Financial fragility in emerging markets: firm balance sheets and the sectoral structure

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Abstract

This paper builds an overlapping generation model of a two-sector small open economy in order to study the evolution of the sectoral structure and its impact on financial fragility. Firms in the economy are subject to a borrowing constraint. It is also assumed that there is a currency mismatch in the balance sheets of the non-tradable sector. Under these two assumptions, multiple within-period equilibria associated with different real exchange rates and investment levels may arise, making self-fulfilling balance-of-payments crises possible. The within-period crisis equilibrium exists when the non-tradable sector is large enough compared to the tradable sector and sufficiently leveraged.

The paper studies the dynamics of the relative size and leverage of the non-tradable sector. It shows that their evolution leads to a financially fragile state in economies sufficiently opened to external finance and in times of high international liquidity.

Keywords: two-sector models, balance-of-payments crises, sunspots, foreign currency debt, borrowing constraint.

JEL Classification Numbers: E44, F32, F34, F43, O41

1 Introduction

The opening of developing economies to international finance in the last three decades has led in a number of cases to severe balance-of-payments crises with large real costs. The Southern Cone crises at the beginning of the nineteen-eighties, the Mexican crisis of 1994, the Asian crises of 1997, and the Argentine crisis of 2001, to mention but a few of them, all took place after the capital account had been liberalized. The literature dedicated to the empirical analysis of these events (Kaminsky & Reinhart 1999, Tornell & Westermann 2002, Calvo, Izquierdo & Mejía 2004, among

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others) has identified a consistent set of stylized facts: the balance-of-payments crises go together with a real depreciation, a sharp drop of investment and a current-account reversal. Financial factors play a crucial role, and a lot of these currency crises were coupled with banking crises.

Some authors have also pointed to the role played by sectoral factors in these crisis episodes. Tornell & Westermann (2002) show that the relative size of the non-tradable sector usually increases before twin crises in middle-income countries. Calvo et al. (2004) find that the probability of a sudden stop is higher in economies where the absorption of tradable goods is small compared to the pre-crisis current-account deficit, a proxy for the size of a possible sudden stop. The rationale behind these findings is that any shock resulting in a lower demand for non-tradable goods has to be accommodated by a real depreciation in the short run. When the demand for non-tradable goods stemming from the tradable sector is large compared to the size of the non-tradable sector, it acts as a stabilizing buffer, so that the real exchange rate needed to close the gap is not very depreciated.

But the sectoral structure of an economy is endogenous and the size of both the tradable and non-tradable sectors evolves over time. Therefore, in order to fully understand these crisis episodes, one has to explain the sectoral dynamics of emerging economies. A first account of the link between financial crises and sectoral dynamics is provided by Schneider & Tornell (2004). Using a finite-time model, they study the growth of the non-tradable sector during a transitory lending boom and show that it can lead to a self-fulfilling crisis.

This paper extends Schneider & Tornell's (2004) framework and builds a model to study how the allocation of resources between the tradable and non-tradable sectors evolves over an infinite time horizon and how it affects the possibility of self-fulfilling balance-of-payments crises. It shows that the sectoral dynamics depends, among other factors, on external financing conditions, namely the financial openness and the international interest rate. In particular, a permanent increase in the supply of international liquidity can lead to a reallocation of resources towards the non-tradable sector. The paper studies whether this sectoral change is sufficient to make balance-of-payments crises possible.

The paper models a two-sector small open economy with an overlapping generation structure. It embeds a static mechanism of self-fulfilling crisis which can produce multiple equilibria within a single time period, including a crisis equilibrium with a depreciated real exchange rate and defaults in the non-tradable sector. The within-period crisis equilibrium exists when (a) the debt repayments of firms producing non-tradable goods are high enough relative to their cash-flow and (b) the non-tradable sector is large enough relative to the tradable sector. Financial fragility thus depends on both a *financial factor*, the firm-level financial structure within the non-tradable sector, and a *real factor*, the sectoral structure of the whole economy. Both factors evolve along dynamic equilibrium paths. Starting from a closed economy, a country slightly opened to external finance reallocates resources towards the tradable sector in the long run in order to pay its external debt.

In more opened economies however, this is compensated by capital inflows which finance a higher demand for non-tradable goods, thus increasing the weight of the non-tradable sector in the long run.[1] I show that for a sufficient degree of financial openness or equivalently a low enough world interest rate, this sectoral evolution leads to financial fragility in the long run so that equilibrium paths experience episodes of self-fulfilling balance-of-payments crises. Since this result is valid along stationary equilibrium paths, the model is well suited to assess the effect of capital account liberalization over time independently of boom-bust cycles induced by transitory shocks.[2]

