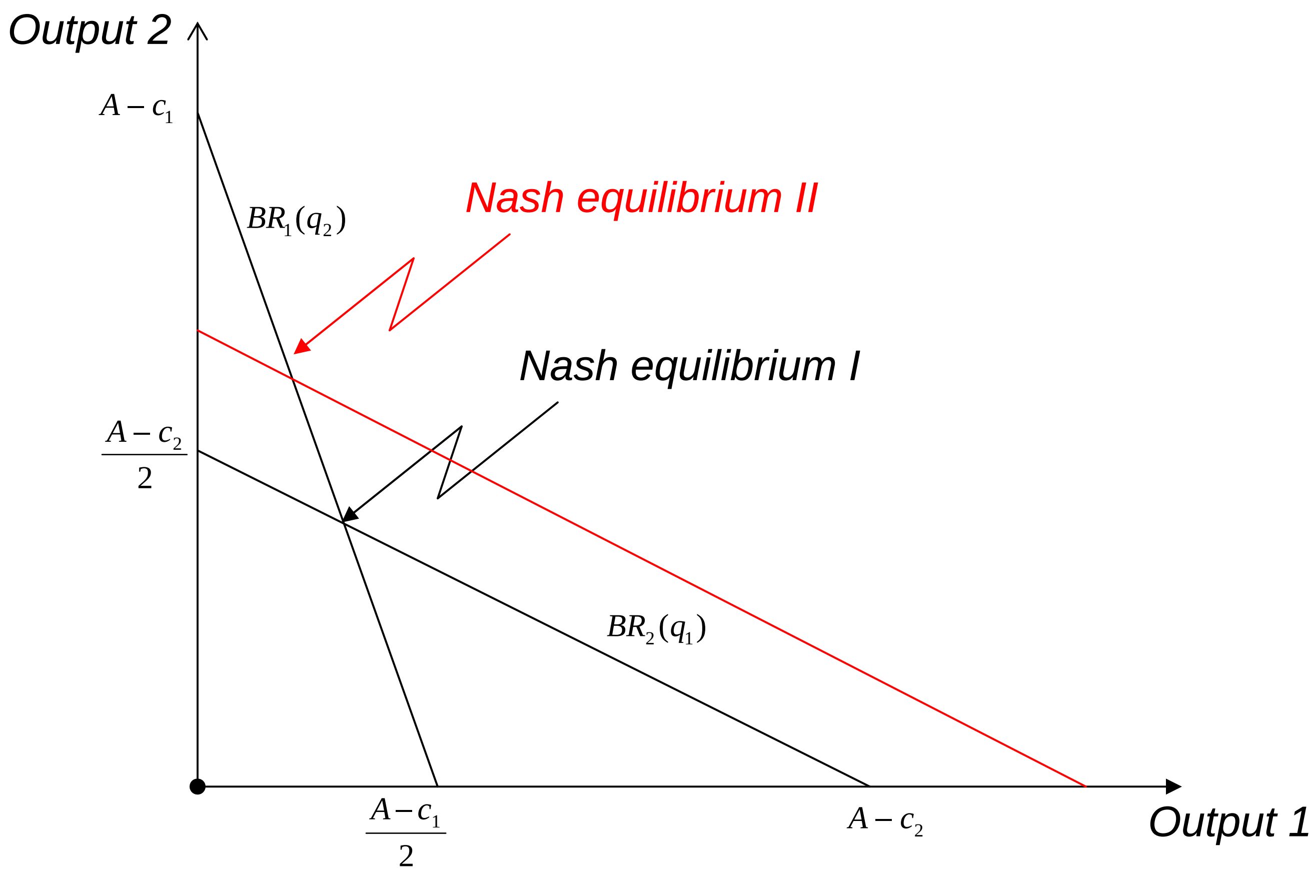
$$q_1^* = \frac{A+c_2-2c_1}{3} \quad \text{and} \quad q_2^* = \frac{A+c_1-2c_2}{3}$$

which is the solution to the two equations above.

The best response functions in the Cournot's duopoly game



Nash equilibrium comparative statics
(a decrease in the cost of firm 2)



A question: what happens when consumers are willing to pay more (*A* increases)?

In summary, this simple Cournot's duopoly game has a unique Nash equilibrium.

Two economically important properties of the Nash equilibrium are (to economic regulatory agencies):

[1] The relation between the firms' equilibrium profits and the profit they could make if they act collusively.

[2] The relation between the equilibrium profits and the number of firms.

[1] Collusive outcomes: in the Cournot's duopoly game, there is a pair of outputs at which *both* firms' profits exceed their levels in a Nash equilibrium.

[2] Competition: The price at the Nash equilibrium if the two firms have the *same* unit cost $c_1 = c_2 = c$ is given by

$$
\begin{aligned}
P^* &= A - q_1^* - q_2^* \\
&= \frac{1}{3}(A + 2c)
\end{aligned}
$$

which is above the unit cost c. But as the number of firm increases, the equilibrium price deceases, approaching c (zero profits).