Econ. 240B, Second Half Daniel McFadden, 2001

EXERCISE 0. HYPOTHESIS TESTING IN A MNL MODEL FITTED BY MLE

(Not to be handed in, for discussion the week of Oct. 23)

This exercise estimates a multinomial logit model by maximum likelihood, and tests several hypotheses using Wald, LR, and LM tests. The application is discrete choice of travel mode for shopping, with bus and auto alternatives, or the alternative of staying at home. The data are artificially generated. The SST program ex0.cmd is in the class data directory, and can be run using the unix command sst ex0. Output from one run of this program, which describes the hypotheses and test procedures, is reproduced below. Using SST, or transcribing this program into TSP, carry out the tests requested below. This exercise may be helpful for practical understanding of the lectures on discrete response models.

```
SST Spool File: ex0.out
Wed Sep 29 07:05:06 1999

AN EXAMPLE OF MAXIMUM LIKELIHOOD ESTIMATION
AND HYPOTHESIS TESTING FOR A MULTINOMIAL LOGIT MODEL


GENERATE SOME MADE-UP DATA, SHOPPING CHOICE OF (1) BUS,
(2) AUTO, OR (3) HOME (NO TRIP)

     range obs[1-400]
     set u = nrnd
     set tt2= 60 + 10*u + 5*nrnd                          # travel time mode 2
     set tt1= 90 + 5*u + 5*nrnd                           # travel time mode 1
     set tc2=2+0.04*tt2-.5*nrnd                           # travel cost mode 2
     set tc1=1.25+0.01*tt1-.5*nrnd                        # travel cost mode 1
     set u = log(-log(urnd))                              # extreme value RV
     set y1=-0.1*tt1-tc1+10+ log(-log(urnd)) - u
     set y2=-0.1*tt2-tc2+10+ log(-log(urnd)) - u
     set one=1
     set z=0
     set i=1*(y1>vmax(y2,0))+2*(y2>=vmax(y1,0))+3*(0>vmax(y1,y2))
     label var[tt1] lab[travel time bus]
     label var[tt2] lab[travel time auto]
     label var[tc1] lab[travel cost bus]
     label var[tc2] lab[travel cost auto]
     label var[i] lab[choice] val[1 bus 2 auto 3 home]
     cova var[tt1 tt2 tc1 tc2] cov


Variable:    tt1    travel time bus

Mean           90.23568    Standard deviation    7.07601
Minimum         70.83353    Skewness          -1.56729e-002
Maximum         1.13171e+002 Kurtosis          2.96962
Valid observations    400

Variable:    tt2    travel time auto
```

```
Mean            60.29345   Standard deviation   11.58801
Minimum          19.05803   Skewness          -0.12238
Maximum          1.02718e+002 Kurtosis          3.41181
Valid observations    400

Variable:   tc1   travel cost bus

Mean            2.14241   Standard deviation   0.51380
Minimum          0.61633   Skewness          -0.14033
Maximum          3.72038   Kurtosis          2.98541
Valid observations    400

Variable:   tc2   travel cost auto

Mean            4.42621   Standard deviation   0.70304
Minimum          2.48229   Skewness          2.39518e-002
Maximum          6.53560   Kurtosis          2.80964
Valid observations    400

Correlation and Covariance matrix

         tt1         tt2          tc1
tt1   49.94471     59.03648      0.51180
tt2    0.72179     1.33946e+002   0.18747
tc1    0.14113     3.15654e-002   0.26333
tc2    0.52834     0.72451     -1.57086e-002

                  tc2
         tt1   2.62176
         tt2   5.88772
         tc1   -5.66009e-003
         tc2   0.49303

freq var[i]


i   choice
400 valid observations

                   bus     auto     home
                   1       2       3
                   ---------- ---------- ----------
Count         57     143      200
Percent      14.25    35.75    50.00


THIS COMPLETES GENERATION OF MADE-UP DATA


ESTIMATION OF BASE MNL MODEL
  mnl dep[i] covmat[cvmat] coef[beta] \
  ivalt[tt: tt1 tt2 z tc: tc1 tc2 z d1: one z z d2: z one z]


  ********** MULTINOMIAL LOGIT **********
  Dependent variable: i

  Value     Label     Count     Percent
    1      bus      57      14.25
```

```
      2     auto     143     35.75
      3     home     200     50.00

ITERATION 1:  OLD LLF =   -4.39445e+002   STEP =      1.29693
NEW LLF =   -3.11731e+002   GRAD*DIREC =   2.30453e+002

ITERATION 2:  OLD LLF =   -3.11731e+002   STEP =      0.99416
NEW LLF =   -3.10431e+002   GRAD*DIREC =   2.61166

ITERATION 3:  OLD LLF =   -3.10431e+002   STEP =      1.00326
NEW LLF =   -3.10429e+002   GRAD*DIREC =   3.90115e-003

   At convergence grad * dir =    1.48498e-009


   Independent     Estimated       Standard         t-
    Variable      Coefficient        Error        Statistic

       tt      -8.57271e-002     1.50024e-002     -5.71422
       tc      -1.25794         0.20111        -6.25483
       d1       9.07420         1.28205        7.07790
       d2      10.17796         0.99883        10.18987



   auxiliary statistics         at convergence    initial
   log likelihood             -310.43       -439.44
   number of observations          400
   percent correctly predicted        64

   calc ll0=_llk
   matrix cvmat
               [ 1]         [ 2]         [ 3]
   [ 1]   2.25072e-004   -1.05793e-003   -1.82733e-002
   [ 2]   -1.05793e-003   4.04472e-002   1.66843e-002
   [ 3]   -1.82733e-002   1.66843e-002   1.64365
   [ 4]   -8.68450e-003   -0.11046      1.00648

                    [ 4]
       [ 1]   -8.68450e-003
       [ 2]   -0.11046
       [ 3]    1.00648
       [ 4]    0.99766


CONSIDER THE FOLLOWING HYPOTHESES
H1: beta_tt = 0.1*beta_tc
H2: beta_d1 = beta_d2
h3: beta_d1 = beta_d2 = 10

HYPTHESIS H1 TESTS WHETHER TRAVEL TIME IS VALUED AT 10
CENTS/MINUTE OR $6/HOUR.  IT IS A ONE-DIMENSIONAL HYPOTHESIS.

HYPOTHESIS H2 TESTS WHETHER THE ALTERNATIVE-SPECIFIC
EFFECTS FOR AUTO AND BUS ARE THE SAME.  THIS WILL BE TRUE
IF THESE REFLECT THE COMMON PAYOFF TO SHOPPING, BUT FALSE
IF THERE IS ANY SYSTEMATIC DIFFERENCE IN THE ATTRACTIVENESS
OF AUTO AND BUS OTHER THAN TRAVEL TIME AND COST DIFFERENCES.
```

HYPOTHESIS H3 TESTS WHETHER THE ALTERNATIVE-SPECIFIC EFFECTS FOR AUTO AND BUS ARE BOTH EQUAL TO 10. THIS IS A 2-DIMENSIONAL HYPOTHESIS

TESTING H1. FIRST, BY REPARAMETERIZATION, THIS CAN BE STATED AS A HYPOTHESIS THAT A PARAMETER IS ZERO. DEFINE A NEW VARIABLE, GENERALIZED COST gc = tc + tt/10 , AND ESTIMATE THE MODEL BELOW. IF H1 IS TRUE, THEN WITH THIS REPARAMETERIZATION, THE COEFFICIENT ON tt SHOULD BE ZERO, AND A T-TEST ON THIS COEFFICIENT IS A TEST OF H1. FOR COMPARISON WITH OTHER TEST STATISTICS, THE SQUARE OF THIS T-STATISTIC IS ALSO COMPUTED.

```
    set gc1 = tc1+tt1/10
    set gc2 = tc2+tt2/10
    mnl dep[i] ivalt[tt: tt1 tt2 z gc: gc1 gc2 z d1: one z z d2: z one z] \
          coef[beta1] covmat[cvmat1]


********** MULTINOMIAL LOGIT **********
Dependent variable: i

Value    Label    Count    Percent
  1       bus      57       14.25
  2       auto     143      35.75
  3       home     200      50.00

ITERATION 1:  OLD LLF =    -4.39445e+002   STEP =      1.29693
NEW LLF =    -3.11731e+002   GRAD*DIREC =    2.30453e+002

ITERATION 2:  OLD LLF =    -3.11731e+002   STEP =      0.99416
NEW LLF =    -3.10431e+002   GRAD*DIREC =   2.61166

ITERATION 3:  OLD LLF =    -3.10431e+002   STEP =      1.00326
NEW LLF =    -3.10429e+002   GRAD*DIREC =    3.90115e-003

  At convergence grad * dir =    1.48498e-009


  Independent     Estimated      Standard        t-
   Variable      Coefficient       Error       Statistic

     tt      4.00669e-002     2.90022e-002     1.38151
     gc      -1.25794         0.20111        -6.25483
     d1       9.07420         1.28205         7.07790
     d2      10.17796         0.99883        10.18986

  auxiliary statistics        at convergence    initial
  log likelihood              -310.43      -439.44
  number of observations          400
  percent correctly predicted       64

    calc beta1[1]^2/cvmat1[1,1]
         1.90857
```

SECOND, A WALD TEST CAN BE CALCULATED FROM THE BASE MODEL.

THE TEST STATISTIC IS ASYMP. CHI-SQUARED WITH 1 D.F. UNDER H1.

```
matrix A = {1; -0.1; 0; 0}
matrix A
               [ 1]
   [ 1]    1.00000
   [ 2]   -0.10000
   [ 3]    0.00000
   [ 4]    0.00000

matrix beta'*A*inv(A'*cvmat*A)*A'*beta
               [ 1]
   [ 1]    1.90857
```

THIRD, A LR TEST CAN BE CALCULATED BY RUNNING THE MODEL UNDER H1. THE TEST STATISTIC IS AGAIN ASYMP. CHI-SQUARED WITH 1 D.F.

```
mnl dep[i] ivalt[gc: gc1 gc2 z d1: one z z d2: z one z] \
     prob[pb2 pb3]


********** MULTINOMIAL LOGIT **********
Dependent variable: i

Value     Label     Count     Percent
     1      bus       57      14.25
     2      auto     143       35.75
     3      home     200       50.00

ITERATION  1:  OLD LLF =    -4.39445e+002   STEP =       1.29878
NEW LLF =    -3.11608e+002   GRAD*DIREC =    2.30446e+002

ITERATION  2:  OLD LLF =    -3.11608e+002   STEP =       0.99555
NEW LLF =    -3.11390e+002   GRAD*DIREC =   0.43751

At convergence grad * dir =    1.73179e-004


Independent     Estimated       Standard         t-
 Variable      Coefficient       Error        Statistic

    gc      -1.01759         9.78282e-002    -10.40186
    d1      10.04989         1.08799         9.23710
    d2      10.09161         0.99991        10.09254

auxiliary statistics         at convergence    initial
log likelihood              -311.39       -439.44
number of observations          400
percent correctly predicted        63

calc lla=_llk
calc lr = 2*(ll0 - lla)
calc lr
```

```
        1.92212
```

FOURTH, A LM TEST CAN BE CALCULATED BY AUXILIARY REGRESSION: CONSTRUCT THE SCORE FOR THE UNRESTRICTED MODEL AT THE RESTRICTED ESTIMATES. REGRESS 1 ON THIS SCORE. THEN LM IS THE SUM OF SQUARED FITTED VALUES.

```
set pb1 = 1 - pb2 - pb3    # fitted probabilities
set r1 = (i==1)-pb1        # generalized residuals
set r2 = (i==2)-pb2
set st = tt1*r1+tt2*r2     # score, tt
set sc = tc1*r1+tc2*r2     # score, tc
set sd1 = r1          # score, d1
set sd2 = r2          # score, d2
reg dep[(1)] ind[st sc sd1 sd2] pred[onehat]
```

```
********* ORDINARY LEAST SQUARES ESTIMATION *********

Dependent Variable:    (1)
```

Independent Variable	Estimated Coefficient	Standard Error	t-Statistic
st	1.54258e-002	1.50540e-002	1.02469
sc	-0.22701	0.19346	-1.17338
sd1	-0.93538	1.27653	-0.73275
sd2	8.49096e-002	0.99540	8.53022e-002

```
Number of Observations            400
R-squared                     1.00000
Corrected R-squared               0.00000
Sum of Squared Residuals           3.98165e+002
Standard Error of the Regression     1.00273
Durbin-Watson Statistic            9.42493e-003
Mean of Dependent Variable          1.00000
```

```
calc sum(onehat^2)
     1.83473
```

THE HYPOTHESIS H1 IS TRUE, AND THE RESULTING TEST STATISTICS ARE SIMILAR. TEST ANOTHER H1 HYPOTHESIS THAT IS FALSE, SAY beta_tt = 0.05*beta_tc. THE STATISTICS IN THIS CASE ARE LIKELY TO ALL REJECT THE NULL, BUT WILL BE MORE SPREAD OUT IN VALUES. FOR THE EXERCISE, FORM THE HYPOTHESES H2 AND H3 AND CALCULATE THE TEST STATISTICS FOR THEM USING THE PROCEDURES GIVEN ABOVE, MODIFIED AS NEEDED.