Econ 240B – Fall 2001
Final Examination Solutions

1. Define $x=[1 \quad p \quad w \quad a \quad s]'$, $\beta=[\beta_1 \quad \beta_2 \quad \beta_3 \quad \beta_4 \quad \beta_5]'$, $\theta=[\beta' \quad \sigma]'$

(a) $L(\theta)=d*\log\left[\Phi\left(\frac{x'\beta-f}{\sigma}\right)\right]+(1-d)*\log\left[1-\Phi\left(\frac{x'\beta-f}{\sigma}\right)\right]$

(b) Define $y_i = \text{Ind}\{i \in \text{sample of actual purchases}\}$. The *LR* statistic below is asymptotically distributed as an χ^2_6 under the null:

$$LR = 2\left[\max_{\beta_A,\sigma_A,\beta_H,\sigma_H}\left\{\sum_{i=1}^{N+M}\left\{d_i*\log\left\{\Phi\left[\frac{y_i(x_i'\beta_A-f)+(1-y_i)(x_i'\beta_H-f)}{y_i\sigma_A+(1-y_i)\sigma_H}\right]\right\}+[1-d_i]*\log\left[1-\Phi\left(\frac{y_i(x_i'\beta_A-f)+(1-y_i)(x_i'\beta_H-f)}{y_i\sigma_A+(1-y_i)\sigma_H}\right)\right]\right\}\right\} - \max_{\beta,\sigma}\left\{\sum_{i=1}^{N+M}d_i*\log\left[\Phi\left(\frac{x_i'\beta-f}{\sigma}\right)\right]+(1-d_i)*\log\left[1-\Phi\left(\frac{x_i'\beta-f}{\sigma}\right)\right]\right\}\right]$$

(c)

$$LR = 2\left[\max_{\beta,\sigma_A,\sigma_H}\left\{\sum_{i=1}^{N+M}\left\{d_i*\log\left[\Phi\left(\frac{x_i'\beta-f}{y_i\sigma_A+(1-y_i)\sigma_H}\right)\right]+[1-d_i]*\log\left[1-\Phi\left(\frac{x_i'\beta-f}{y_i\sigma_A+(1-y_i)\sigma_H}\right)\right]\right\}\right\} - \max_{\beta,\sigma}\left\{\sum_{i=1}^{N+M}d_i*\log\left[\Phi\left(\frac{x_i'\beta-f}{\sigma}\right)\right]+(1-d_i)*\log\left[1-\Phi\left(\frac{x_i'\beta-f}{\sigma}\right)\right]\right\}\right] \xrightarrow{D} \chi^2_1$$

2. Using WESML approach:

$$\hat{\mu} = \frac{\sum_{y_i<\log(20K)}\frac{y_i}{.9}+\sum_{y_i>\log(20K)}\frac{y_i}{.6}}{\sum_{y_i<\log(20K)}\frac{1}{.9}+\sum_{y_i>\log(20K)}\frac{1}{.6}}$$

3.

(a) It will not be consistent as long as z is not clean. This condition does not depend on the true value of γ.

(b) Do the omitted variable version of the Hausman Test: Regress y on 1, x, z and $z^{\#}$ and test whether the coefficient of $z^{\#}$ is zero by means of a Wald test.

4.

(a) If $X_1=X_2=X$, then: $\hat{\varepsilon}_1=Q\varepsilon_1$, $\hat{\varepsilon}_2=Q\varepsilon_2$, $Q=I-X(X'X)^{-1}X'$ and $tr(E[\hat{\varepsilon}_1\hat{\varepsilon}_2']) = tr(Q)\sigma_{12} = (T-K)\sigma_{12} \neq 0$

(b) $\hat{\beta}_j = \beta_j-(X_j'X_j)^{-1}X_j'\varepsilon_j$, $j=1,2$. If $X_1'X_2 = 0$, then $E[(\hat{\beta}_1-\beta_1)(\hat{\beta}_2-\beta_2)'] = (X_1'X_1)^{-1}X_1'X_2(X_2'X_2)^{-1}\sigma_{12} = 0$

5.

(a) Let z_t be any random variable. Then the following must hold:
$cov(z_t,\hat{\varepsilon}_{1t}) = cov(z_t,\varepsilon_{1t})-(\hat{\beta}_{12}-\beta_{12})cov(z_t,w_t)$. And, asymptotically, $cov(z_t,\hat{\varepsilon}_{1t}) = cov(z_t,\varepsilon_{1t})$. Thus, for $z_t = \varepsilon_{2t}$: $cov(\varepsilon_{2t},\hat{\varepsilon}_{1t}) = cov(\varepsilon_{2t},\varepsilon_{1t}) = 0$; and for $z_t = w_t$: $cov(w_t,\hat{\varepsilon}_{1t}) = cov(w_t,\varepsilon_{1t}) \neq 0$.

(b) From (a) we can see that $\hat{\varepsilon}_{1t}$ is a proper instrument for w_t, then supply equation can be estimated by 2SLS using 1, m_t, and $\hat{\varepsilon}_{1t}$ as instruments.