Price Setting in the Corsetti-Pesenti[1] Framework

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There are two cases, PCP (producer-currency pricing, price sticky in home currency of the producer) and LCP (local-currency pricing, price sticky in currency of the buyer, so that a producer must set domestic as well as foreign prices). In both cases, prices are set on date $t-1$ to be charged buyers in period t.

PCP case. Worldwide profits of a Home producer (say, in terms of domestic currency) are given by

$$\Pi = [p(h) - MC]\left[\frac{p(h)}{P_H}\right]^{-\theta} C_H + [\mathcal{E}p^*(h) - MC]\left[\frac{p^*(h)}{P_H^*}\right]^{-\theta} C_H^*,$$

where $p^*(h)$ is the Foreign-currency price at which goods are sold in Foreign. Under PCP, the Foreign price will simply be $p(h)/\mathcal{E}$, where $p(h)$ is set a period in advance. Thus, ex post nominal profits under sticky prices (once date t variables including the exchange rate $\mathcal{E}$ are realized) will be:

$$\Pi = [p(h) - MC]\left[\frac{p(h)}{P_H}\right]^{-\theta} C_H + [p(h) - MC]\left[\frac{p(h)/\mathcal{E}}{P_H^*}\right]^{-\theta} C_H^*. \tag{1}$$

Because the firm sets the price a period in advance and asset markets are *complete*, the payoff to the firm in a given date-t state of nature s_t, valued in terms of date $t-1$ money, will be

$$\frac{\pi(s_t)\beta u'[C(s_t)]/P(s_t)}{u'(C_{t-1})/P_{t-1}}\Pi(s_t),$$

where $\pi(s_t)$ is the probability of occurrence of state s_t. (Recall that the ratio

$$\frac{\pi(s_t)\beta u'[C(s_t)]/P(s_t)}{u'(C_{t-1})/P_{t-1}}$$

is the value of a unit of money delivered on date t contingent on state s_t, measured in terms of money on date $t-1$.) The firm maximizes, with respect to its date $t-1$ choice of $p_t(h)$, the sum of the preceding state-contingent payoffs, and therefore solves the problem

$$\max_{p_t(h)} \mathrm{E}_{t-1}\left\{\frac{\beta u'(C_t)/P_t}{u'(C_{t-1})/P_{t-1}}\Pi_t\right\} \Leftrightarrow \max_{p_t(h)} \mathrm{E}_{t-1}\left\{\frac{\beta P_{t-1}C_{t-1}}{P_t C_t}\Pi_t\right\}.$$

(The equivalence is a consequence of log utility.)

Substituting eq. (1) into the preceding maximization, one expresses the firm's problem (after dividing by $P_{t-1}C_{t-1}$, which is exogenous to the individual

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producer and known as of date $t-1$, and multiplying by P_H, which also is known as of date $t-1$), as

$$\max_{p_t(h)} \mathrm{E}_{t-1}\left\{[p_t(h) - MC_t]\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}\frac{P_{H,t}C_{H,t}}{P_tC_t}\right.$$
$$\left. + [p_t(h) - MC_t]\left[\frac{p_t(h)/\mathcal{E}_t}{P^*_{H,t}}\right]^{-\theta}\frac{\mathcal{E}_tP^*_{H,t}C^*_{H,t}}{P_tC_t}\right\}.$$

Above, we have used the fact that, under PCP, P_H will always equal $\mathcal{E}P^*_H$ (since that relationship holds for each individual Home good $h \in [0,1]$).

Differentiating with respect to $p_t(h)$ yields the first-order condition

$$\mathrm{E}_{t-1}\left\{\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}\frac{P_{H,t}C_{H,t}}{P_tC_t} - \theta\frac{[p_t(h) - MC_t]}{p_t(h)}\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}\frac{P_{H,t}C_{H,t}}{P_tC_t}\right.$$
$$\left. + \left[\frac{p_t(h)/\mathcal{E}_t}{P^*_{H,t}}\right]^{-\theta}\frac{\mathcal{E}_tP^*_{H,t}C^*_{H,t}}{P_tC_t} - \theta\frac{[p_t(h) - MC_t]}{p_t(h)}\left[\frac{p_t(h)/\mathcal{E}_t}{P^*_{H,t}}\right]^{-\theta}\frac{\mathcal{E}_tP^*_{H,t}C^*_{H,t}}{P_tC_t}\right\}$$
$$= \quad 0.$$

Under the Corsetti-Pesenti preference assumptions, $\frac{P_{H,t}C_{H,t}}{P_tC_t} = \frac{1}{2}$, and, as we have noted, $P_H = \mathcal{E}P^*_H$ under PCP. Furthermore, $P^*_{H,t}C^*_{H,t} = \frac{1}{2}P^*_tC^*_t$, and under complete markets, $P^*_tC^*_t = P_tC_t/\mathcal{E}_t$, so $\frac{\mathcal{E}_tP^*_{H,t}C^*_{H,t}}{P_tC_t} = \frac{1}{2}$. Thus, the preceding first-order condition reduces to

$$\mathrm{E}_{t-1}\left\{\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta} - \theta\frac{[p_t(h) - MC_t]}{p_t(h)}\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}\right\} = 0.$$

Because, moreover, $p_t(h)$ and $P_{H,t}$ are known as of date $t-1$, the term $\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}$ may be factored out above, leaving

$$\mathrm{E}_{t-1}\left\{1 - \theta\frac{[p_t(h) - MC_t]}{p_t(h)}\right\} = 0,$$

or

$$p_t(h) = \frac{\theta}{\theta - 1}\mathrm{E}_{t-1}\left\{MC_t\right\}.$$

LCP case. Following steps analogous to those above, but recognizing that the producer now can choose independently $p_t(h)$ and $p^*_t(h)$, we express the maximization problem of the price-setting firm as

$$\max_{p_t(h), p^*_t(h)} \mathrm{E}_{t-1}\left\{[p_t(h) - MC_t]\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}\frac{C_{H,t}}{P_tC_t}\right.$$
$$\left. + \ [\mathcal{E}_tp^*_t(h) - MC_t]\left[\frac{p^*_t(h)}{P^*_{H,t}}\right]^{-\theta}\frac{C^*_{H,t}}{P_tC_t}\right\}.$$

The first-order condition with respect to $p_t(h)$ is

$$\mathrm{E}_{t-1}\left\{\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}\frac{C_{H,t}}{P_tC_t}-\theta\frac{[p_t(h)-MC_t]}{p_t(h)}\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}\frac{C_{H,t}}{P_tC_t}\right\}=0.$$

Multiplying through by $P_{H,t}$ as above, which is known at date $t-1$, we get

$$\mathrm{E}_{t-1}\left\{1-\theta\frac{[p_t(h)-MC_t]}{p_t(h)}\right\}=0,$$

or

$$p_t(h)=\frac{\theta}{\theta-1}\mathrm{E}_{t-1}\left\{MC_t\right\}.$$

The first-order condition with respect to $p_t^*(h)$ is

$$\mathrm{E}_{t-1}\left\{\mathcal{E}_t\left[\frac{p_t^*(h)}{P_{H,t}^*}\right]^{-\theta}\frac{C_{H,t}^*}{P_tC_t}-\theta\frac{[\mathcal{E}_tp_t^*(h)-MC_t]}{p_t^*(h)}\left[\frac{p_t^*(h)}{P_{H,t}^*}\right]^{-\theta}\frac{C_{H,t}^*}{P_tC_t}\right\}=0.$$

Now, $P_{H,t}^*$ also is known with certainty as of date $t-1$, so we may multiply through the expectations operator in the preceding equation and rearrange terms to get

$$\mathrm{E}_{t-1}\left\{\left[\frac{p_t^*(h)}{P_{H,t}^*}\right]^{-\theta}\frac{\mathcal{E}_tP_{H,t}^*C_{H,t}^*}{P_tC_t}-\theta\frac{[\mathcal{E}_tp_t^*(h)-MC_t]}{\mathcal{E}_tp_t^*(h)}\left[\frac{p_t^*(h)}{P_{H,t}^*}\right]^{-\theta}\frac{\mathcal{E}_tP_{H,t}^*C_{H,t}^*}{P_tC_t}\right\}=0,$$

which reduces to

$$\mathrm{E}_{t-1}\left\{1-\theta\frac{[\mathcal{E}_tp_t^*(h)-MC_t]}{\mathcal{E}_tp_t^*(h)}\right\}=0$$

(because $\frac{\mathcal{E}_tP_{H,t}^*C_{H,t}^*}{P_tC_t}=\frac{1}{2}$ under complete markets). We may multiply $p_t^*(h)$ through the expectations operator to yield

$$\mathrm{E}_{t-1}\left\{p_t^*(h)-\theta\left[p_t^*(h)-\frac{MC_t}{\mathcal{E}_t}\right]\right\}=0$$

or, solving for $p_t^*(h)$,

$$p_t^*(h)=\frac{\theta}{\theta-1}\mathrm{E}_{t-1}\left\{\frac{MC_t}{\mathcal{E}_t}\right\}.$$