Economics 100A
Fall 2001
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PROBLEM SET #10 SOLUTIONS

(Prepared by: Peter Adams)

1 Monopoly Behavior (Ch. 25)

1.1 Price Discrimination

Problem 1 *Suppose Napster is considering selling music via email. There are two types of users, Students and Non-Students. Each Non-Student has an inverse demand function,* $p_n(x) = 200 - x$, *and each Student has an inverse demand function,* $p_s(x) = 160 - x$, *where x is the number of songs delivered by email (p is measured in cents). The marginal cost to Napster of sending an additional song via email is ZERO. (For the following questions, it may be helpful to graph the two demand curves)*

See Figure 1

1. **Suppose Napster can identify all users as either Students or Non-Students. If Napster offers a fixed number of songs per year to each person, what is the profit maximizing level of songs offered to a Student (x_s) and a Non-Student (x_n)? In other words what is the equilibrium level of output for each type of person under first-degree price discrimination (i.e. perfect price discrimination).**

Napster would like to capture the entire consumer surplus from each individual. Since it can perfectly identify Students and Non-Students, Napster will supply the maximum quantity that each individual demands.

$$(x_n, x_s) = (200, 160)$$

2. **What is the dollar price charged to Students (p_s) and Non-Students (p_n) per year (Note: the price in the demand function is in cents)?**

Napster will charge the value of consumer surplus for each type of individual. In other words, Napster will charge the area under each demand curve up to the *quantity supplied.*

$$\begin{aligned}
Students &: \frac{1}{2}(160)^2 = 12800¢ = \$128 \\
Non-Students &: \frac{1}{2}(200)^2 = 20000¢ = \$200 \\
(p_n, p_s) &= (200, 128)
\end{aligned}$$

3. **Suppose Napster CANNOT identify users as Students or Non-Students. If Napster offers a bundle of 160 songs, what is the maximum price that Students will pay?**

Although Napster CANNOT identify Students from Non-Students, the maximum that Students are willing to pay is still the value of their consumer surplus at x=160.

$$p_s(160) = \$128$$

4. **What is the gross consumer surplus (i.e. area under the demand curve up to $x = 160$) that Non-Students enjoy if they consume 160 songs per year at the price from (3)? What is the net consumer surplus?**

Gross Consumer Surplus for Non-Students is the area under their demand curve, Area A+B.

$$\begin{aligned}
Gross\ CS_N(160) &= Area(A+B) \\
&= Area(A+B+C) - Area(C) \\
&= 20000 - \frac{1}{2}(40)^2 \\
&= 20000 - 800 \\
&= 19200¢ = \$192
\end{aligned}$$

Net Consumer Surplus for Non-Students is the Gross CS minus the cost of the bundle of songs, $Area(A)$.

$$\begin{aligned}
Net\ CS_N(160) &= Gross\ CS_N - Area(A) \\
&= 19200 - 12800 = 6400¢ = \$64
\end{aligned}$$

5. **What is the maximum price Napster can charge for 200 songs per year if it offers 160 songs per year at the highest price Students are willing to pay?**

If Napster is offering 160 songs at a price of \$128, then the most it can charge Non-Students, and get them to self-select the 200 song bundle over the 160 song bundle, is the $Area(A+C)$.

$$\begin{aligned} p_n(200 \mid 160) &= Area\,(A + C) \\ &= 12800 + 800 \\ &= 13600¢ = \$136 \end{aligned}$$

6. **If Napster offers a "Student Package" of 140 songs per year, what is the most it can charge and still get Students to buy?**

See Figure 2

The most Napster can charge Students is their Gross Consumer Surplus at a quantity of 140 songs.

$$\begin{aligned} p_s(140) &= 12800 - \frac{1}{2}(20)^2 \\ &= 12800 - 200 \\ &= 12600¢ = \$126 \end{aligned}$$

7. **How much net consumer surplus does a Non-Student get from buying the "Student Package"?**

$$\begin{aligned} Net\ CS_N(140) &= Max\ CS_N - Area\,(A + C + D) \\ &= Max\ CS_N - p_s(140) - Area\,(C + D) \\ &= 20000 - 12600 - \frac{1}{2}(60)^2 \\ &= 20000 - 12600 - 1800 \\ &= 5600¢ = \$56 \end{aligned}$$

8. **What is the most Napster can charge for the 200 song "Non-Student Package" if it is offering the 140 song "Student Package" at the price from (6)?**

If Napster is offering 140 songs at a price of \$126, then the most it can charge Non-Students and get them to self-select the 200 song bundle over the 140 song bundle is equal the $Area\,(A + C + D)$.

$$\begin{aligned} p_n(200 \mid 140) &= Area\,(A + C + D) \\ &= 12600 + 1800 \\ &= 14400¢ = \$144 \end{aligned}$$

9. **Assuming there is only one Student and one Non-Student in the population, does Napster make more profit by offering the 160 song "Student Package" or the 140 song "Student Package"?**

For simplicity, assume FC=0. Because MC=0, all of the revenue that Napster collects we can treat as profit. So the total profit for Napster under each scenario is the revenue collected from the Student and the Non-Student.

$$\begin{aligned}\pi(200,160) &= p_n(200 \mid 160) + p_s(160) \\ &= \$136 + \$128 \\ &= \$264\end{aligned}$$

$$\begin{aligned}\pi(200,140) &= p_n(200 \mid 140) + p_s(140) \\ &= \$144 + \$126 \\ &= \$270\end{aligned}$$

$$\pi(200,140) = \$270 > \$264 = \pi(200,160)$$

10. **(Difficult) In general, if Napster cannot distinguish between Students and Non-Students, what areas of the graph is Napster trying to maximize (see page 437, Figure 25.3.C)? What are the optimal (profit maximizing) bundles (x_s, x_n) for Napster to offer and prices (p_s, p_n) for Napster to charge assuming the number of Students equals the number of Non-Students? [Hint: Napster wants to maximize the area $2A + C + D$. Write this area as a function of output (x), and solve for the maximum (x^*)].**

See Figure 3

With an equal number of Students and Non-Students, Napster would like to maximize the sum of revenue collected from a Student and a Non-Student. Napster will sell 200 units to the Non-Students at a price of $Area\,(A + C + D)$. *The problem for Napster is to choose the Student quantity,* x_s, *and sell it at a price* $p_s(x_s) = Area\,(A)$.

$$\begin{array}{rcl}
\underset{x_s}{max}\ \pi\left(200, x_s\right) & = & Area\left(A + C + D\right) + Area\left(A\right) \\
Area\left(A\right) & = & 12800 - \frac{1}{2}\left(160 - x_s\right)^2 \\
Area\left(C + D\right) & = & \frac{1}{2}\left(200 - x_s\right)^2 \\
\underset{x_s}{max}\ \pi\left(200, x_s\right) & = & \left[12800 - \frac{1}{2}\left(160 - x_s\right)^2 + \frac{1}{2}\left(200 - x_s\right)^2\right] \\
 & & + \left[12800 - \frac{1}{2}\left(160 - x_s\right)^2\right] \\
 & = & 25600 - \frac{1}{2}\left(160 - x_s\right)^2 + \frac{1}{2}\left(200 - x_s\right)^2 \\
FOC & : & 2\left(160 - x_s\right) - \left(200 - x_s\right) = 0 \\
x_s & = & 120
\end{array}$$

In this problem we have linear demand curves, so we can the areas of triangles to define the profit function. In general, however, we can define the area under each demand curve using integral calculus.

$$\begin{array}{rcl}
\underset{x_s}{max}\ \pi\left(200, x_s\right) & = & Area\left(A + C + D\right) + Area\left(A\right) \\
Area\left(A\right) & = & \int\limits_0^x \left(160 - t\right) dt \\
Area\left(C + D\right) & = & \int\limits_{x_s}^{200} \left(200 - t\right) dt \\
\underset{x_s}{max}\ \pi\left(200, x_s\right) & = & \left[2\int\limits_0^{x_s} \left(160 - t\right) dt\right] + \left[\int\limits_{x_s}^{200} \left(200 - t\right) dt\right] \\
 & = & 2\left(160x - \frac{x_s^2}{2}\right) + \left(20000 - 200x + \frac{x_s^2}{2}\right) \\
 & = & 20000 - 120x - \frac{x_s^2}{2} \\
FOC & : & -120 - x_s = 0 \\
x_s & = & 120
\end{array}$$

2 Game Theory (Ch. 28)

Definition 1 *A Nash Equilibrium (for a 2-player game) is a set of strategies* (s_1^*, s_2^*) *such that the strategy for player 1,* s_1^**, is the best response (in terms of payoffs) to the strategy of player 2,* s_2^**, and vice versa.*

Problem 2 *Consider the following game.*

		Player 2	
		d	e
Player 1	A	$(4, 9)$	$(4, 9)$
	B	$(6, 5)$	$(0, 1)$
	C	$(8, 7)$	$(3, 2)$

1. **What are the pure strategy Nash equilibria of this game?**

The best response of each player to a strategy of the other player is underlined in the table below. The Nash equilibria are those cells (or strategy pairs) in which both players are playing a best response.

