Economics 100A
Fall 2001
Prof. Daniel McFadden

PROBLEM SET #2 SOLUTIONS

(Prepared by: Peter Adams)

1 Univariate Calculus Review

1.1 Differentiation

Problem 1 *Find the first derivative of the following functions:*

(a) $3x^2 - 9x + 7x^{2/5} - 3x^{1/2}$

Apply the ***Power Rule****,* $f(x) = Ax^k \Longrightarrow f'(x) = kAx^{k-1}$*, where* A *is a scalar.*

$$6x - 9 + \frac{14}{5}x^{-3/5} - \frac{3}{2}x^{-1/2} \qquad (*)$$

(b) $\left(x^{1/2} + x^{-1/2}\right)\left(4x^5 - 3\sqrt{x}\right)$

This is the product of two functions of x, so apply the ***Product Rule****,* $h(x) = f(x)\,g(x) \Longrightarrow \frac{dh}{dx} = \frac{df}{dx}g(x) + \frac{dg}{dx}f(x)$.

$$\left(\frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}\right)\left(4x^5 - 3\sqrt{x}\right) + \left(x^{1/2} + x^{-1/2}\right)\left(20x^4 - \frac{3}{2\sqrt{x}}\right) \qquad (*)$$

(c) $\dfrac{e^{2x}}{(x^2+1)}$

This is the ratio of two functions of x, so apply the ***Quotient Rule*** *which is a special case of the* ***Product Rule****,* $h(x) = \frac{f(x)}{g(x)} \Longrightarrow \frac{dh}{dx} = \frac{\frac{df}{dx}g(x) - \frac{dg}{dx}f(x)}{g(x)^2}$.

$$\frac{2e^{2x}\left(x^2+1\right) - 2xe^{2x}}{\left(x^2+1\right)^2} \qquad (*)$$

(d) $\ln(x + 3x + 1) + xe^{3-x} = \ln(4x+1) + xe^{3-x}$

This first part is a composite function of x, so apply the ***Chain Rule.*** *Apply the* ***Product Rule*** *and* ***Chain Rule*** *to the second part,* $h(x) = f[g(x)] + a(x)\,b(x) \Longrightarrow \frac{dh}{dx} = \frac{df}{dg}\frac{dg}{dx} + \frac{da}{dx}b(x) + \frac{db}{dx}a(x)$.

$$\frac{4}{(4x+1)} + e^{3-x} - xe^{3-x} \qquad (*)$$

1.2 Maximization

Problem 2 *Consider a firm's profit function,* $\Pi(x) = R(x) - C(x)$, *where* $R(x)$ *is total revenue as a function of output* (x), *and* $C(x)$ *is total cost as a function of output* (x).

(a) Under perfect competition, each firm is a price taker. Assuming a competitive market price, $p^* = 10$, and a cost function, $C(x) = (x-5)^2$, express the firm's profit as a function of x.

$$\Pi(x) = R(x) - C(x) = px - (x-5)^2 = 10x - (x-5)^2 \qquad (*)$$

(b) Find the competitive firm's profit maximizing level of output, x^* (Hint: maximize the firm's profit by taking the first derivative of the profit function, setting it equal to zero, and solving for the level of output, x^*).

$$\begin{aligned} \underset{x}{max}\ \Pi(x) &= 10x - (x-5)^2 \\ \frac{d\Pi}{dx} &= 0 \\ 10 - 2(x-5) &= 0 \\ 2x &= 20 \\ x^* &= 10 \qquad (*) \end{aligned}$$

(c) If the firm were only interested in minimizing costs, what level of output would it choose?

$$\begin{aligned} \underset{x}{min}\ C(x) &= (x-5)^2 \\ \frac{dC}{dx} &= 0 \\ 2(x-5) &= 0 \\ 2x &= 10 \\ \hat{x} &= 5 \qquad (*) \end{aligned}$$

(d) Under monopoly conditions, the firm is no longer a price taker. Rather, the firm now faces the entire market demand curve and sets price to the level that maximizes its profit given the market demand curve. Suppose the market demand curve (output) as a function of price is, $x = D(p) = 40 - 2p$. Write an equation for the profit of the firm as a function of p.

$$\begin{aligned}
\Pi(p) &= R(p) - C(p) \\
&= px - C(x) \\
&= pD(p) - C[D(p)] \\
&= pD(p) - (D(p) - 5)^2 \\
&= p\,(40 - 2p) - [(40 - 2p) - 5]^2 \\
&= \left(40p - 2p^2\right) - (35 - 2p)^2 \qquad (*)
\end{aligned}$$

(e) Solve for the monopoly firm's profit maximizing price,p^M. How much output, x^M, is supplied at the monopoly price? Assuming the same market demand curve as in *(d)*, how much output, x^*, would be supplied to the market at the competitive price, $p^* = 10$?

$$\begin{aligned}
\underset{p}{max}\ \Pi(p) &= \left(40p - 2p^2\right) - (35 - 2p)^2 \\
\frac{d\Pi}{dx} &= 40 - 2p - 2\,(35 - 2p) = 0 \\
2p &= 30 \\
p^M &= 15 \qquad (*) \\
x^M &= D(p^M) = 40 - 2(15) = 10 \qquad (*)
\end{aligned}$$

To find the level of output, x^*, supplied at the competitive price, p^*, we substitute the competitive price, $p^* = 10$, into the market demand curve. Keep in mind that in parts (a) and (b) we are only considering one (1) competitive firm and the quantity it produces. In general, the quantity each competitive firm produces is only a small part of the total output for the industry. In part (d) you need to calculate the total output as determined by the market demand curve if the price is the competitive price, $p^* = 10$.

$$x^* = D(p^*) = 40 - 2(10) = 20 \qquad (*)$$

(f) If this example is representative of monopolistic versus competitive markets, what can you say, in general, about the relative prices and relative quantities of output available to consumers?

The monopolist typically charges a HIGHER PRICE, (*)

$$p^M = 15 > p^* = 10$$

and supplies LESS OUTPUT.

$$x^M = 10 < x^* = 20$$

2 Consumer Choice

Problem 3 and Problem 4 cover budget constraints (Varian, Ch. 2) and Problem 5 addresses preferences (Varian, Ch. 3). This material serves as the underpinnings of utility theory (Varian, Ch. 4) which characterizes optimal consumer choice (Varian, Ch. 5) in a constrained setting.

2.1 Budget Constraints

Problem 3 *Billy Madison consumes 100 units of X and 50 units of Y. The price of X rises from 2 to 3. The price of Y remains at 4.*

(a) How much must Billy's income rise so that he can exactly afford 100 units of X and 50 units of Y?

$$\begin{aligned} BC &: p_X X + p_Y Y = m \\ &\Rightarrow 2(100) + 4(50) = 400 \\ &\mathit{IfthepriceofX,}\ p_X\mathit{,rises\ from\ 2\ to\ 3,} \\ &\Rightarrow 3(100) + 4(50) = 500 \\ &\Rightarrow \Delta m = 100 \end{aligned} \qquad (*)$$

(b) Draw Billy's original budget set when the price of X is 2.

See Figure 1 (*)

(c) Draw Billy's budget set when the price of X is 3, but his income is unchanged.

See Figure 2 (*)

(d) Draw Billy's budget set when the price of X is 3 and his income increases by the amount in *(a)* so that he can afford his original consumption bundle.

See Figure 3 (*)

Problem 4 *If Veronica Vaughn spends all of her daily income on cigarettes and Yoo-Hoo, she can afford 10 packs of cigarettes and 10 bottles of Yoo-Hoo. She can also afford 6 packs of cigarettes and 22 bottles of Yoo-Hoo.*

(a) What is the relative price of cigarettes in terms of bottles of Yoo-Hoo (i.e. what is the ratio of prices, $\frac{p_c}{p_y}$).

You know that if Veronica spends all of her income on each bundle, then each must satisfy the budget contraint, $p_cC + p_yY = m$, with equality.

$$p_c10 + p_y10 = m$$
$$p_c6 + p_y22 = m$$

Now set these two equations equal to each other and solve for the ratio of prices.

$$p_c10 + p_y10 = p_c6 + p_y22$$
$$4p_c = 12p_y$$
$$\frac{p_c}{p_y} = 3 \qquad (*)$$

(b) Exactly how much income does Veronica earn in one week if the price of cigarettes is \$6? Write a budget equation for Veronica that is a function of the packs of cigarettes, C, and the number of bottles of Yoo-Hoo, Y.

$$p_c = 6 \Rightarrow p_y = 2$$
$$6(10) + 2(10) = m$$
$$m = 80 \qquad (*)$$
$$6C + 2Y = 80 \qquad (*)$$

(c) Draw Veronica's daily budget set with packs of cigarettes on the *x-axis* and bottles of Yoo-Hoo on the *y-axis*.

See Figure 4 (*)

(d) Now suppose the price of cigarettes falls by \$1 and the price of Yoo-Hoo rises by \$1. If Veronica was consuming 5 packs of cigarettes and 30 bottles of Yoo-Hoo prior to the price changes, how much must her income rise under the new prices in order for her to just afford the old bundle, $(5, 30)$? Draw this new budget line.

$$\textit{BC (original prices):}\ 6C + 2Y = m$$
$$\Rightarrow m = 90 = 6(5) + 2(30)$$
$$\textit{BC (new prices):}\ 5C + 3Y = m$$
$$\Rightarrow m = 115 = 5(5) + 3(30)$$
$$\Delta m = 25 \qquad (*)$$

NOTE: *Veronica's bundle of* $(5, 30)$ *is not affordable under the original prices and an income,* $m = 80$. *This is a mistake I made in writing the question. However, assuming Veronica's income is $90 as required by the question, her change in income must be* $\Delta m = 25$ *in order to just afford her old bundle of* $(5, 30)$ *at the new prices.*

See Figure 5 (*)

2.2 Preferences

Problem 5 *Assume, aside from yourself, that there are only two types of people in the world, Stanford students and UCLA students. Also assume that you, as a Berkeley student, dislike both. This suggests that from your perspective Stanford students and UCLA students are "bads."*

(a) If both Stanford (S) students and UCLA (U) students are bads, will the indifference curves have a positive or negative slope? Draw some "smooth" indifference curves and indicate the direction of increasing preference. Label the number of Stanford students (S) on the *x-axis* and UCLA students (U) on the *y-axis*.

See Figure 6 (*)

(b) Now suppose your dislike for people can be characterized by $D = max(S, 3U)$, where D is the level of dislike associated with a bundle of Stanford and UCLA students. In effect, your dislike is determined by the maximum of a particular type, with 3 UCLA students as tolerable as 1 Stanford student. Draw some indifference curves characterized by these preferences.

For those of you who are not familiar with the function, $D = max(x, y)$, *the value of the function,* D, *is simply the larger of the two arguments,* (x, y). *For example, if you have a bundle,* $(S, U) = (3, 2)$, *then* $D = max(3, 6) = 6$. *In order to draw some indifference curves, simply pick any point in the S-U plane,* (S_1, U_1), *and find the associated value of* $D_1 = max(S_1, 3U_1)$. *An indifference curve that passes through that point,* (S_1, U_1), *must include all points such that the value of* $D = D_1$. *So find all other points that give you a value of* $D = D_1$ *and that will map out an indifference curve. The dislike function is essentially a utility function, and, by definition, the level of utility is constant along any indifference curve.*

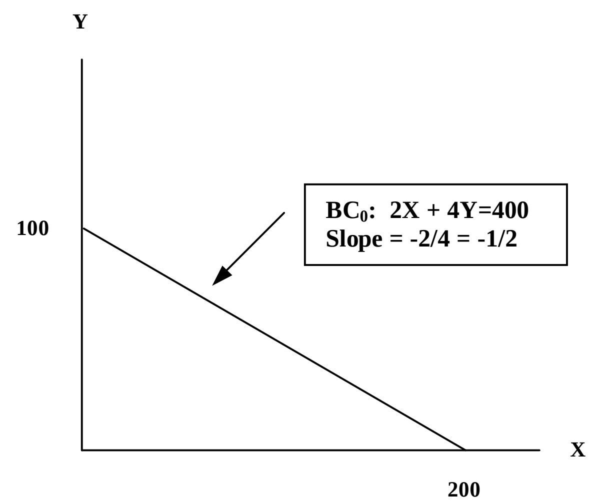
See Figure 7 (*)

NOTE: *In order for the dislike function* $D = max(x, y)$ *to be consistent with the fact that "3 UCLA students are as tolerable as 1 Stanford student," the function* ***SHOULD*** *be* $D = max(3S, U)$.*rather than* $D = max(S, 3U)$. *This is another mistake made by me in writing the question.*

(c) Assume now that you do not care one way or the other about being around UCLA students (i.e. you are neutral or indifferent). Draw some indifference curves associated with these preferences.

See Figure 8 (*)

3 Figures



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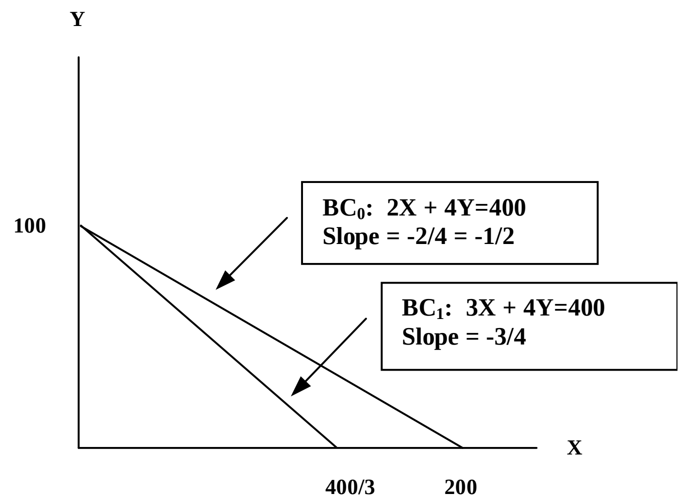

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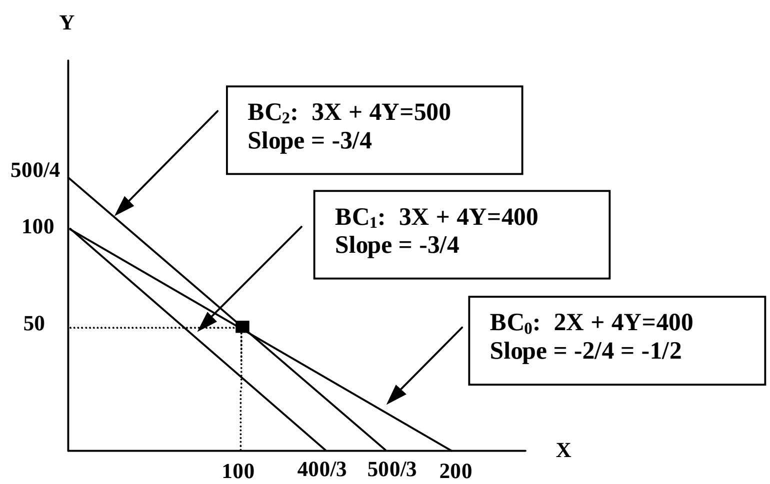

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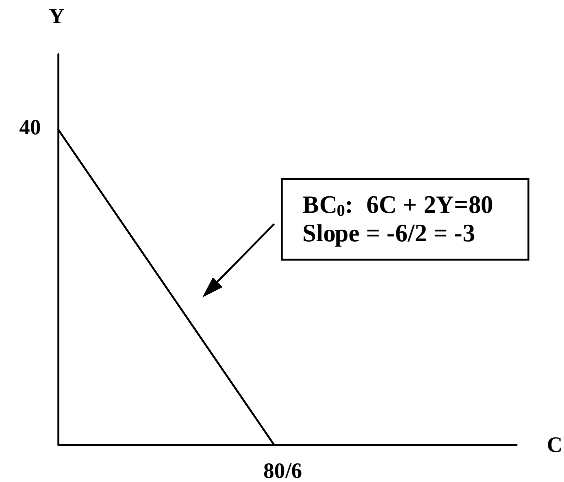

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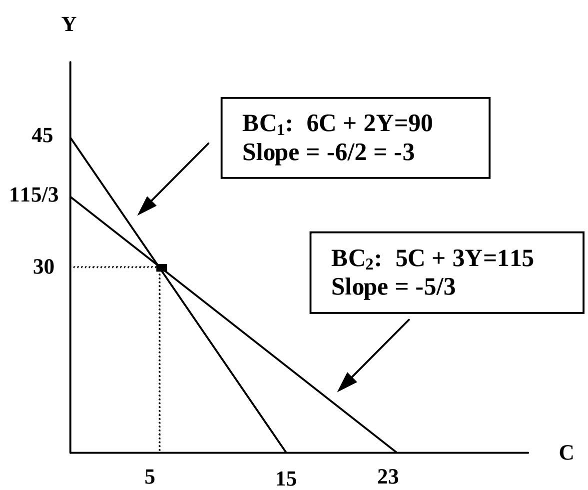

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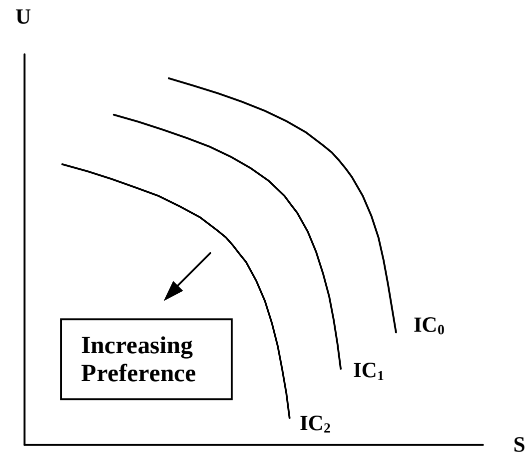

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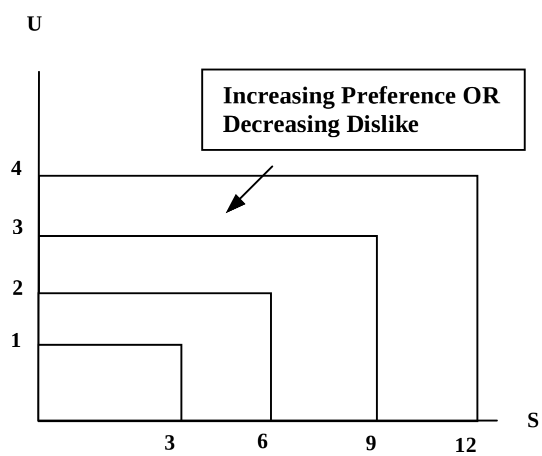

Figure 7:

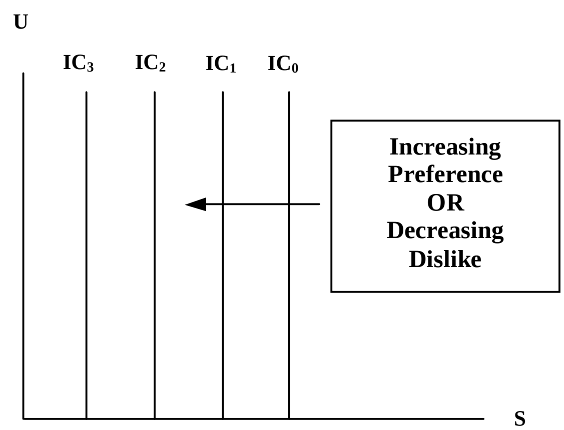
U
IC3
IC2
IC1
IC0
Increasing
Preference
OR
Decreasing
Dislike
S

Figure 8: