Econ. 240A, Spring 1998 D. McFadden

PROBLEM SET 2 (Properties of Special Distributions)

(Due Monday, Feb. 16, with discussion in section on Feb. 11)

1. Suppose that the duration of a spell of unemployment (in days) can be described by a geometric distribution, Prob(k) = $p^k(1-p)$, where $0 < p < 1$ is a parameter and k is a non-negative integer. What is the expected duration of unemployment? What is the probability of a spell of unemployment lasting longer than K days? What is the conditional expectation of the duration of unemployment, given the event that $k > m$, where m is a positive integer?

2. Using the characteristic function, find $\mathbf{E}X^3$ and $\mathbf{E}X^4$ for a standard normal X.

3. A log normal random variable Y is one that has log(Y) normal. If log(Y) has mean μ and variance σ^2, find the mean and variance of Y. [Hint: It is useful to find the moment generating function of Z = log(Y).]

4. If X and Y are independent normal, then X+Y is again normal, so that one can say that *the normal family is closed under addition*. (Addition of random variables is also called convolution, from the formula for the density of the sum.) Now suppose X and Y are independent and have extreme value distributions,

$$\text{Prob}(X \leq x) = \exp(-e^{a-x}) \text{ and } \text{Prob}(Y \leq y) = \exp(-e^{b-y}) \ ,$$

where a and b are location parameters. Show that max(X,Y) once again has an extreme value distribution (with location parameter $c = \log(e^a+e^b)$), so that *the extreme value family is closed under maximization*.

5. If X is standard normal, derive the density and characteristic function of $Y = X^2$, and confirm that this is the same as the tabled density of a chi-square random variable with one degree of freedom. If X is normal with variance one and a mean μ that is not zero, derive the density of Y, which is non-central chi-square distributed with one degree of freedom and noncentrality parameter μ^2.