		Player 2	
		d	e
Player 1	A	$(4, \underline{9})$	$(\underline{4}, \underline{9})$
	B	$(6, \underline{5})$	$(0, 1)$
	C	$(\underline{8}, \underline{7})$	$(3, 2)$

So, the pure strategy Nash equilibria of this game are:

$$\begin{aligned} N.E. &= (A, e) \\ &= (C, d) \end{aligned}$$

2. **Write this game in extensive form as a sequential game with Player 1 moving first.**

See Figure 4

Definition 2 *The backwards induction outcome of a sequential game (under complete and perfect information) is solved by finding Players 2's best strategy (e.g. d or e) under EACH strategy (e.g. A, B, and C) for Player 1. Player 1 then selects the strategy (A, B, or C) that gives the highest payoff among Player 2's best strategies.*

3. **Solve for the backwards induction outcome.**

The Backwards induction outcome is determined by finding Player 2's best response given each strategy of Player 1. Player 1 knows Player 2 will choose strategies in this manner. So, Player 1 then chooses the strategy with the highest payoff among Player 2's best responses. The best responses of each player are highlighted in gray (above).

$$\begin{aligned} B.I.\ outcome &= (C, d) \\ Payoff &= (8, 7) \end{aligned}$$

Problem 3 *(Difficult) A dollar is to be divided between two people. The two people simultaneously announce shares,* s_1 *and* s_2*. If* $s_1 + s_2 \leq \$1$ *then each person gets his announced share. Otherwise, if* $s_1 + s_2 > \$1$*, then the payoff to each is* $\$0$*. What is the SET of pure-strategy Nash equilibria for this game? (Hint: Think about the best response of each player given a particular announcement (or share) from the other player).*

(NOTE: The problem does not state explicitly that each player must choose a share, $s \epsilon [0,1]$*. However, clearly there is no incentive to choose are share greater than $1, as you are guaranteed a payoff = 0). Now, suppose Player 1 proposes a share,* $s_1 < \$1$*. Player 2's best response is* $s_2 = \$1 - s_1$*, and nothing less. Likewise, if Player 2 proposes a share* $s_2 < \$1$*, then Player 1's best response is* $s_1 = \$1 - s_2$*, and nothing less. Therefore, any combination of shares such that they exactly sum to* $\$1$ *is a Nash equilibrium. This implies a continuum of Nash equilibria such that the two shares exactly sum to $1.*

What about the case when a player proposes a share, $s = \$1$*? If Player 1 proposes* $s_1 = \$1$*, then player 2's best response is any share in the closed unit interval,* $s_2 \epsilon [0,1]$*, because the payoff to Player 2 is zero in all cases. The same is true for Player 1 if Player 2 proposes* $s_2 = \$1$*. So, the only case where these best responses overlap is when both players propose* $\$1$*. So, the strategy pair,* $(1,1)$*, is also a Nash equilibrium of this game. The full set of Nash Equilibria can be summarized as follows:*

$$N.E. = (1,1) \cup \{(s_1, s_2) : s_1 + s_2 = \$1\}$$

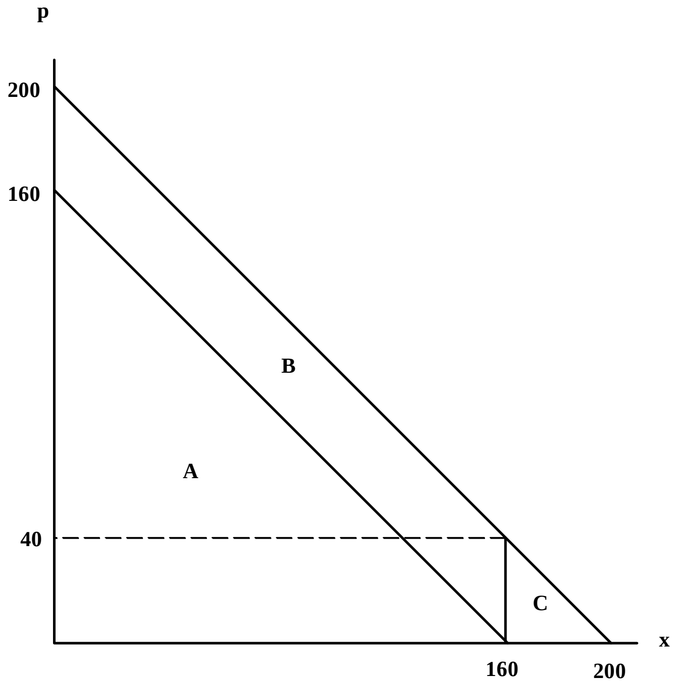

Figure 1:

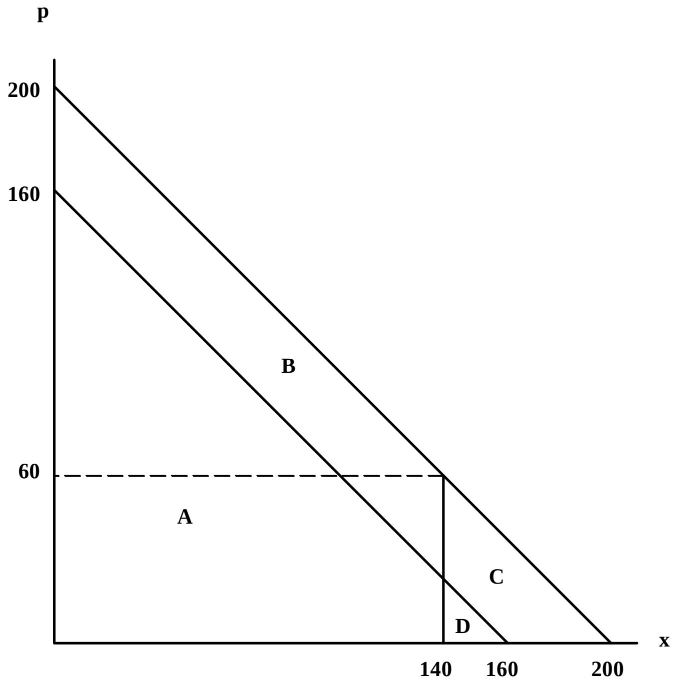

Figure 2:

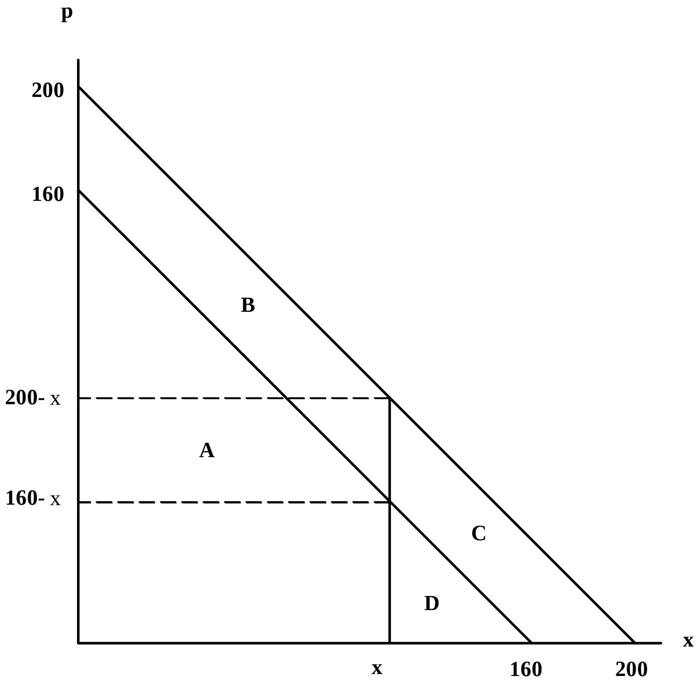

Figure 3:

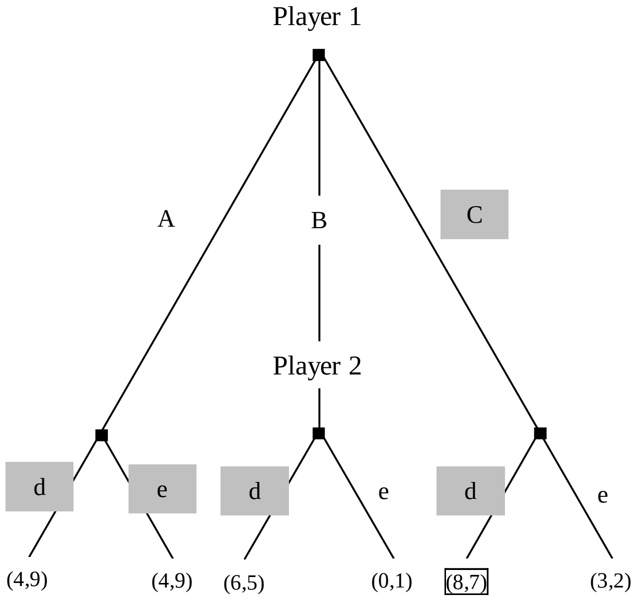

Figure 4: