UC Berkeley
Haas School of Business
Game Theory
(EMBA 296 & EWMBA 211)
Summer 2017

Risk preferences

Block 4
Jul 20-22, 2017

The fundamental tradeoffs in life

People's attitudes towards risk, time and other people enter every realm of (financial) decision-making:

$$\begin{aligned} \text{risk} &\iff \text{return} \\ \text{today} &\iff \text{tomorrow} \\ \text{self} &\iff \text{others} \end{aligned}$$

Risk, time and social preferences are thus important inputs into any broader measure of welfare and enter virtually every field of economics.

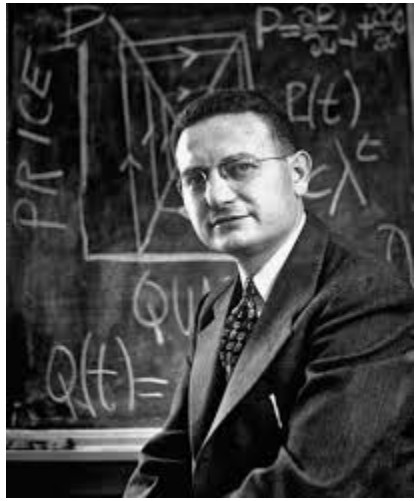
The touchstones of (financial) decision-making

Rational choice 'simply' requires consistent preferences over all possible alternatives, and choices that correspond to the most preferred alternative from the feasible set.

Insofar as preferences are rational, then the techniques of economic analysis may be brought to bear on modeling the decisions governed by these preferences.

Foundations of Economic Analysis (1947)

Paul A. Samuelson (1915-2009) – the first American Nobel laureate in economics and the foremost (academic) economist of the 20th century (and the uncle of Larry Summers…).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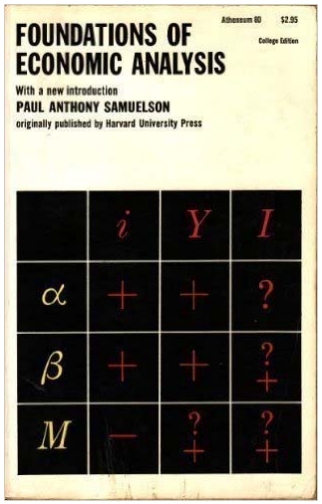
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College Edition
FOUNDATIONS OF ECONOMIC ANALYSIS
With a new introduction
PAUL ANTHONY SAMUELSON
originally published by Harvard University Press
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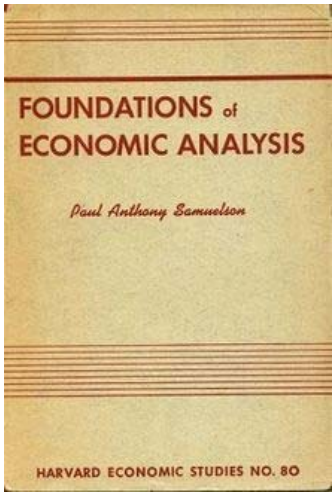

FOUNDATIONS of ECONOMIC ANALYSIS
Paul Anthony Samuelson
HARVARD ECONOMIC STUDIES NO. 80

Formally, we represent the consumer's preferences by a binary relation $\succsim$ defined on the set of consumption bundles.

For any pair of bundles x and y, if the consumer says that x is at least as good as y, we write

$$x \succsim y$$

and say that x is *weakly preferred* to y.

Bear in mind: economic theory often seeks to convince you with simple examples and then gets you to extrapolate. This simple construction works in wider (and wilder circumstances).

The basic assumptions about preferences

The theory begins with three assumptions about preferences. These assumptions are so fundamental that we can refer to them as "axioms" of decision theory.

[1] Completeness

$$x \succsim y \text{ or } y \succsim x$$

for any pair of bundles x and y.

[2] Transitivity

$$\text{if } x \succsim y \text{ and } y \succsim z \text{ then } x \succsim z$$

for any three bundles x, y and z.

Together, completeness and transitivity constitute the formal definition of *rationality* as the term is used in economics. Rational economic agents are ones who

> have the ability to make choices [1], and whose choices display a logical consistency [2].

(Only) the preferences of a rational agent can be represented, or summarized, by a *utility function* (more later).

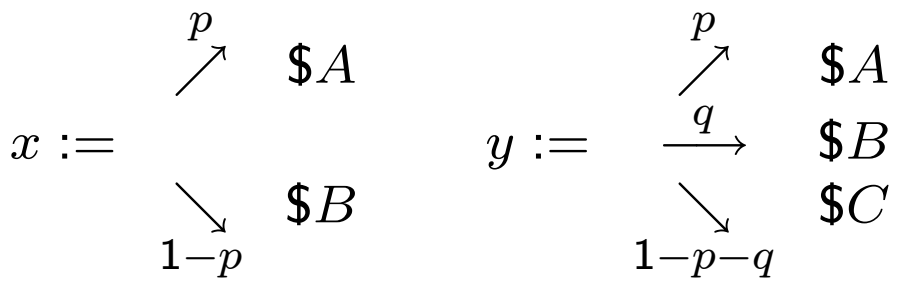
Decision making under uncertainty

- Uncertainty is a fact of life so people's attitudes towards risk enter every realm of economic decision-making.

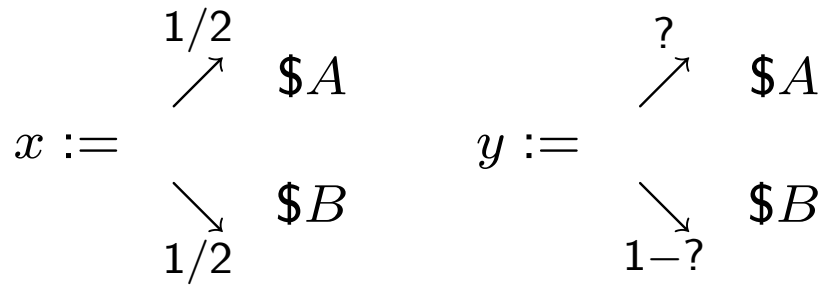
- We *must* study individual behavior with respect to choice involving uncertainty.

- Models of decision making under uncertainty play a key role in every field of economics.

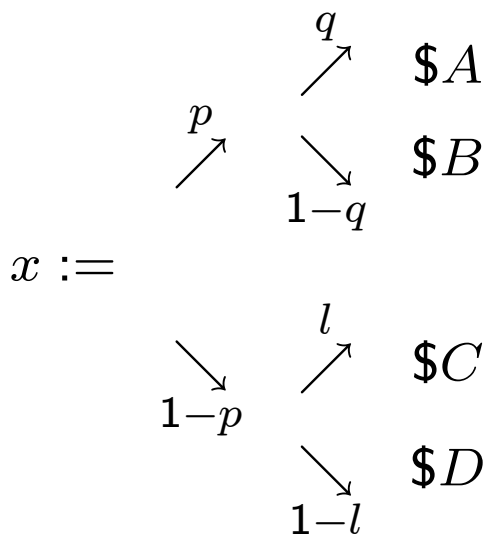
Life is full of lotteries :-(



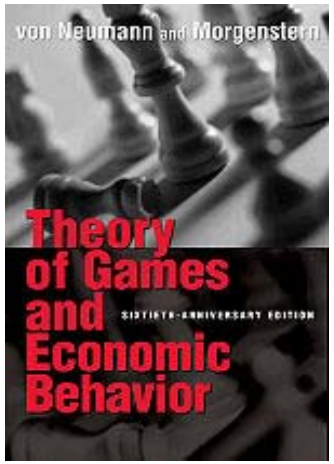
A risky lottery (left) and an ambiguous lottery (right)

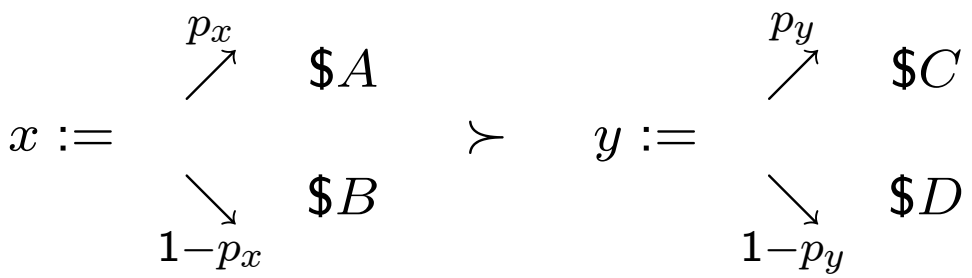


A compounded lottery



The paternity of decision theory and game theory (1944)

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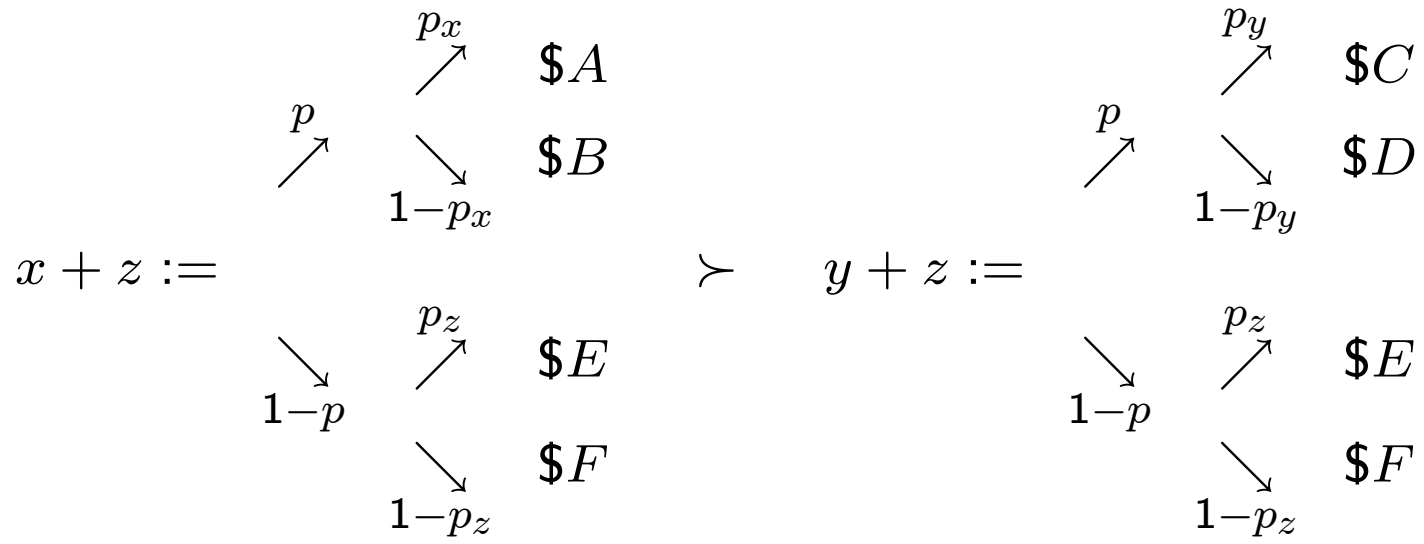
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$y + z :=$
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$\$C$
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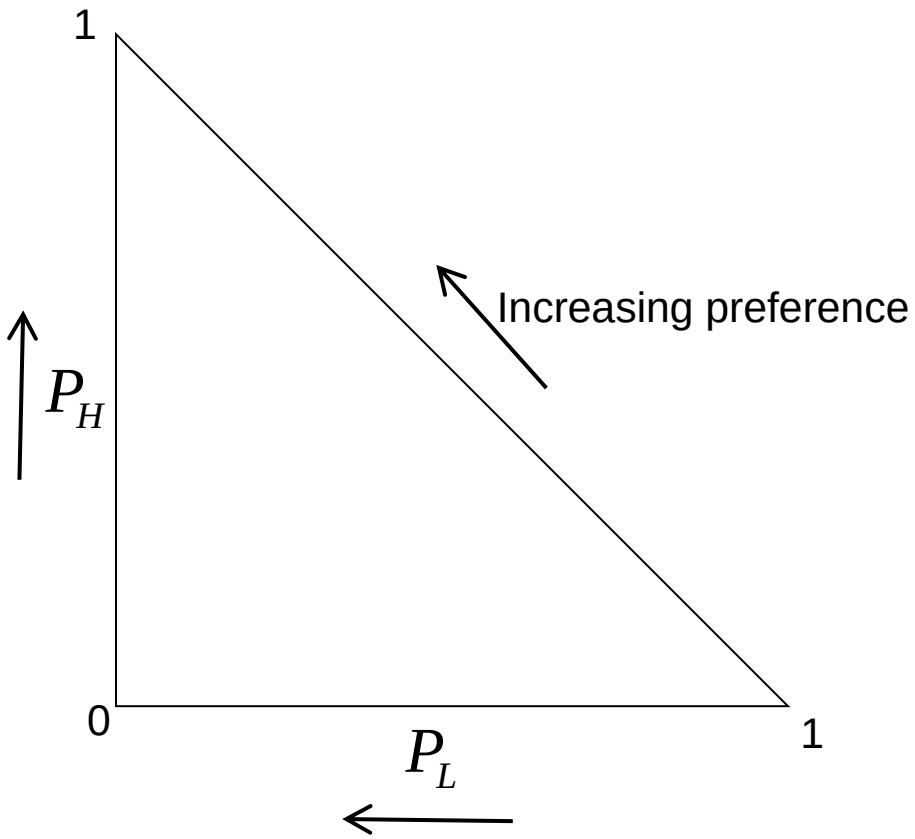
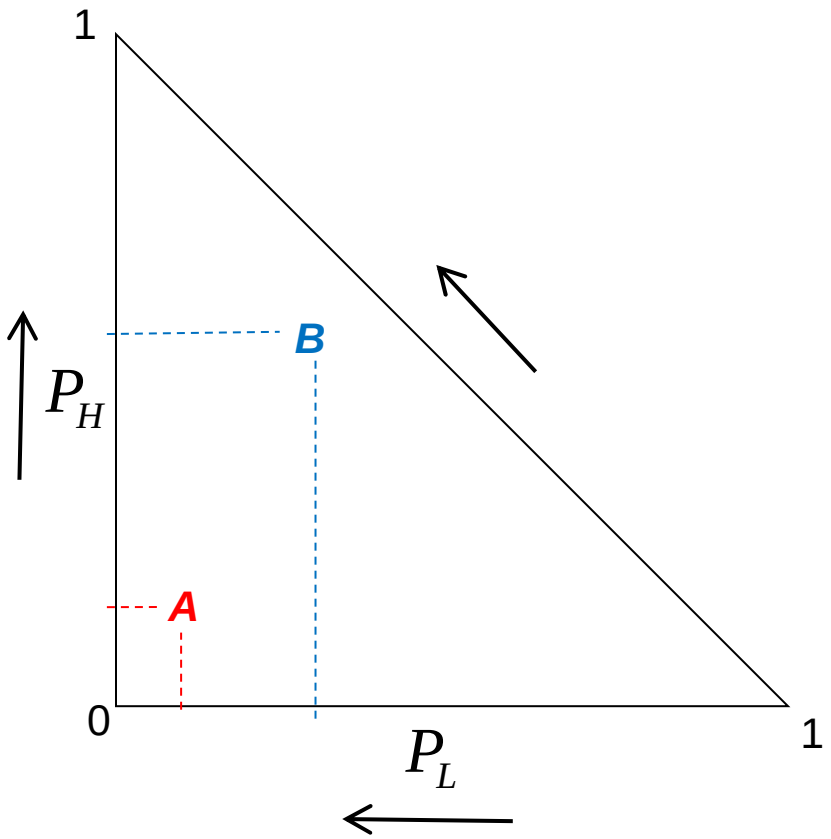
The (Marschak-Machina) probability triangle



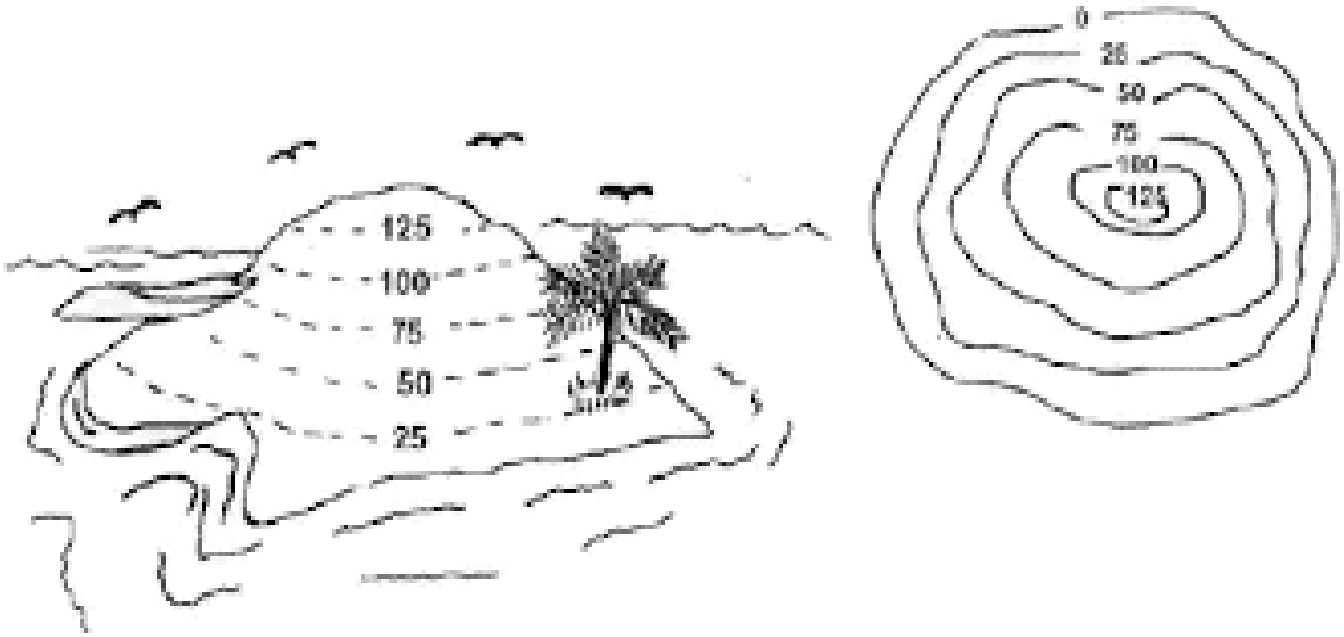
Consider three monetary payouts H, *M*, and *L* where *H>M>L*

Risk profiling

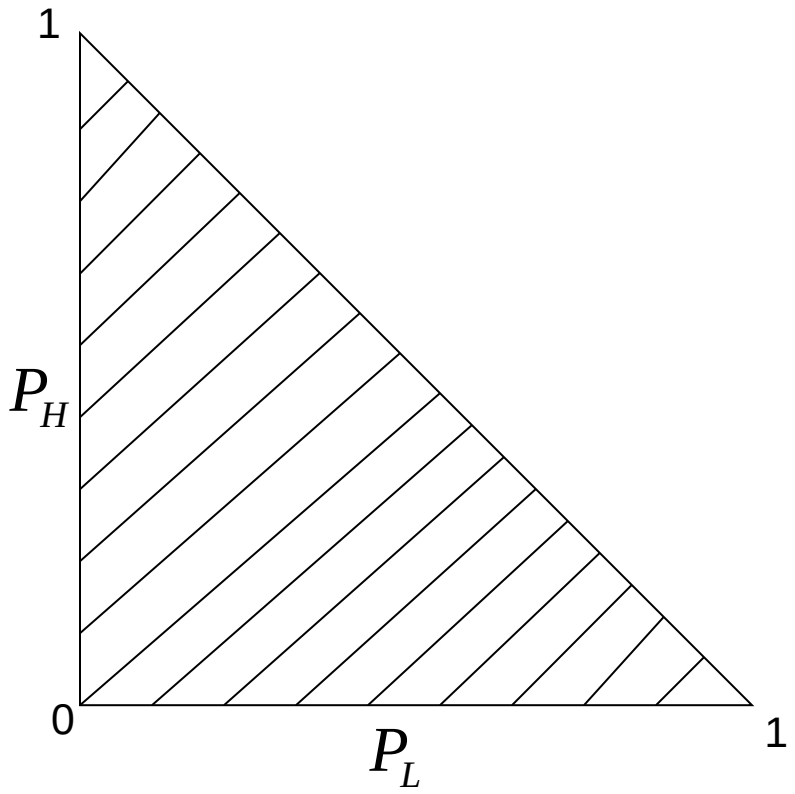


A “complete” risk profiling requires knowing all possible comparisons like between *A* and *B*.

A topographic map

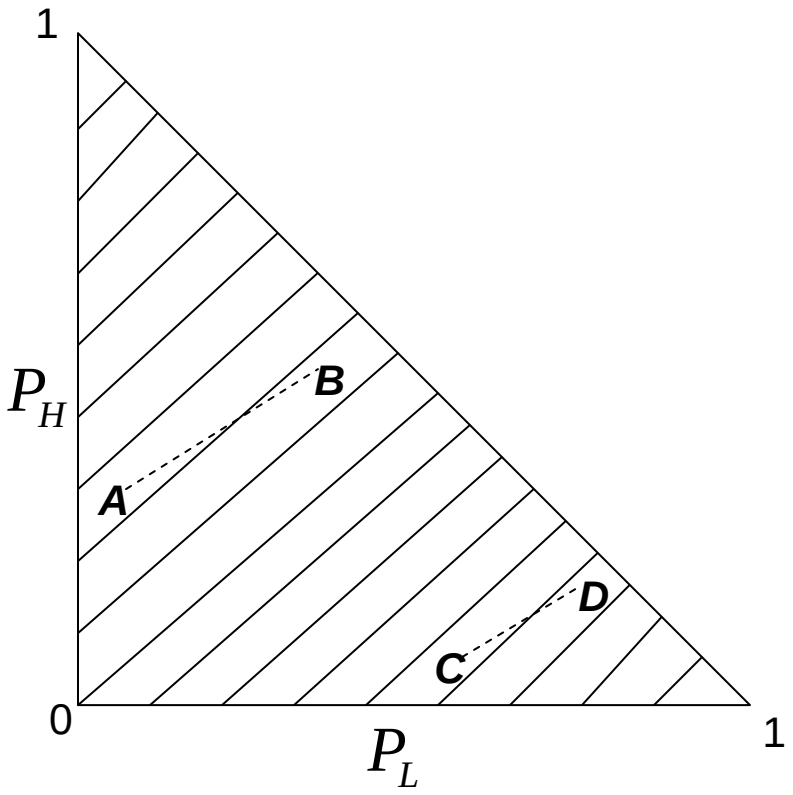


An indifference map of a loss-neutral (expected utility) individual



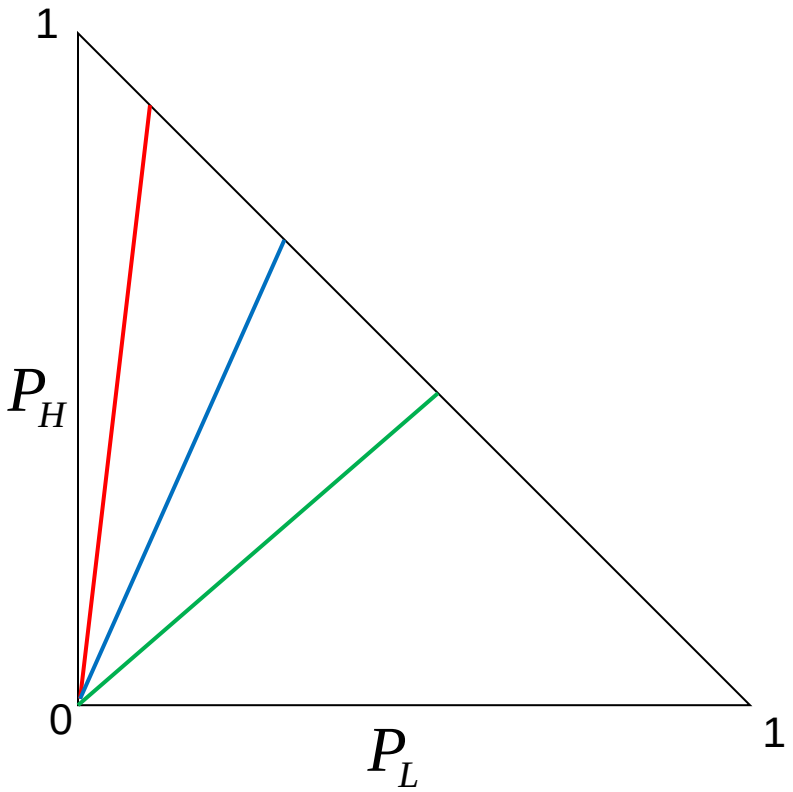
Expected Utility Theory (EUT) requires that indifference lines are parallel

A test of Expected Utility Theory (EUT)



EUT requires that indifference lines are parallel so one must choose either ***A*** and ***C***, or ***B*** and ***D***.

Loss neutral and more risk tolerant



Mr. Green is more risk tolerant than Mr. Blue who is more risk tolerant than Mr. Red. The gentlemen are loss neutral.

von Neumann and Morgenstern Expected Utility Theory (EUT)

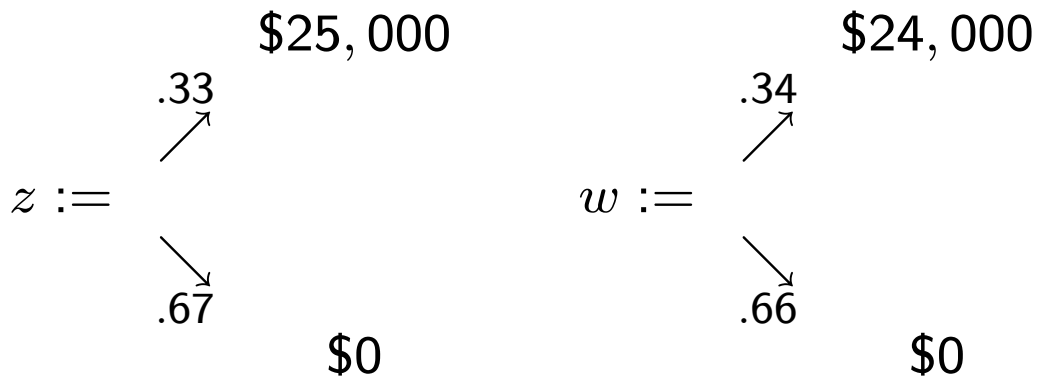
Allais (1953) I

– Choose between the two gambles:

$$x := \begin{array}{cc} & \$25,000 \\ \overset{.33}{\nearrow} & \\ \xrightarrow{.66} & \$24,000 \\ \underset{.01}{\searrow} & \\ & \$0 \end{array} \qquad y := \xrightarrow{1} \$24,000$$

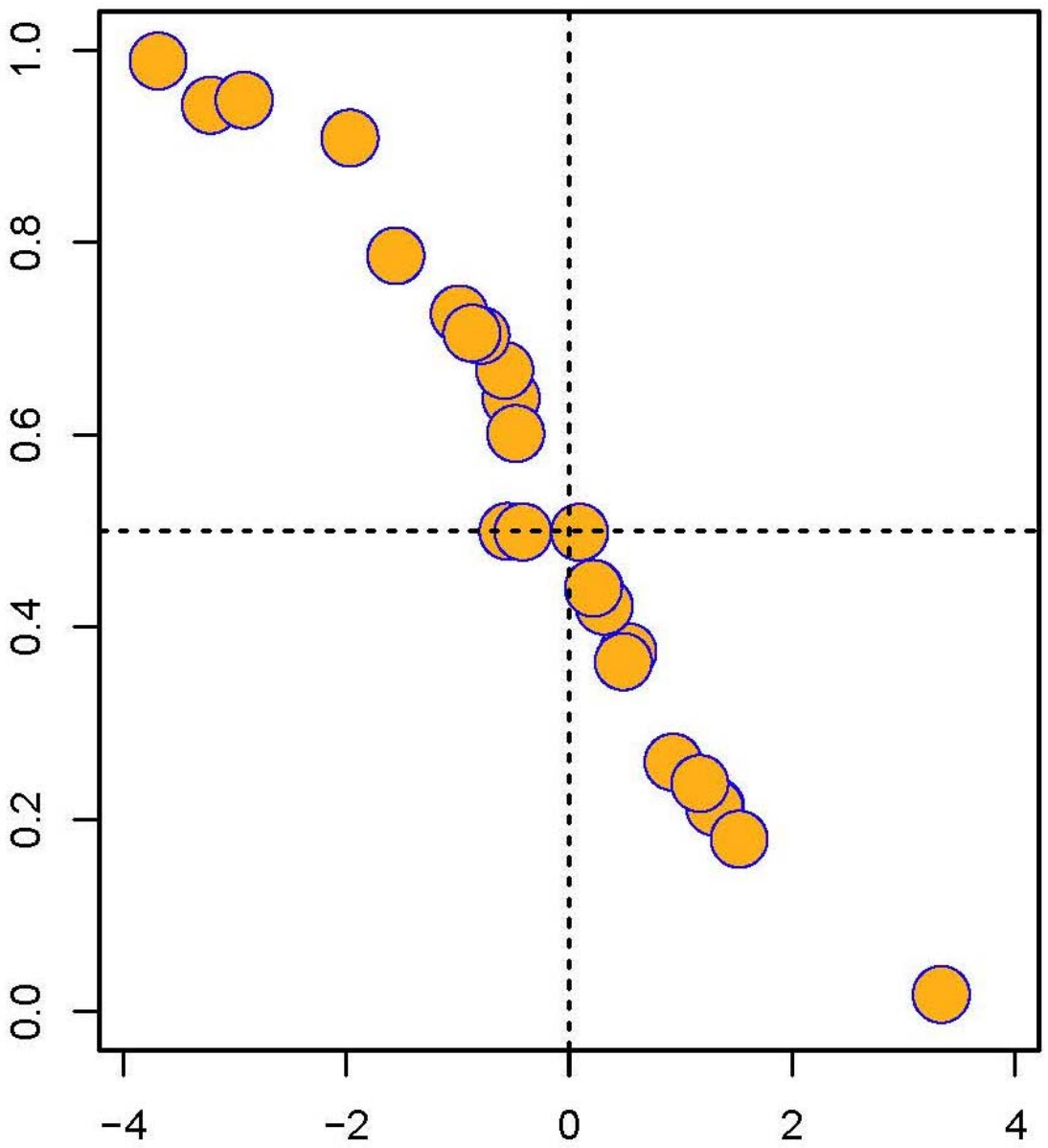
Allais (1953) II

– Choose between the two gambles:



Putting (risk) preferences under the microscope

Some “fingerprints” of individual behaviors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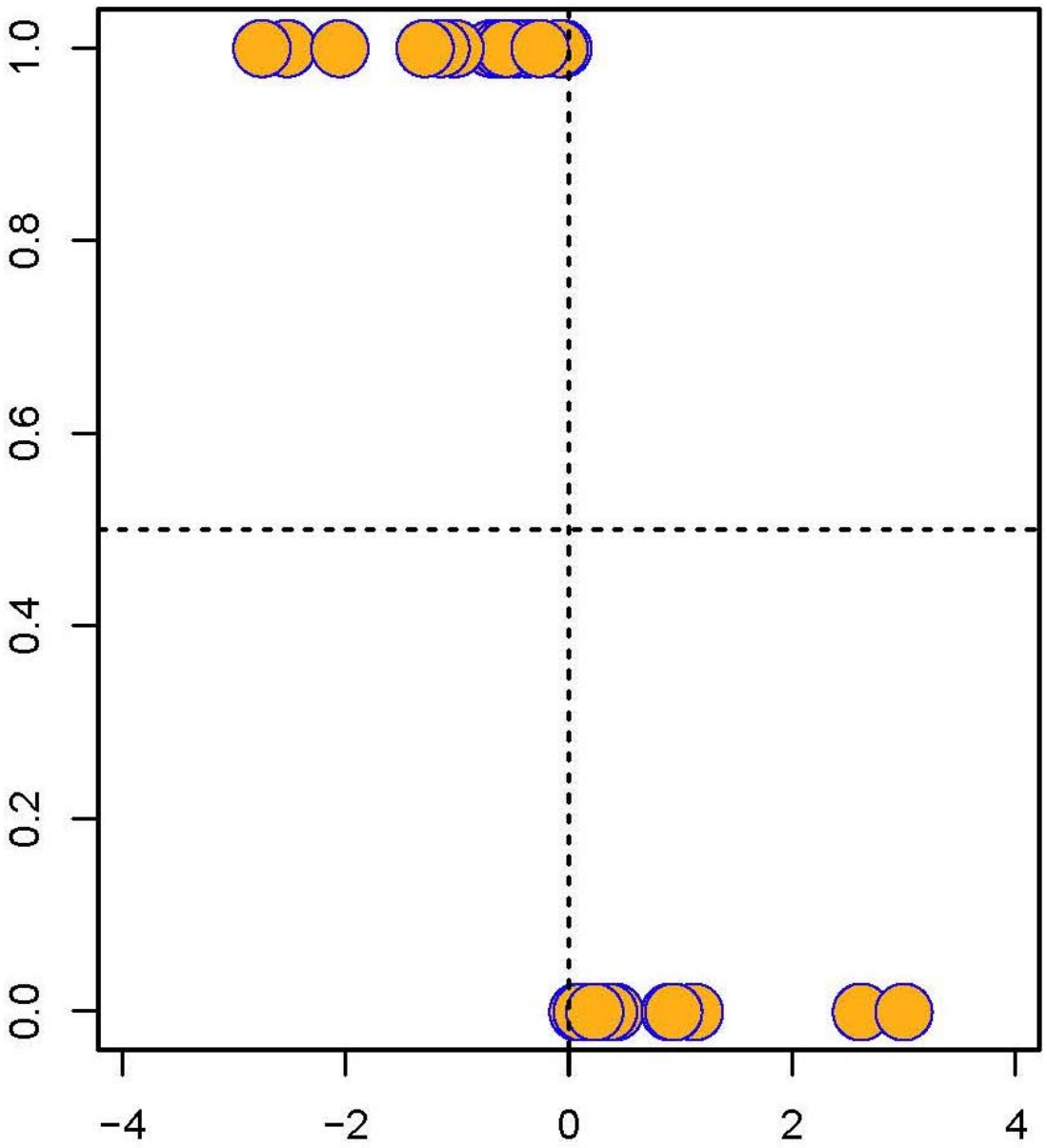
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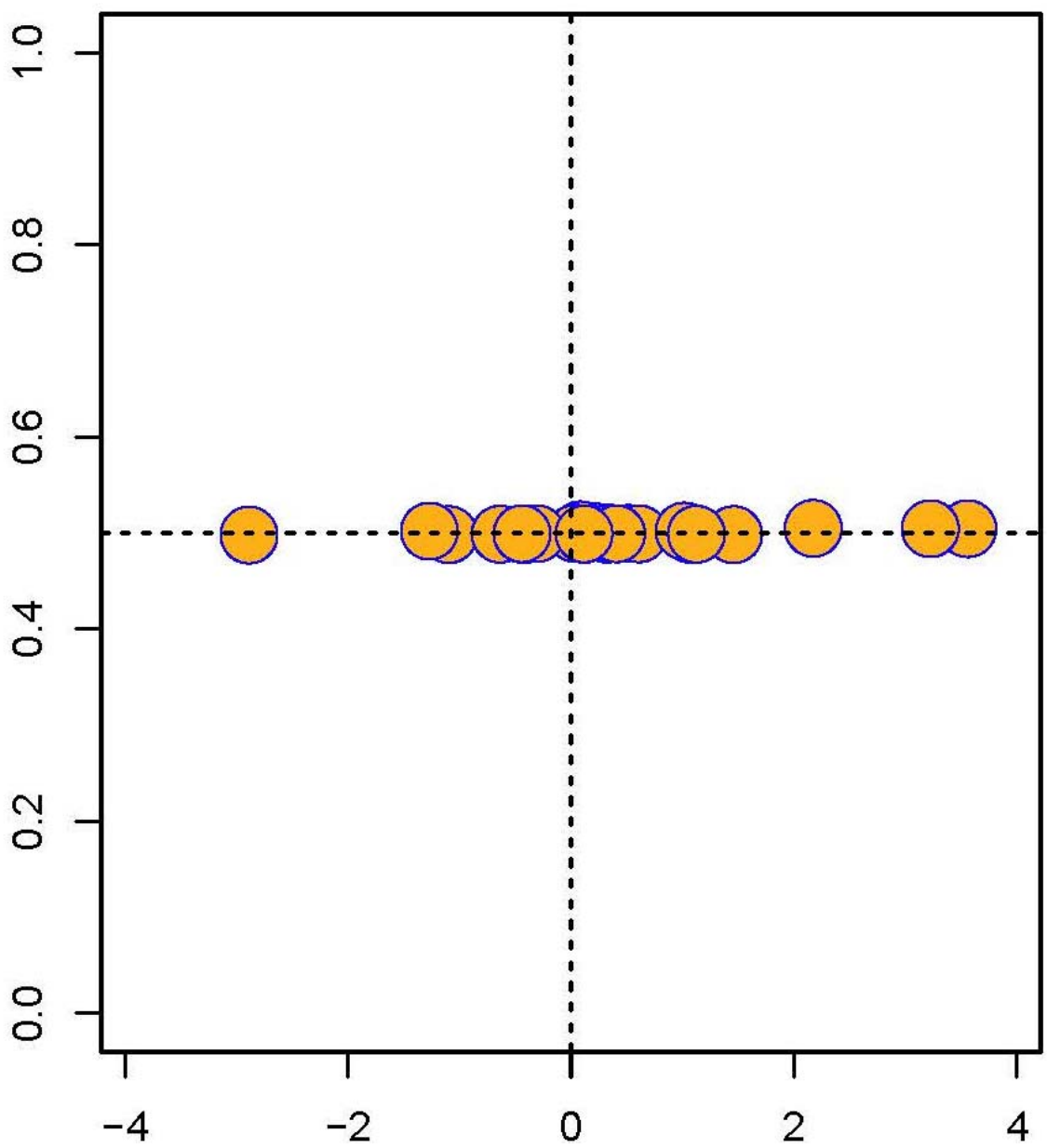
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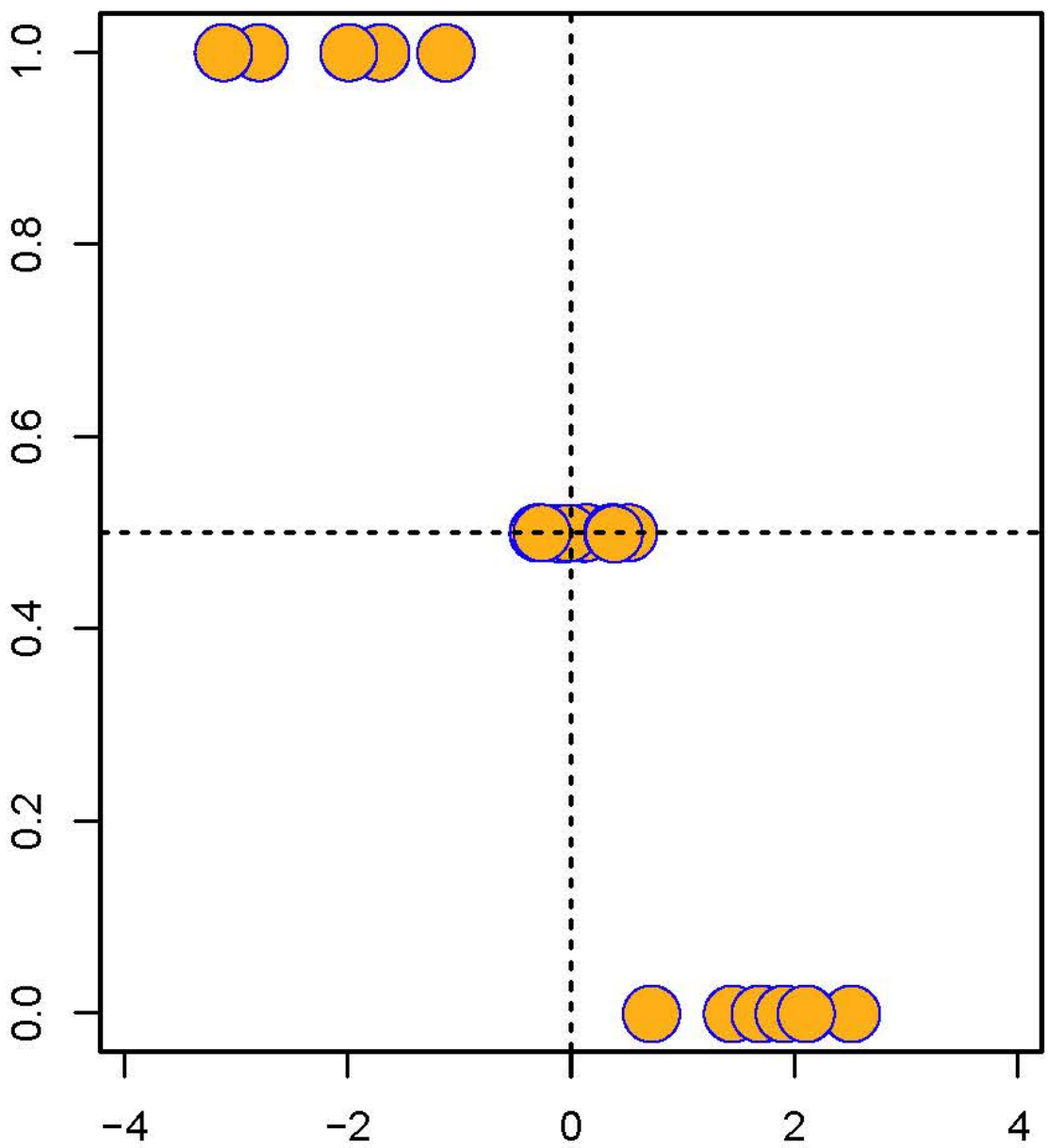
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Who is Homo-Economicus?

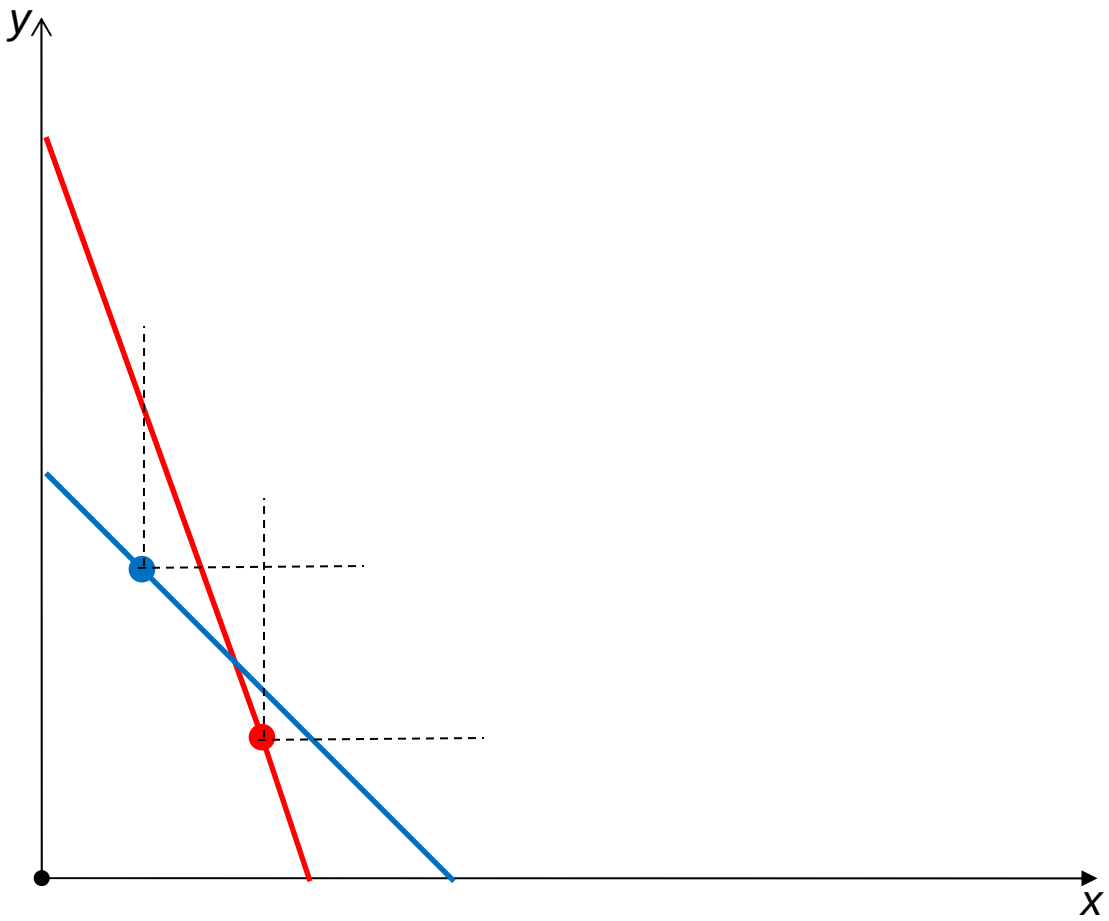
Testing rationality

- Classical revealed preference theory (Afriat, 1967) provides a direct test:

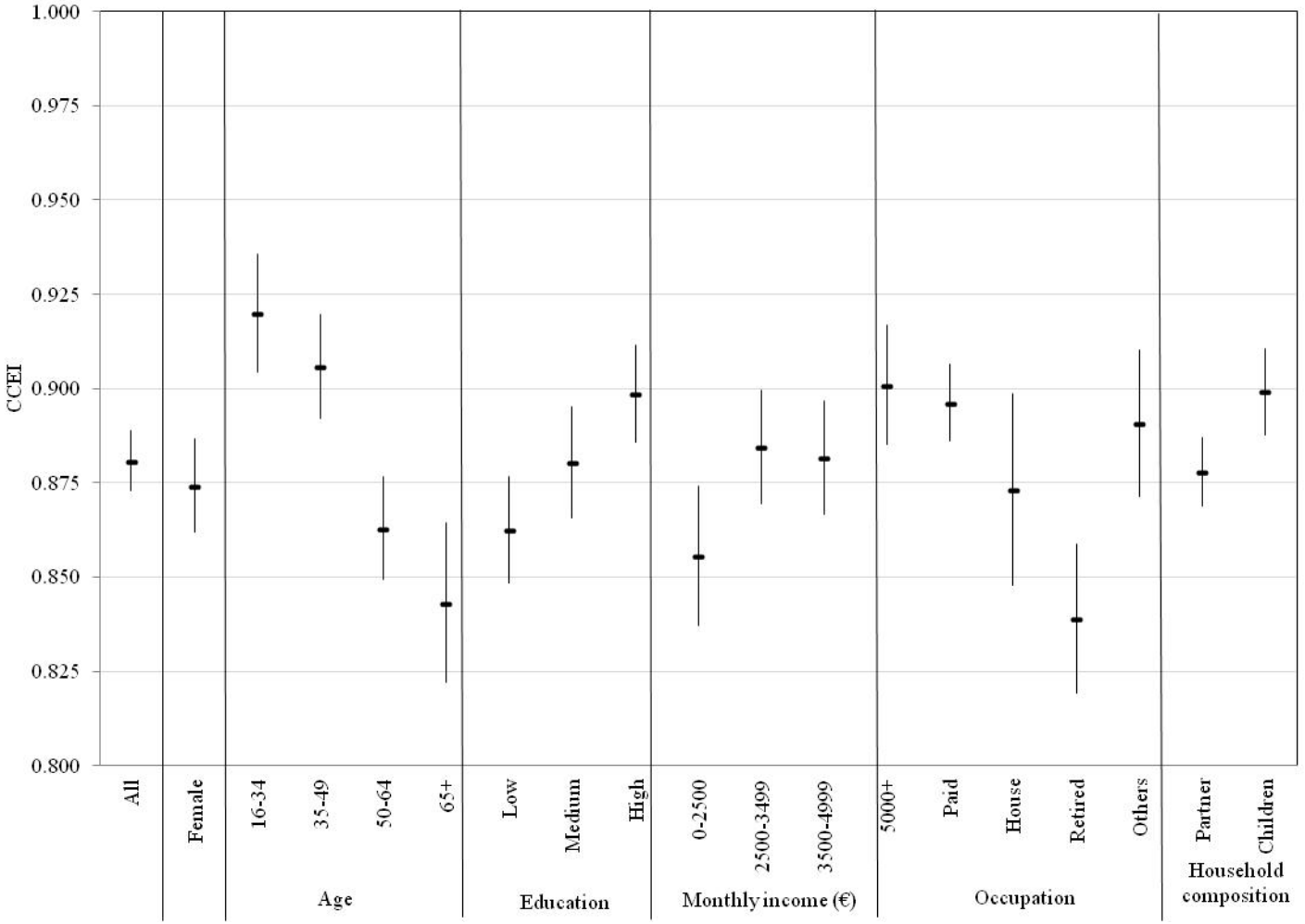
  choices are consistent with maximizing a *well-behaved* utility function if and only if they satisfy the Generalized Axiom of Revealed Preference (GARP).

- Since GARP offers an exact test, we assess how nearly individual choice behavior complies with GARP by using Afriat's (1972) Critical Cost Efficiency Index (CCEI).

The construction of a Homo-Economicus score



Homo Economicus: equiprobable lotteries



Wealth differentials

$\Longrightarrow$ The heterogeneity in wealth is not well-explained either by standard observables (income, education, family structure) or by standard unobservables (intertemporal substitution, risk tolerance).

$\Longrightarrow$ If consistency with utility maximization in the experiment is a good proxy for (financial) DMQ then the degree to which consistency differ across subjects should help explain wealth differentials.

The relationship between CCEI scores and wealth

	(1)	(2)	(3)
CCEI	1.351**	1.109**	101888.0*
	(0.566)	(0.534)	(52691.9)
Log 2008 household income	0.584***	0.606***	
	(0.132)	(0.126)	
2008 household income			1.776***
			(0.4)
Female	-0.313*	-0.356**	-32484.3*
	(0.177)	(0.164)	(17523.9)
Partnered	0.652***	0.595***	46201.9***
	(0.181)	(0.171)	(17173.7)
# of children	0.090	0.109	14078.6*
	(0.093)	(0.086)	(8351.5)
Age	Y	Y	Y
Education	Y	Y	Y
Occupation	Y	Y	Y
Constant	6.292	0.469	76214.4
	(6.419)	(3.598)	(559677.5)
R^2	0.179	0.217	0.188
# of obs.	517	566	568

The robustness of the correlation -- controls for constraints

	(1)	(2)	(3)	(4)	(5)
CCEI	1.322**	1.318**	1.925***	1.888***	1.441**
	(0.570)	(0.574)	(0.672)	(0.652)	(0.578)
Log household income					
2008	19.770	1.000	0.544***	0.285*	0.616***
	(14.629)	.	(0.137)	(0.165)	(0.128)
2008^2	-2.194				
	(1.533)				
2008^3	0.082				
	(0.053)				
2006				0.232	
				(0.231)	
2004				0.215	
				(0.174)	
Female	-0.291	-0.201	-0.337*	-0.296	-0.321*
	(0.181)	(0.173)	(0.185)	(0.186)	(0.176)
Partnered	0.598***	0.561***	0.734***	0.707***	0.641***
	(0.181)	(0.178)	(0.192)	(0.193)	(0.179)
# of children	0.091	0.101	0.018	0.031	0.088
	(0.092)	(0.096)	(0.099)	(0.095)	(0.093)
Age	Y	Y	Y	Y	Y
Education	Y	Y	Y	Y	N
Occupation	Y	Y	Y	Y	Y
Constant	-47.059	0.864	5.354	3.016	6.398
	(46.275)	(6.545)	(6.93)	(7.109)	(6.484)
R^2	0.187		0.205	0.217	0.177
# of obs.	517	517	449	449	517

The robustness of the correlation -- controls for preferences and beliefs

	(1)	(2)	(3)	(4)	(5)
CCEI	1.379**	1.396**	1.404**	1.214*	1.237**
	(0.568)	(0.568)	(0.569)	(0.625)	(0.623)
Risk tolerance					
Quantitative (experiment)	-0.768	-0.808	-0.766		
	(0.714)	(0.711)	(0.718)		
Qualitative (survey)		0.017	0.023		
		(0.074)	(0.076)		
Qualitative (survey) missing		-0.190	-0.162		
		(0.335)	(0.482)		
Conscientiousness			0.089		
			(0.072)		
Conscientiousness missing			-0.040		
			(0.668)		
Longevity expectations					-0.034
					(0.040)
Log 2008 household income	0.589***	0.578***	0.572***	0.443***	0.434***
	(0.132)	(0.131)	(0.133)	(0.123)	(0.123)
Female	-0.316*	-0.310*	-0.323*	-0.415**	-0.417**
	(0.177)	(0.181)	(0.181)	(0.186)	(0.186)
Partnered	0.655***	0.658***	0.642***	0.686***	0.687***
	(0.181)	(0.181)	(0.182)	(0.204)	(0.205)
# of children	0.086	0.087	0.083	0.075	0.083
	(0.093)	(0.093)	(0.093)	(0.102)	(0.102)
Age	Y	Y	Y	Y	Y
Education	Y	Y	Y	Y	Y
Occupation	Y	Y	Y	Y	Y
Constant	6.840	6.883	6.496	3.777	4.411
	(6.361)	(6.357)	(6.395)	(15.258)	(15.256)
R^2	0.179	0.176	0.176	0.163	0.163
# of obs.	517	517	517	414	414

	(1)	(2)	(3)	(4)
CCEI	1.253*	1.401*	1.269*	1.177**
	(0.712)	(0.729)	(0.729)	(0.583)
CCEI (combined dataset)	0.099			
	-0.38			
von Gaudecker et al. (2011)			0.927*	
			(0.485)	
Cognitive Reflection Test (CRT)				0.120*
				(0.071)
CRT missing				-0.203
				(0.237)
Log 2008 household income	0.586***	0.388*	0.383*	0.577***
	(0.132)	(0.155)	(0.154)	(0.132)
Female	-0.314*	-0.218	-0.207	-0.292*
	(0.177)	(0.212)	(0.211)	(0.176)
Partnered	0.653***	0.907***	0.926***	0.690***
	(0.181)	(0.230)	(0.228)	(0.181)
# of children	0.089	0.105	0.096	0.091
	(0.093)	(0.114)	(0.113)	(0.092)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	6.237	10.056	8.355	6.855
	(6.424)	(6.976)	(6.990)	(6.464)
R^2	0.177	0.225	0.232	0.181
# of obs.	517	326	326	517

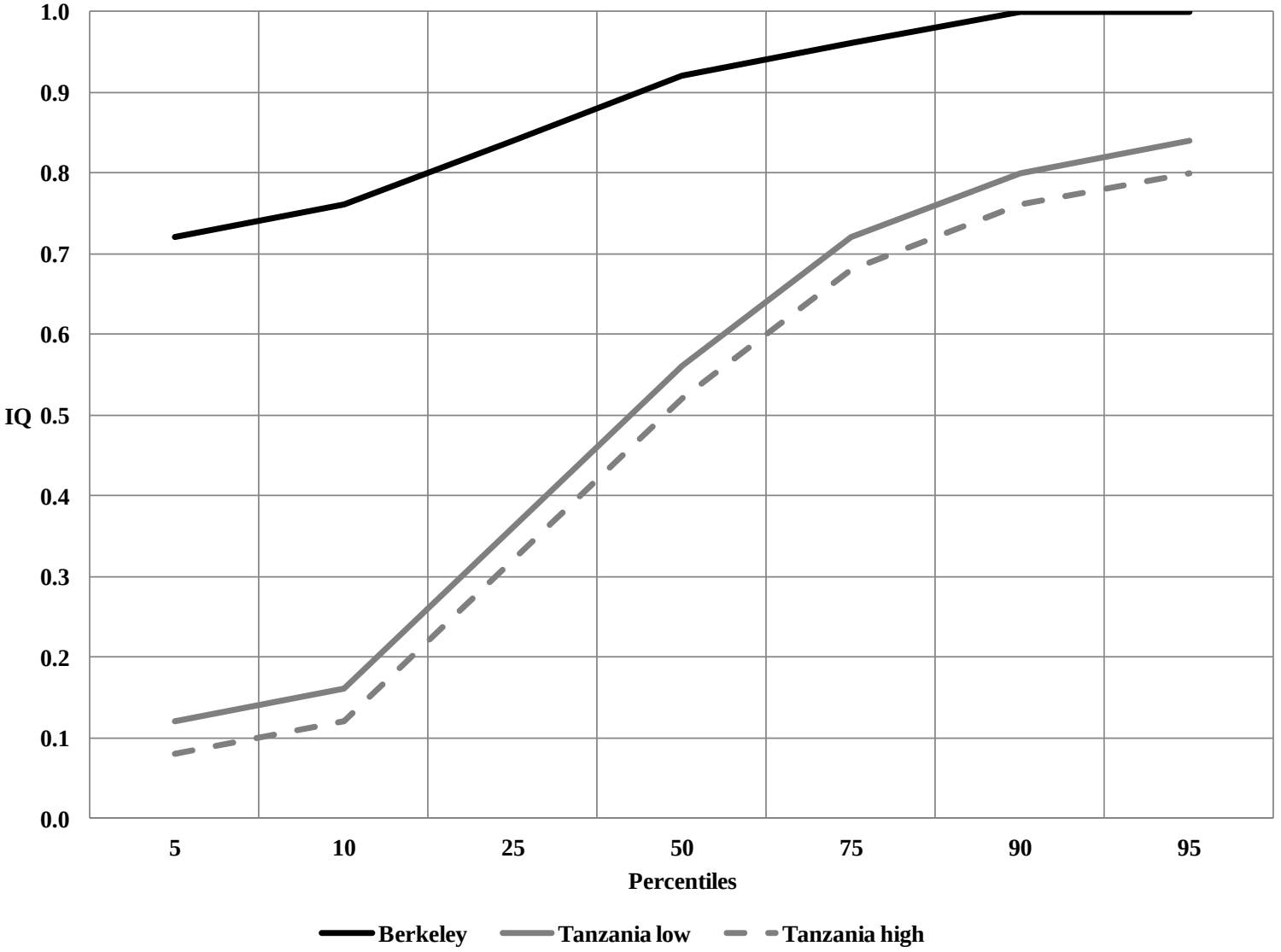
The sources of the relationship

	(1)	(2)	(3)	(4)
	Have checking	Fraction in checking	Have saving	Fraction in saving
CCEI	0.03	-0.098*	-0.047	-0.162*
	(0.032)	(0.057)	(0.053)	(0.097)
Log 2008 household income	0.001	-0.029**	0.003	-0.068***
	(0.002)	(0.013)	(0.010)	(0.021)
Female	0.007	0.023	0.014	0.038
	(0.005)	(0.020)	(0.019)	(0.033)
Partnered	-0.005	-0.031	0.017	-0.054
	(0.004)	(0.020)	(0.022)	(0.033)
# of children	0.000	-0.004	-0.025*	-0.043***
	(0.001)	(0.010)	(0.014)	(0.013)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	0.998***	0.106	1.126	1.448
	(0.172)	(0.822)	(0.848)	(1.288)
R^2	-0.007	0.021	-0.011	0.083
# of obs.	512	512	502	502

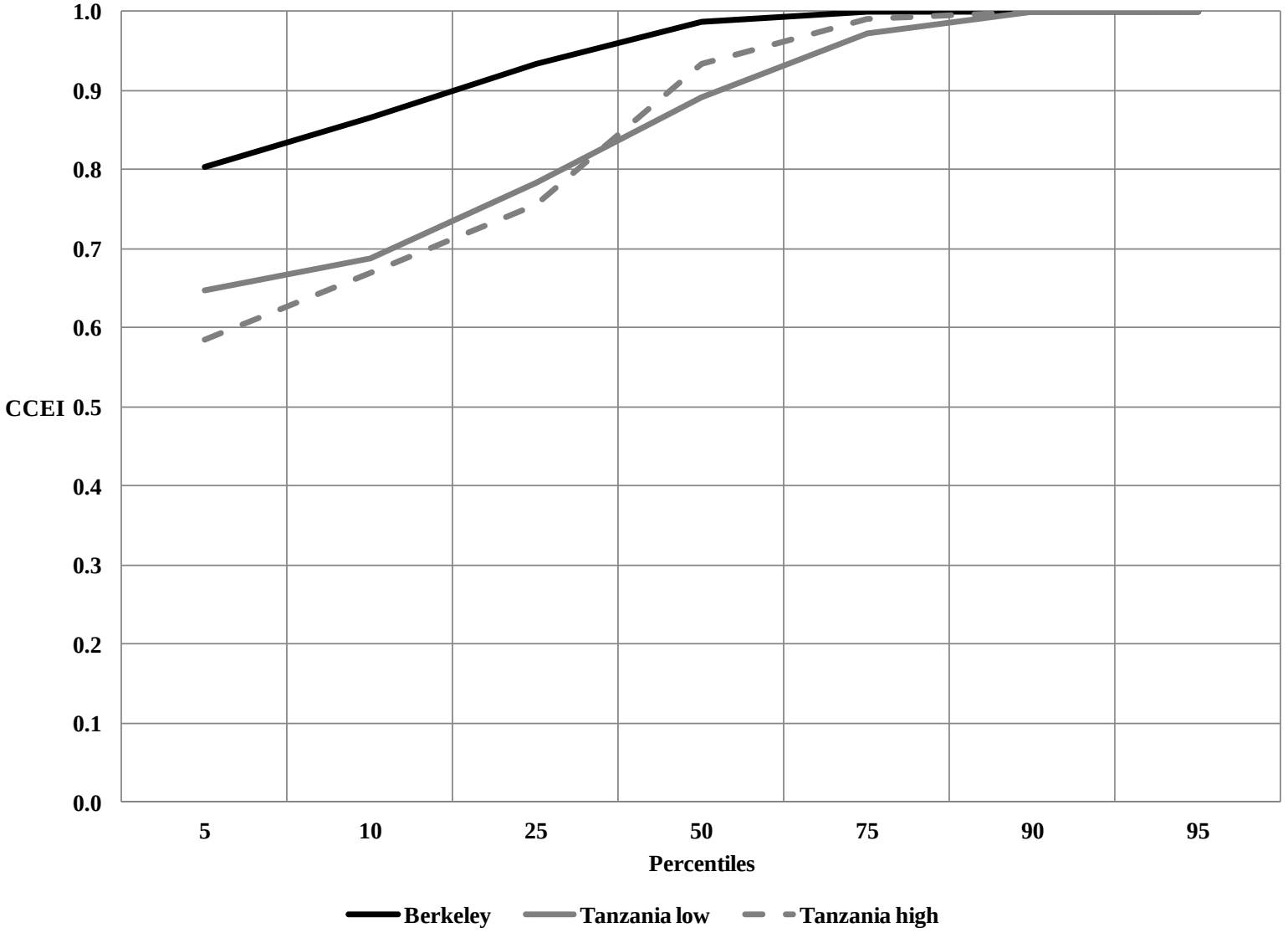
The sources of the relationship (cont.)

	(5)	(6)	(7)	(8)
	Have stocks	Fraction in stocks	Have a house	Fraction in house
CCEI	0.167	0.001	0.352**	0.324**
	(0.163)	(0.050)	(0.152)	(0.129)
Log 2008 household income	0.148***	0.013	0.134***	0.096***
	(0.031)	(0.009)	(0.029)	(0.024)
Female	0.007	0.009	-0.038	-0.066
	(0.050)	(0.013)	(0.050)	(0.043)
Partnered	0.005	-0.007	0.207***	0.127***
	(0.049)	(0.014)	(0.051)	(0.044)
# of children	0.003	0.000	0.048**	0.063***
	(0.026)	(0.007)	(0.020)	(0.019)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	-3.152*	-0.317	-1.047	-1.151
	(1.856)	(0.398)	(1.760)	(1.419)
R^2	0.079	0.002	0.148	0.123
# of obs.	514	514	479	479

Is there a development gap in rationality (IQ)?



Is there a development gap in rationality (CCEI)?



Risk aversion and loss aversion

Loss aversion/tolerance

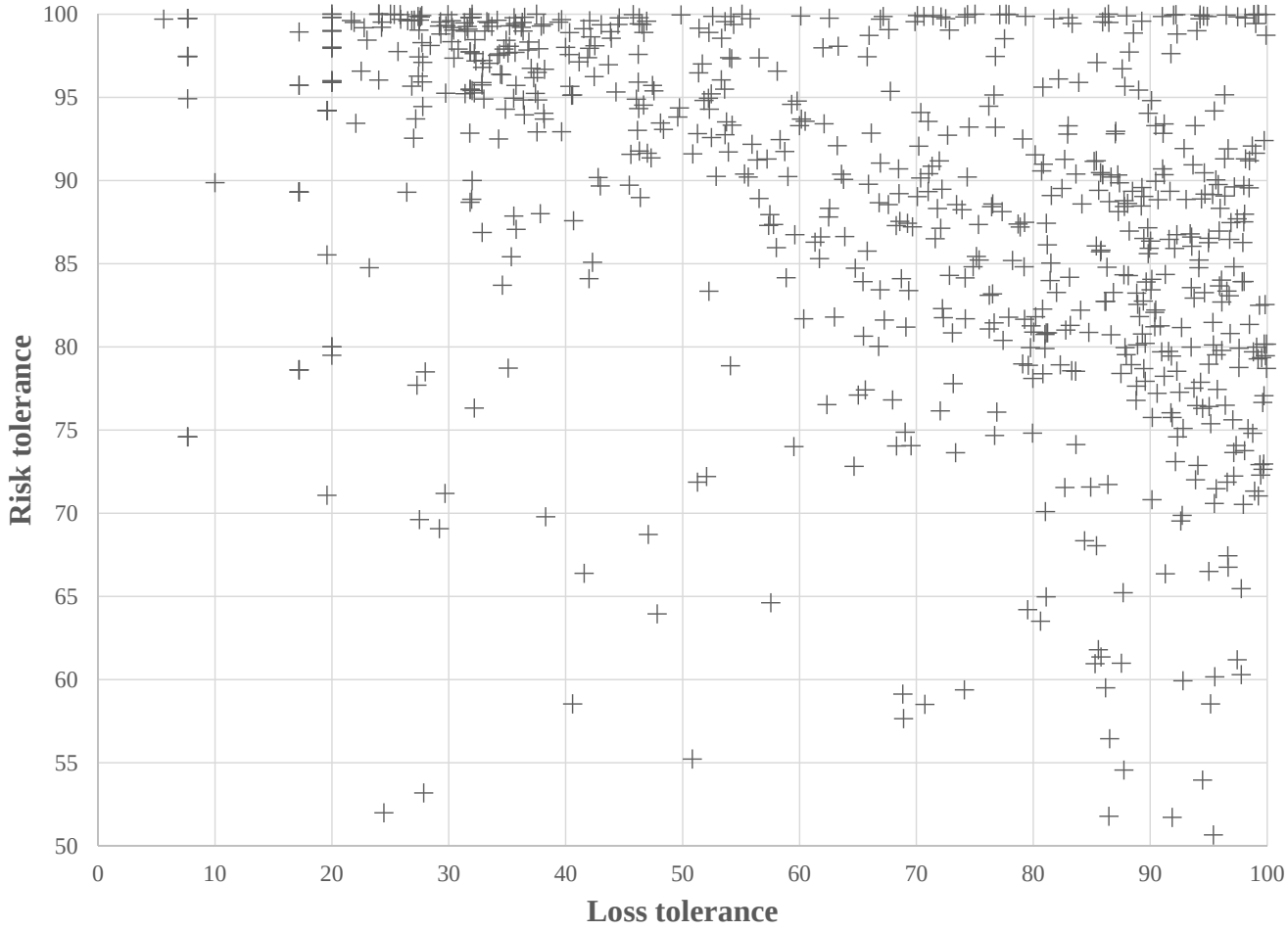
Suppose the underlying utility function over portfolios takes the form

$$\min \{\alpha u(x) + u(y), u(x) + \alpha u(y)\},$$

where $\alpha \geq 1$ measures loss aversion and $u(\cdot)$ measures risk aversion using CRRA or CARA.

If $\alpha > 1$ there is a kink at the point where $x = y$ and if $\alpha = 1$ we have loss neutrality (standard EUT representation).

Risk and loss tolerance



Ambiguity aversion

- The distinction between settings with risk and ambiguity dates back to at least the work of Knight (1921).

- Ellsberg (1961) countered the reduction of subjective uncertainty to risk with several thought experiments.

- A large theoretical literature (axioms over preferences) has developed models to accommodate this behavior.

Experiments à la Ellsberg

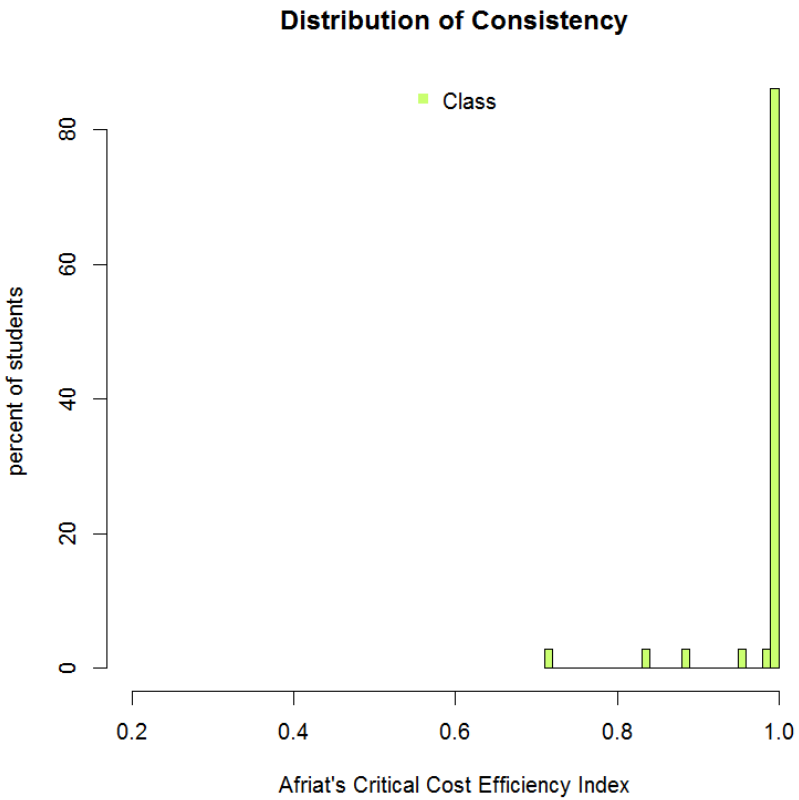
Consider the following four two-color Ellsberg-type urns (Halevy, 2007):

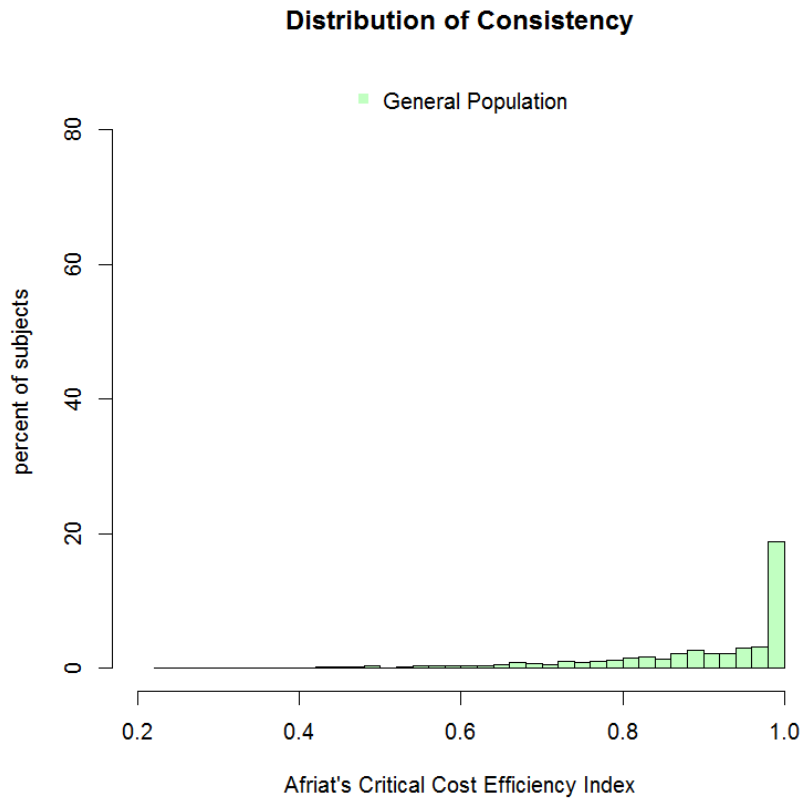
I. 5 red balls and 5 black balls

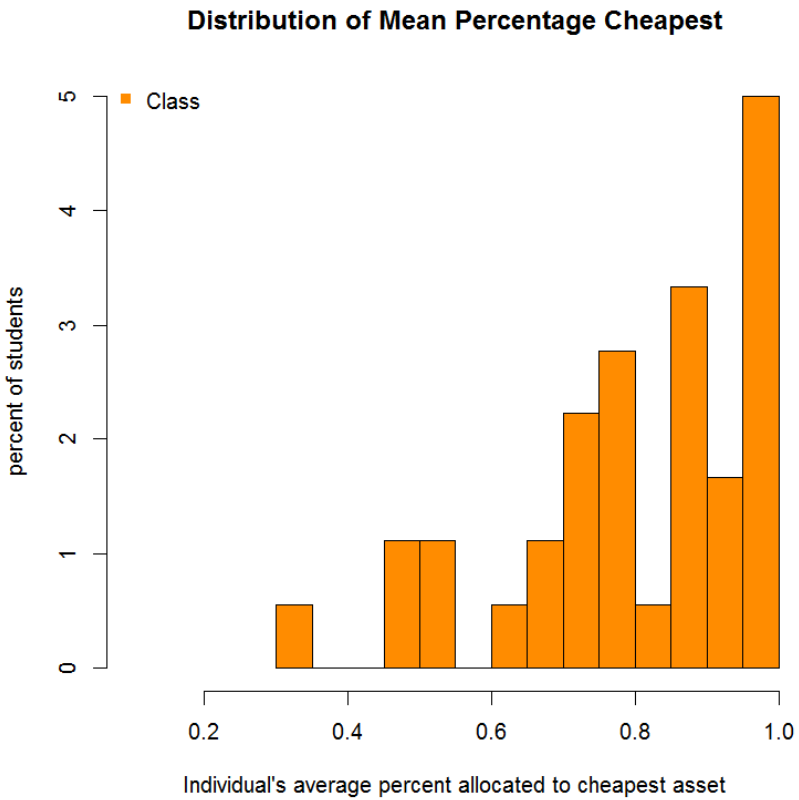
II. an unknown number of red and black balls

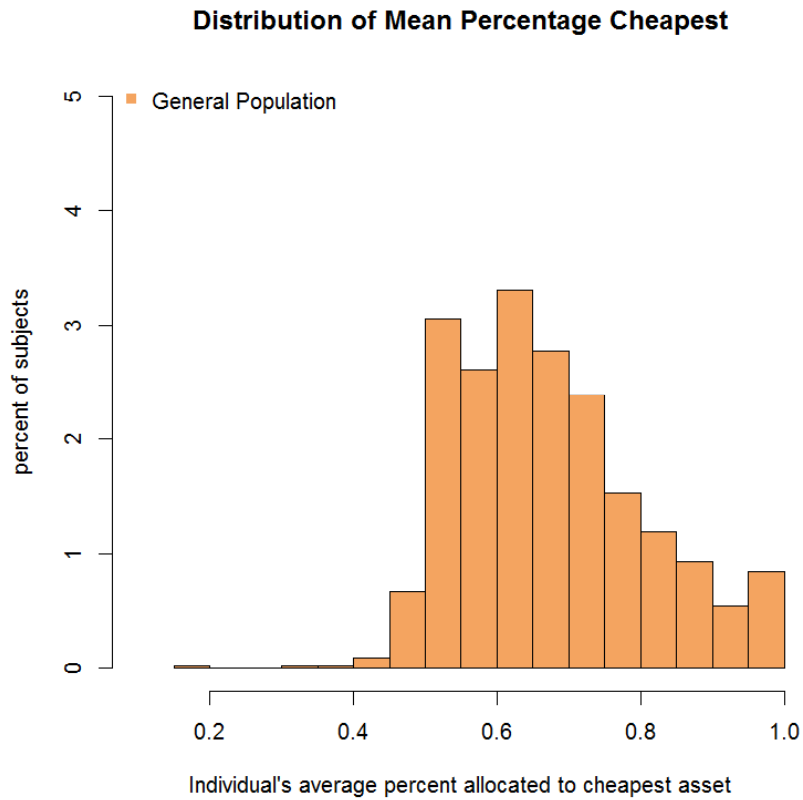
III. a bag containing 11 tickets with the numbers 0-10; the number written on the drawn ticket determines the number of red balls

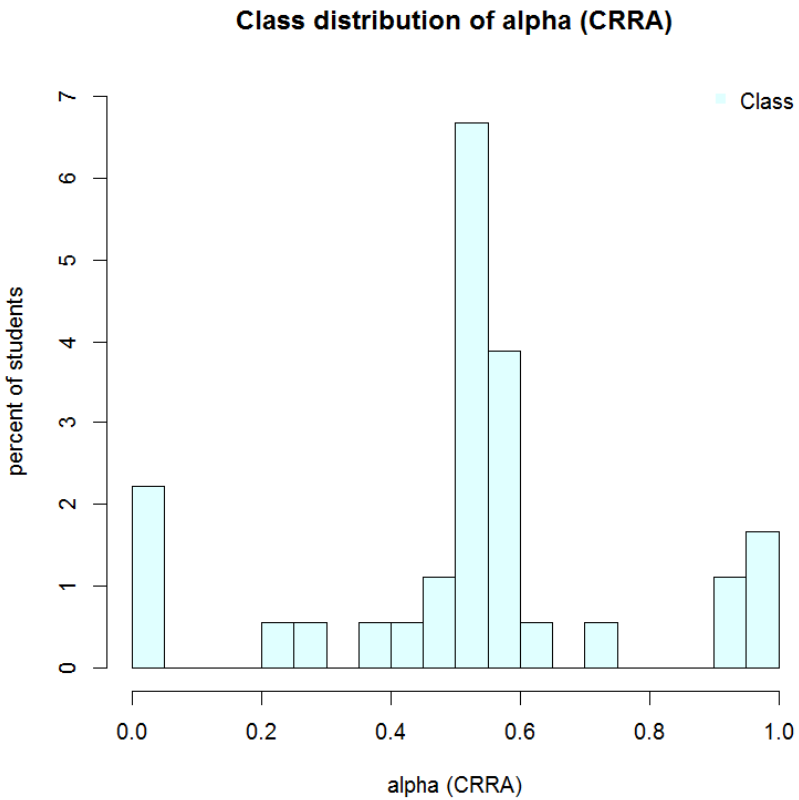
IV. a bag containing 2 tickets with the numbers 0 and 10; the number written on the drawn ticket determines the number of red balls

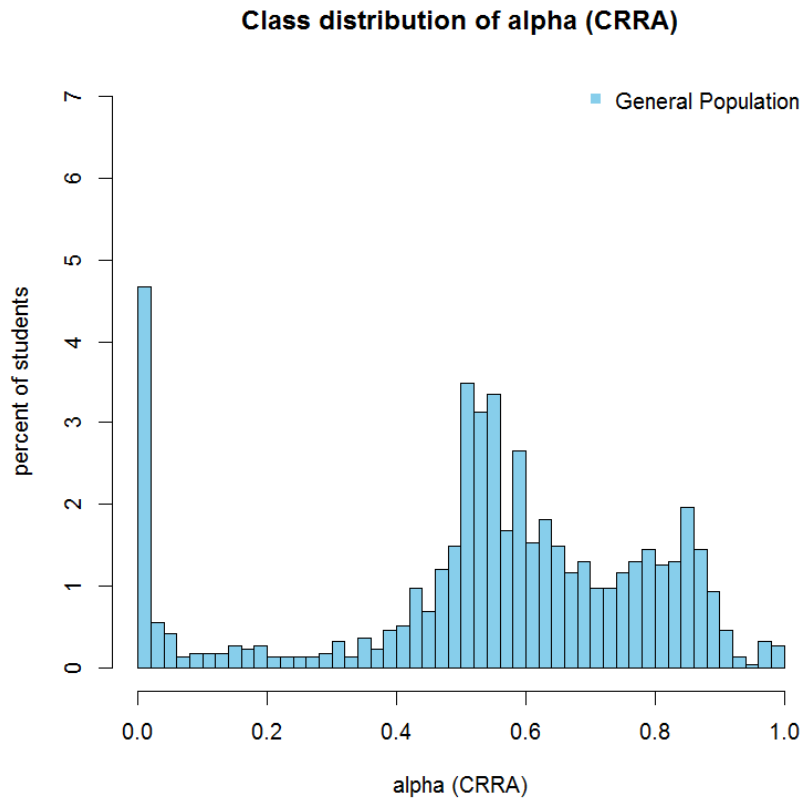

Distribution of Consistency
Class
percent of students
0
20
40
60
80
0.2
0.4
0.6
0.8
1.0
Afriat's Critical Cost Efficiency Index

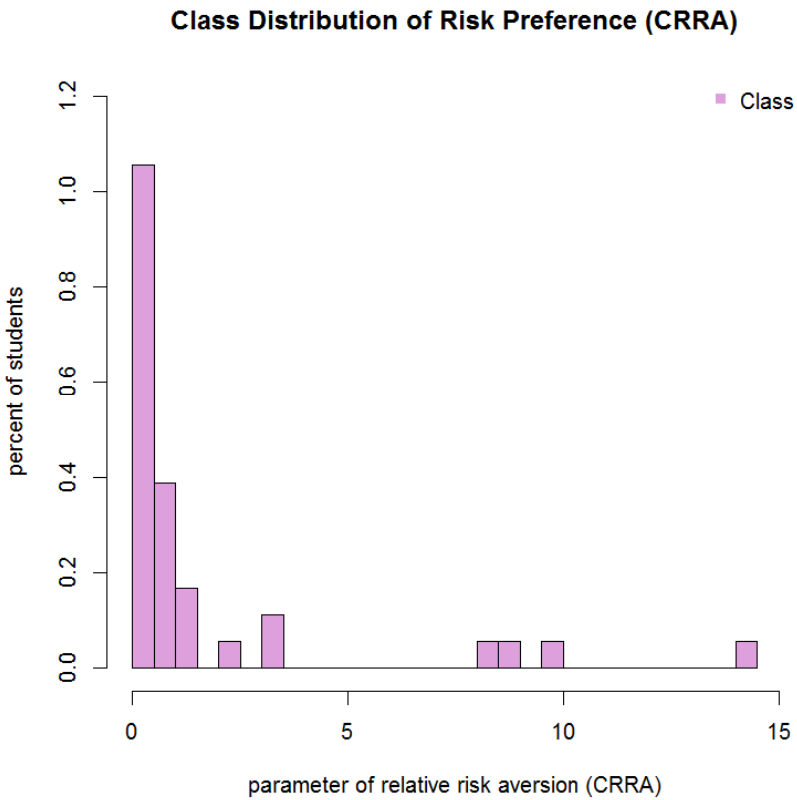

Distribution of Consistency
General Population
percent of subjects
0
20
40
60
80
0.2
0.4
0.6
0.8
1.0
Afriat's Critical Cost Efficiency Index

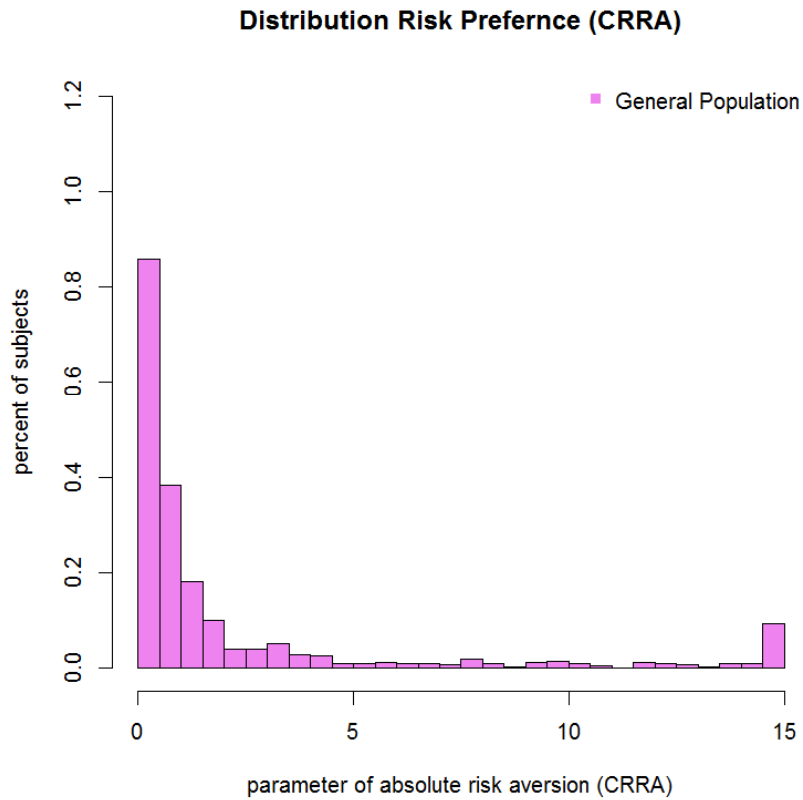

Distribution of Mean Percentage Cheapest
Class
percent of students
0
1
2
3
4
5
0.2
0.4
0.6
0.8
1.0
Individual's average percent allocated to cheapest asset


Distribution of Mean Percentage Cheapest
General Population
percent of subjects
0
1
2
3
4
5
0.2
0.4
0.6
0.8
1.0
Individual's average percent allocated to cheapest asset


Class distribution of alpha (CRRA)
Class
percent of students
0
1
2
3
4
5
6
7
0.0
0.2
0.4
0.6
0.8
1.0
alpha (CRRA)


Class distribution of alpha (CRRA)
General Population
percent of students
0
1
2
3
4
5
6
7
0.0
0.2
0.4
0.6
0.8
1.0
alpha (CRRA)


Class Distribution of Risk Preference (CRRA)
Class
percent of students
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
5
10
15
parameter of relative risk aversion (CRRA)


Distribution Risk Prefernce (CRRA)
General Population
percent of subjects
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
5
10
15
parameter of absolute risk aversion (CRRA)