The precise mechanism underlying the existence of multiple equilibria within a single time period involves a self-reinforcing link between the real exchange rate and the level of investment expenditures, typical of balance-sheet approaches. First of all, firms are subject to a borrowing constraint. The amount they are able to borrow is limited by their cash-flows. Second, the economy is subject to *Original Sin* and firms cannot contract debt in domestic currency, which generates a currency mismatch in the balance sheets of the non-tradable sector. Together, these two market imperfections create a balance-sheet effect in the non-tradable sector, whereby movements in the real exchange rate affect firms' balance sheets, their capacity to raise external funds, and their level of investment. Third, investment partly consists of expenditures in non-tradable goods so that an increase in investment provokes a real appreciation. Thus, a real appreciation increases the cash-flow of non-tradable firms and loosens their borrowing constraint so that they can invest more. The higher level of investment reinforces the real appreciation until the borrowing constraint does not bind any more. On the contrary a real depreciation has a negative impact on their balance sheets, which limits the investment expenditures they can finance and further depreciates the real exchange rate until the non-tradable firms eventually default on their loans. To make this reinforcing mechanism possible, the borrowing constraint has to be sufficiently weak.

The crisis equilibrium only exists when the relative size of the non-tradable sector is high enough for the following reason. When the tradable sector is large compared to the non-tradable sector, a large fraction of the demand for non-tradable goods stems from the tradable sector. Suppose firms in the non-tradable sector stop investing. The residual demand for non-tradable goods, stemming from the tradable sector, can be large enough to sustain an appreciated real exchange rate, so that firms in the non-tradable sector do not default on their loans. Then, these firms had no reason to stop investing in the first place and the crisis equilibrium is impossible. This argument supposes that the demand for non-tradable goods stemming from the tradable sector increases with the size of the tradable sector. In the setting of this paper, this is in part the consequence of a borrowing constraint.

[1]This is consistent with a stylized fact established by Tornell, Westermann & Martinez (2004). These authors find that the ratio of non-tradables over tradables grows at a higher rate after the financial liberalization, defined *de facto* by an increase in capital inflows.

[2]Kaminsky & Schmukler (2003) argue from empirical evidence that the large amplitude of boom-bust cycles in the stock market following financial liberalization might be a transitory phenomenon and disappear in the long run.

Relation to the existing literature

This paper is related to both the literature studying the sectoral evolution of open economies and the more recent works on financial crises in emerging markets based on balance-sheet effects in firms. As regards the former, several works studied the so-called *Dutch disease* phenomenon. The reader may for example refer to Corden & Neary (1982), Bruno & Sachs (1982), and van Wijnbergen (1984). More recently, Hausmann & Rigobon (2002) show how a high concentration of capital in the non-tradable sector increases the volatility of the real exchange rate. This in return induces a shift of resources from the tradable to the non-tradable sector, eventually leading to a complete specialization in non-tradable goods.

As for the literature on crises in emerging markets, most existing works build static models of crises and do not study the dynamics that leads to the crisis.[3] The crisis mechanism used in the present paper comes from Krugman (1999) who models a real economy with multiple equilibria. Aghion, Bacchetta & Banerjee (2004*a*) construct a multi-period monetary model with nominal rigidities where multiple equilibria arise in the first period if the subsequent productivity is sufficiently large. Both works rely upon a balance-sheet effect with borrowing constraints and currency mismatches in firms' balance sheets. Jeanne & Zettelmeyer (2002) propose a simple and unified framework that encompasses several static balance-sheet approaches based on either currency mismatches or maturity mismatches and bank runs.

The dynamic model I use in this paper builds on Schneider & Tornell (2004), who insert Krugman's (1999) static model in a dynamic framework. While the formal structure of my model is closed to theirs, there are several differences. First of all, these authors focus their analysis on the growth of the non-tradable sector whereas I am interested in the allocation of capital across the tradable and non-tradable sectors. Therefore, I explicitly model the two sectors. They are introduced in the model in a symmetric way and any difference between them arises endogenously. I also use weaker technological assumptions: production functions are concave and there is a finite desired level of investment. As a result, borrowing constraints need not bind in the equilibrium. Finally, I consider an infinite number of periods and study both transitory dynamics and stationary equilibrium paths whereas Schneider & Tornell's (2004) model has a finite number of periods and therefore only studies transitory dynamics. These differences come from the different mechanism underlying the sectoral dynamics. In Schneider & Tornell (2004), the expectation of a future increase in the demand for non-tradable goods induces a credit boom that finances the non-tradable sector. If the boom is large enough, it can make self-fulfilling crises possible during the transition phase. On the contrary, the model presented here focuses on the long-run evolution of the sectoral structure driven by external financing conditions and does not require exogenous shocks.

Rancière, Tornell & Westermann (2003) also analyze the occurrence of crises in a dynamic

[3] A notable exception is Aghion, Bacchetta & Banerjee (2004*b*) who develop a dynamic model of financial instability where endogenous cycles can arise because of a balance-sheet effect.

framework. They develop an endogenous growth model with self-fulfilling crises and use it to study the relationship between the possibility of crisis and the growth rate and welfare of the economy. In their work, the non-tradable sector produces with increasing returns an intermediate good that is used as an input by the sector producing tradable consumption goods. They show that investment in the intermediate non-tradable sector is higher along equilibrium paths with crises than without crises.

The paper is organized as follows. The model is presented in section 2. Section 3 solves the within-period equilibrium and shows that multiple equilibria may arise, making self-fulfilling crises possible. Section 4 studies the long-run dynamics of the model in the absence of crisis. Section 5 determines the conditions under which an equilibrium path displays financial fragility in the long run. The plausibility of the results are assessed thanks to a calibration exercise with data from Argentina in the nineteen-nineties. Section 6 extends the analysis to different kinds of unexpected shocks. Section 7 concludes.

2 The model

Consider a small open economy with an overlapping generation structure. Time is discrete. There are three kinds of agents: households, entrepreneurs, and deep-pocket foreign lenders. All agents live two periods. There are three goods: a consumption good C, an intermediate tradable good T, and an intermediate non-tradable good N. The tradable good T is chosen as the numeraire. Denote p_t (p_t^C) the relative price of the non-tradable intermediate good (consumption good) in period t. The relative price p_t is a measure of the real exchange rate. A high value of p_t corresponds to an appreciated real exchange rate and *vice versa*.

2.1 General framework

Production

In period t, the consumption good C_t is produced by a competitive sector using labor in quantity L_t and the two intermediate goods in quantities T_t and N_t. The production function is a Cobb-Douglas function with constant returns to scale,

$$C_t = \left[(N_t)^{\mu}(T_t)^{1-\mu}\right]^{\alpha} L_t^{1-\alpha}$$

where $\alpha, \mu \in (0,1)$.

The tradable input T is produced by a tradable sector (sector T). It can also be imported and any excess production of tradable goods can be exported. The non-tradable input N is exclusively produced by a domestic non-tradable sector (sector N) and the whole production has to be used domestically. Each intermediate sector is composed of a continuum of firms of measure one.

A firm in sector N produces in period $t+1$ a quantity Y^N_{t+1} of non-tradable intermediate goods using a capital composed of K^N_t units of tradable goods and J^N_t units of non-tradable goods.[4] The production function is a concave Cobb-Douglas function given by

$$Y^N_{t+1} = (A^N_{t+1})^{1-\delta} \left[\left(\frac{K^N_t}{1-\eta} \right)^{1-\eta} \left(\frac{J^N_t}{\eta} \right)^{\eta} \right]^{\delta}$$

where $\eta, \delta \in (0,1)$. Both types of capital fully depreciate from one period to the next.

Likewise, the production function for a firm in sector T is

$$Y^T_{t+1} = (A^T_{t+1})^{1-\delta} \left[\left(\frac{K^T_t}{1-\eta} \right)^{1-\eta} \left(\frac{J^T_t}{\eta} \right)^{\eta} \right]^{\delta} .$$

There is an exogenous and homogenous growth trend in the productivity of both sectors:

$$A^i_t = a^i (1+g)^t , \qquad i = N, T.$$

The structure of production is illustrated by figure 1.

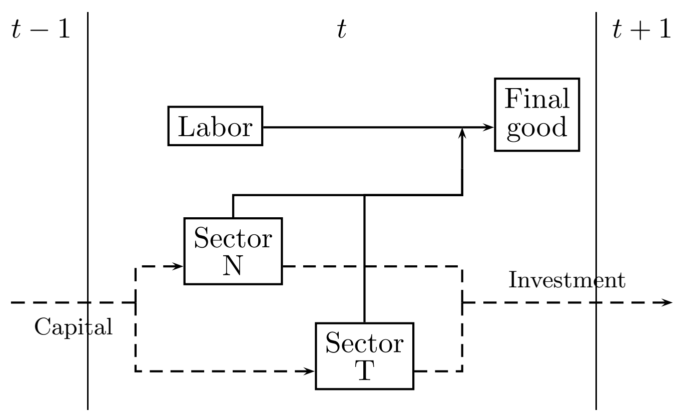


Figure 1: Structure of production.

Households

Households are endowed with one unit of labor in their first period of life only. They derive utility from the consumption good. Their preferences are given by the utility function

$$U = \log\left(c^y_t\right) + \beta\, \mathrm{E}_t \left[\log\left(c^o_{t+1}\right) \right]$$

[4] K^N_t mainly consists of machinery, transportation, etc. J^N_t represents buildings but also all possible non-tradable goods and services necessary to the installation of tradable capital.

where c_t^y is the consumption level of a household born in period t, c_{t+1}^o the consumption level of the same household in period $t+1$, $\beta \in (0,1)$ is a discount factor, and E_t denotes the expected value in period t. As households do not work in their second period of life, their consumption c_{t+1}^o comes from the returns on what they saved during their first period of life. The number of households is constant and equal to L.

Entrepreneurs

Each firm in the intermediate sectors is run by successive generations of risk-neutral entrepreneurs. Entrepreneurs consume the consumption good in their second period of life only. In period t a new entrepreneur in sector i starts with internal funds W_t^i and makes investment decisions to maximize the expected next period profit $\mathrm{E}_t \Pi_{t+1}^i$. If the firm's profit is strictly positive in period $t+1$, the incumbent entrepreneur gets a fixed fraction $\gamma \in (0,1)$ of it as dividends for her own consumption and gives the remaining proceeds to her successor. If on the contrary the firm's income exceeds its promised debt repayment, the incumbent entrepreneur defaults and does not consume. Then, the new entrepreneur starts with an exogenous endowment $Z_{t+1} = z(1+g)^{t+1}$ provided by international institutions as a rescue package. Thus, we have $W_t^i = (1-\gamma)\Pi_t^i$ in the absence of default and $W_t^i = Z_t$ when there is a default.

Foreign lenders

Foreign lenders are risk-neutral and have large endowments of tradable goods, which they can lend to domestic agents. The world riskless interest rate is exogenous and equal to R^*.

Financial contracts

Agents can trade one-period bonds denominated in tradable goods. The financial market is subject to several imperfections.

To begin with, there is an iceberg cost $\tau \geq 0$ to international financial transactions. When a foreign lender lends $1+\tau$ units of tradable good to a domestic agent, the domestic agent only gets 1 unit, and *vice versa*. This iceberg cost allows us to model different level of financial openness in a simple way. The case $\tau = 0$ corresponds to an economy entirely opened to international finance.

Then, bonds denominated in non-tradable goods are not permitted. Consequently, there is a currency mismatch in the balance sheets of the non-tradable sector and entrepreneurs producing non-tradable goods cannot insure against real exchange rate risk (except by choosing not to issue any debt). The fact that the domestic agents of a developing country are unable to issue debt denominated in non-tradable goods on international financial markets has been dubbed the *Original Sin*.[5] My assumption is slightly stronger than that since I also exclude domestic lending in non-

[5]See Eichengreen & Hausmann (1999). Eichengreen, Hausmann & Panizza (2005*b*) investigate the empirical relevance of this concept. According to Hausmann & Panizza (2003), Original Sin might be the result of transaction

tradable goods.[6]

The third imperfection is that debt contracts involving entrepreneurs are subject to a borrowing constraint. An entrepreneur with internal funds W_t^i can at most borrow $(\lambda - 1)W_t^i$, with $\lambda \geq 1$. I shall refer to coefficient λ as the *financial multiplier*. Appendix A.1 proposes a possible microfoundation for this borrowing constraint based on the imperfect enforcement of debt contracts, whereby the financial multiplier λ can be interpreted as the level of domestic financial development. The case $\lambda = 1$ corresponds to a fully financially repressed economy while the case $\lambda = \infty$ corresponds to a perfect domestic financial system.

Finally, when an entrepreneur defaults on her loan I assume that the entire production is then wasted as a bankruptcy cost so that the lenders do not get anything either.[7] Furthermore, the new entrepreneur of a defaulting firm has no access to the financial market.

Sunspot variable

As we will see in the next section, multiple equilibria may arise under certain circumstances within a given time period. When this is the case the agents need to coordinate on one of the two possible stable equilibria. I introduce an exogenous sunspot variable S_t that can possibly play the role of an external coordination device.

The sunspot variable S_t takes the value 1 with probability ω and 0 with probability $1-\omega$ $(\omega < 1)$. When the agents use S_t as a coordination device, $S_t = 0$ corresponds to the equilibrium with a depreciated real exchange rate (crisis times) and $S_t = 1$ to the equilibrium with an appreciated real exchange rate (tranquil times).

Low savings

I define an emerging market as an economy where international capital flows into, after a capital account liberalization. In such an economy domestic savings are lower than domestic investment or, equivalently, the autarky interest rate (the rate of interest that would equalize the supply and demand of loanable funds if the economy were closed to international finance) is higher than the interest rate of the open economy.

A simple way to model this is to assume that the world interest rate is lower than the stationary autarky rate. This is the case if households are more impatient in the domestic economy than in

costs in international finance which set a finite number of currencies in the world's portfolio. The cost to detain the marginal currency should compensate the benefit derived from risk diversification. As large countries offer more diversification than small ones, they argue that one should expect the currencies of large countries to be dominant in international portfolios, and provide empirical evidence to support this view.

[6] Several authors have proposed arguments to explain why domestic firms choose to take a risky position by issuing debt denominated in foreign currency: moral hazard induced by expected bail-outs (Schneider & Tornell 2004), borrowing constraints in the domestic financial system (Caballero & Krishnamurthy 2000), commitment problems (Jeanne 2000) or the lack of credibility of the domestic monetary policy (Jeanne 2003).

[7] This assumption yields a simple expression for the risky interest rate.

the rest of the world or if their share of income $1-\alpha$ is lower.[8]

To simplify the saving problem of households I make a slightly stronger assumption.

Assumption 1 (Low savings). *It is assumed that the discount factor of households β is sufficiently low and that the entrepreneurs' share of income α is sufficiently high so that in any period t total savings from households are lower than the demand for loanable funds from the tradable sector alone.*

2.2 Optimization behaviors

Final good sector

Profit maximization by firms in the consumption good sector gives the usual first order conditions

$$(1-\alpha)p_t^C C_t = w_t L \,, \tag{1a}$$
$$\alpha\mu p_t^C C_t = p_t N_t \,, \tag{1b}$$
$$\alpha(1-\mu)p_t^C C_t = T_t \,, \tag{1c}$$

where w_t denotes the wage rate in terms of the tradable good.

Investment behavior of entrepreneurs

Entrepreneurs decide how much to borrow and invest in order to maximize the expected next period profit. The only source of uncertainty comes from the sunspot variable and manifests itself, when multiple equilibria arise, in an uncertain real exchange rate p_t. This has no effect on the expected profits of sector T. However, it leads to possible defaults in sector N because firms in this sector produce non-tradable goods and are indebted in tradable goods. Denote $\frac{B_{t+1}^i}{R_t^i}$ the amount of tradable goods a firm in sector i borrows in period t, where B_{t+1}^i is the promised debt repayment at $t+1$. The proceeds from the sales of an N firm in period t are strictly lower than its debt repayment when $p_t < p_t^D$, where

$$p_t^D = \frac{B_t^N}{Y_t^N}.$$

Profits in both sectors are thus given by

$$\Pi_t^T = Y_t^T - B_t^T \,, \tag{2}$$

$$\Pi_t^N = \begin{cases} p_t Y_t^N - B_t^N & \text{if } p_t \geq p_t^D, \\ 0 & \text{if } p_t < p_t^D. \end{cases} \tag{3}$$

[8] The stationary autarky rate is computed in appendix A.4. It is decreasing with the share of households' income $1-\alpha$ and their discount factor β.

Consider first the maximization program of an entrepreneur in sector N when the incumbent entrepreneur has not defaulted. She will not get anything if the firm defaults in period $t+1$. Therefore, she maximizes the expected next period profit in the state of nature where there is no default.

$$\max_{K_t^N,\, J_t^N,\, I_t^N,\, B_{t+1}^N} \mathrm{E}_t\big[p_{t+1} | p_{t+1} \geq p_{t+1}^D\big] Y_{t+1}^N - B_{t+1}^N \tag{4}$$

$$\begin{aligned} \text{s. t.} \quad & Y_{t+1}^N = (A_{t+1}^N)^{1-\delta} \left[\left(\tfrac{K_t^N}{1-\eta} \right)^{1-\eta} \left(\tfrac{J_t^N}{\eta} \right)^{\eta} \right]^{\delta} && \text{(i)} \\ & I_t^N = K_t^N + p_t J_t^N && \text{(ii)} \\ & I_t^N = \tfrac{B_{t+1}^N}{R_t^N} + W_t^N && \text{(iii)} \\ & \tfrac{B_{t+1}^N}{R_t^N} \leq (\lambda - 1) W_t^N && \text{(iv)} \end{aligned}$$

The expression $\mathrm{E}_t\big[p_{t+1} | p_{t+1} \geq p_{t+1}^D\big]$ denotes the conditional expectation in period t of p_{t+1} given that there is no default in period $t+1$ and I_t^N is the investment expenditure in period t. Equation (i) is the production function. Equation (ii) defines the investment expenditure. Equations (iii) and (iv) are the budget constraint and the borrowing constraint. Of course, B_{t+1}^N could be negative, in which case R_t^N would be the rate of return on the internal funds not invested in production, but the remaining of the paper only considers situations where $B_{t+1}^N \geq 0$.

The optimal composition of capital is given by

$$K_t^N = (1-\eta) I_t^N \,, \tag{5a}$$

$$p_t J_t^N = \eta I_t^N \,, \tag{5b}$$

which implies

$$Y_{t+1}^N = (A_{t+1}^N)^{1-\delta} \left(\frac{I_t^N}{p_t^{\eta}} \right)^{\delta} . \tag{5c}$$

The amount invested depends on whether the borrowing constraint binds or not. If it does, the investment expenditure is limited by internal funds and we have $I_t^N = \lambda W_t^N = \lambda(1-\gamma)(p_t Y_t^N - B_t^N)$. If it doesn't, we obtain $I_t^N = \bar{I}_t^N$ with

$$\bar{I}_t^N = A_{t+1}^N \left(\frac{\delta\, \mathrm{E}_t\big[p_{t+1} | p_{t+1} \geq p_{t+1}^D\big]}{p_t^{\eta\delta} R_t^N} \right)^{\frac{1}{1-\delta}} . \tag{5d}$$

Finally, if the incumbent entrepreneur has defaulted, the young entrepreneur starts with the exogenous endowment Z_t and has no access to financial markets so that $I_t^N = \min(\bar{I}_t^N, Z_t)$. I suppose that Z_t is low enough so that the investment expenditure in sector N is on the whole given by

$$I_t^N = \begin{cases} \bar{I}_t^N(p_t) & \text{if } p_t \geq p_t^B\,, \\ \lambda(1-\gamma)(p_t Y_t^N - B_t^N) & \text{if } p_t^D \leq p_t < p_t^B\,, \\ Z_t & \text{if } p_t < p_t^D\,. \end{cases} \tag{5e}$$

where p_t^B is the value of the relative price p_t for which $\bar{I}_t^N(p_t) = \lambda W_t^N(p_t) = \lambda(1-\gamma)(p_t Y_t^N - B_t^N)$.

The entrepreneur of the tradable sector faces a similar problem, except that the return on investment, measured in tradable goods, is certain as both the debt repayment and the sales are tradable goods. The solution of the maximization program is then given by the following equations.

$$K_t^T = (1-\eta) I_t^T \tag{6a}$$

$$p_t J_t^T = \eta I_t^T \tag{6b}$$

$$Y_{t+1}^T = (A_{t+1}^T)^{1-\delta} \left(\frac{I_t^T}{p_t^\eta} \right)^\delta \tag{6c}$$

$$\bar{I}_t^T = A_{t+1}^T \left(\frac{\delta}{p_t^{\eta\delta} R_t^T} \right)^{\frac{1}{1-\delta}} \tag{6d}$$

$$I_t^T = \min(\bar{I}_t^T, \lambda W_t^T) \tag{6e}$$

Loans from foreign lenders

The risk-neutrality of foreign lenders and the fact that they have deep pockets determine the interest rates faced by domestic agents borrowing or lending abroad. The riskless interest rate faced by a domestic agent *borrowing* abroad is $R^D = R^*(1+\tau)$. On the contrary, a domestic agent *lending* abroad would get a return equal to $R^*/(1+\tau)$.

Because of assumption 1 (low savings), the interest rate of bonds issued by the tradable sector is set by the foreign lenders. Therefore, we get

$$R_t^T = R^D = (1+\tau)R^*.$$

As we will see, this is also the case in the non-tradable sector. Denote ρ_t the probability that sector N does not default in period t. The expression of ρ_t will depend on the type of equilibrium considered and I leave it undefined for the time being. Because of the bankruptcy cost, the lender does not get anything in case of default. Therefore, the interest rate of bonds issued by the non-tradable sector is equal to

$$R_t^N = \frac{R^D}{\mathrm{E}_t[\rho_{t+1}]} = \frac{(1+\tau)R^*}{\mathrm{E}_t[\rho_{t+1}]}. \tag{7}$$

Households' savings

Each household maximizes its expected utility under a budget constraint. This maximization program is solved in appendix A.2 and I just give the basic idea here. Because utility is logarithmic, the saving problem can be decomposed into two independent decisions: how much to save and what kind of assets to hold. The saving rate is given by the usual formula:

$$s = \frac{\beta}{1+\beta}\,.$$

The household can hold three different assets: riskless bonds bought on the international market and bonds issued by the domestic entrepreneurs of either the tradable or the non-tradable sector. Because of the iceberg cost τ, bonds bought on the international market are strictly dominated by bonds issued by the tradable sector $\left(R^*/(1+\tau) < (1+\tau)R^*\right)$. Furthermore, bonds issued by the non-tradable sector are risky and return nothing in some states of nature. Since the household is risk-averse, it requires a higher return than risk-neutral foreign lenders in the states of nature where they yield a strictly positive return. Therefore, entrepreneurs from the non-tradable sector borrow all their external funds abroad and the household's portfolio only consists of bonds issued by the tradable sector, which is possible under assumption 1.

3 Within-period equilibrium

In this section I study the temporary equilibrium in period t. Given the optimal individual behaviors that were derived in the previous section, and for given predetermined and expected variables, the equilibrium is determined by market clearing conditions. In the whole section therefore I consider the predetermined variables w_{t-1}, Y_t^N, Y_t^T, B_t^N, B_t^T and the expected variables $\mathrm{E}_t[p_{t+1}|p_{t+1} \geq p_{t+1}^D]$, $\mathrm{E}_t[\rho_{t+1}]$, and $R_t^N = \frac{R^D}{\mathrm{E}_t[\rho_{t+1}]}$ as exogenous. Then, the variables p_t, I_t^N, I_t^T, W_t^N, W_t^T, B_{t+1}^N, B_{t+1}^T, w_t, p_t^C, and C_t are endogenously determined.

Market clearing conditions

The demand for non-tradable intermediate goods stems from both the final good sector and the investment expenditures from the intermediate sectors: $Y_t^N = N_t + J_t^N + J_t^T$. From equations (1b), (5b), and (6b), we have

$$p_t Y_t^N = \alpha\mu p_t^C C_t + \eta\left(I_t^T + I_t^N\right)\,. \tag{8}$$

The demand for final consumption goods stems from young and old workers and old entrepreneurs of both sectors.

$$p_t^C C_t = (1-s)w_t L + R^D s w_{t-1} L + \gamma\left(\Pi_t^T + \Pi_t^N\right) \tag{9}$$

Using equations (1a) and (8), we get the expression of the wage rate w_t.

$$w_t L = \frac{1-\alpha}{\alpha\mu}\left[p_t Y_t^N - \eta(I_t^N + I_t^T)\right] \tag{10}$$

The NN and II schedules

By plugging (9) and (10) into (8), we get an increasing relationship between the real exchange rate p_t and the investment expenditures I_t^N, given by

$$p_t Y_t^N = \frac{\mu}{1+s\frac{1-\alpha}{\alpha}}\left[\gamma\left(\Pi_t^T + \Pi_t^N(p_t)\right) + R^D s w_{t-1} L\right] + \eta\left(I_t^T(p_t) + I_t^N\right), \tag{NN}$$

where the dependence on p_t is made explicit. The profits Π_t^T and $\Pi_t^N(p_t)$ are given by equations (2) and (3) and $I_t^T(p_t)$ is given by equations (6e) and (6d). This relationship is represented by the NN schedule in figure 2. It is increasing because the supply of non-tradable goods Y_t^N is predetermined. Higher investment expenditures I_t^N increase the demand for non-tradable goods. With a predetermined supply of non-tradable goods, this increase has to be met by a real appreciation.

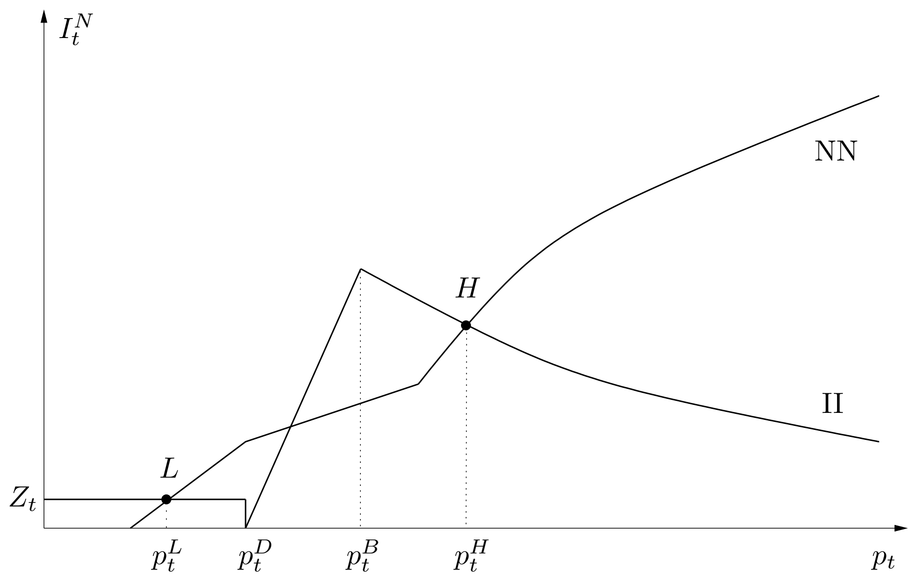


Figure 2: Within-period multiple equilibria.

A second relationship between p_t and I_t^N comes from the investment behavior of the N firms described by equation (5e) and represented by the II schedule in figure 2.

An intersection of these two schedules fully determines a within-period equilibrium.[9]

[9]In the tradable sector, we have $W_t^T = (1-\gamma)(Y_t^T - B_t^T)$ and the investment expenditures I_t^T are given by equation (6e). Then, we have $W_t^N = (1-\gamma)(p_t Y^N - B_t^N)$ if $p_t \geq p_t^D$ and $W_t^N = Z_t$ if $p_t < p_t^D$. The future debt repayment is given by $B_{t+1}^i = R_t^i(I_t^i - W_t^i)$ for $i = T, N$. The wage rate w_t comes from equation (10). Finally, equations (1), together with the production function of the final good sector, implicitly determine the price of the final good p_t^C as a function of p_t and w_t. The quantity C_t of final goods follows from (9).

Multiple within-period equilibria

I now focus on the determination of p_t and I_t^N. The II schedule is composed of three distinctive parts (see figure 2). For $p_t < p_t^D$, the N firms default. The new cohort of entrepreneurs starts with the exogenous endowment Z_t and has no access to the financial market. Therefore, $I_t^N = Z_t$ on that interval. For $p_t^D \leq p_t < p_t^B$, N firms have insufficient internal funds and face a binding borrowing constraint. On this interval, I_t^N is linearly increasing with p_t. Because N firms are indebted in tradable goods, a real appreciation improves their balance sheets and allows them to borrow more. For $p_t \geq p_t^B$, the internal funds of N firms are sufficiently high so that the borrowing constraint does not bind. They borrow less than the maximum amount possible and invest the optimal quantity $\bar{I}_t^N$. Then, I_t^N is decreasing with p_t on that interval.

As it can be seen in figure 2, it is possible that the II and NN schedules intersect three times, with one intersection on each of these three intervals, thus yielding multiple equilibria. The equilibrium located in the interval $[p_t^D, p_t^B]$ is then unstable (in the sense of any virtual out-of-equilibrium dynamics corresponding to the walrasian auctioneer's *tatonnement*) and we are left with two stable equilibria:

- a high equilibrium H with an appreciated real exchange rate p_t^H and high investment expenditures, where N firms have high internal funds and are not constrained,
- a low equilibrium L with a depreciated real exchange rate p_t^L and low investment expenditures, where N firms default on their loans.

Equilibrium H can be identified to tranquil times and equilibrium L to crisis times. This framework allows us to construct a balance-of-payments crisis event as a transition from the high equilibrium p_t^H to the low equilibrium p_{t+1}^L. Such a crisis manifests itself by a real depreciation, a decrease in investment expenditures and widespread defaults on foreign debt in the non-tradable sector.

A necessary condition for the existence of multiple equilibria is that the slope of the II schedule has to be steeper than the slope of the NN schedule at their point of intersection on the interval $[p_t^D, p_t^B]$, as established in the following proposition.

Proposition 1. *A necessary condition for the existence of the two equilibria H and L is given by the inequality*

$$\frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha}} + \eta(1-\gamma)\lambda > 1\,. \tag{11}$$

Proof. See Appendix A.3. □

I make the following assumption to ensure that the economy may be subject to financial fragility.

Assumption 2 (Necessary condition for financial fragility). *It is assumed that inequality (11) holds.*

Note that this condition is satisfied when the financial multiplier λ is large enough, *i.e.* when the borrowing constraint is weak enough. If coefficient λ is interpreted as the level of financial development, it means that the domestic financial system has to be sufficiently developed. The kind of crisis described here would not happen in an economy subject to financial repression.[10]

To get the intuition behind this result, it is useful to look at the special case when entrepreneurs do not get dividends ($\gamma = 0$). Condition (11) then simply becomes $\eta\lambda > 1$. The parameter η determines how the demand for N goods, and consequently the real exchange rate, react to changes in investment expenditures, as can be seen from equations (5b) and (6b). The financial multiplier λ determines how the maximum level of investment expenditures in sector N reacts to changes in internal funds driven by changes in the real exchange rate. The mechanism behind the existence of multiple equilibria is the following. A real appreciation feeds into higher investment expenditures through λ and higher investment expenditures feed into a real appreciation through η. When both effects are strong enough, *i.e.* when η and λ are large enough, it leads to the existence of two equilibria. As an example, consider the case of $\eta = 1$ (the capital consists only of N goods). The condition $\eta\lambda > 1$ would then always be satisfied. If on the contrary $\eta = 0$ (the capital consists only of T goods), a change in investment has no effect on the real exchange rate and the condition is never satisfied.

Financial fragility

An economy is said to be *financially fragile* when a balance-of-payments crisis is possible, *i.e.* when equilibrium L exists. This is the case when the NN schedule intersects the horizontal line $I_t^N = Z_t$ on the left of p_t^D. Using the fact that $I_t^T \leq \lambda(1-\gamma)\Pi_t^T$, a sufficient condition for this can be derived:

$$\frac{B_t^N}{\Pi_t^T} > \frac{\mu}{1 + s\frac{1-\alpha}{\alpha}} \left[\gamma + \frac{R^D s w_{t-1} L}{\Pi_t^T}\right] + \eta(1-\gamma)\lambda + \frac{\eta Z_t}{\Pi_t^T}\,. \tag{12}$$

This condition states that the crisis equilibrium exists whenever the debt repayment of sector N is large enough compared to the profits of sector T.

The existence of equilibrium L in period t is sufficient for unpredicted crises to occur. Inequality (12) is therefore a sufficient condition for the possibility of unexpected crises. Modeling expected crises is slightly more complicated and is delayed in section 4.

Let us examine condition (12) in more details. The ratio $\frac{B_t^N}{\Pi_t^T}$ can be decomposed in the product of two factors.

$$\frac{B_t^N}{\Pi_t^T} = \frac{B_t^N}{\Pi_t^{NH}} \frac{W_t^{NH}}{W_t^T}$$

The first factor B_t^N/Π_t^{NH} relates debt service to tranquil times profits and reflects the financial

[10]This is a usual result in the literature on balance sheets and financial crises. See for example Aghion et al. (2004*b*) and Schneider & Tornell (2004).

structure of N firms' balance sheets. As debt is denominated in tradable goods, it also measures the extent of the currency mismatch. The second factor W_t^{NH}/W_t^T describes the relative size of both sectors and is an indicator of the sectoral structure of the whole economy. This sectoral structure is what determines the level of the real exchange rate needed to adjust a shock on the demand for N goods. Thus, highly leveraged N firms and a sectoral structure largely oriented toward the production of non-tradable goods are conditions that favor the possibility of crises.

Consider now the right-hand side of inequality (12). It consists of the stable components of the demand for N goods which sustain the real exchange rate during a crisis. These components are (a) the second period consumption of both T firms' entrepreneurs and households,[11] (b) the investment expenditures of sector T, which is limited by the financial multiplier λ, and (c) the rescue package Z_t.

The variables entering this condition endogenously evolve with the model dynamics and this evolution will be studied in section 4.

4 Long-run dynamics

4.1 Equilibrium paths

The long-run dynamics consists of successive within-period equilibria and depends on the way agents form their expectations and on the coordination rule they use to choose the within-period equilibria. The remaining part of the paper mainly considers rational expectation equilibria, although the possibility of unexpected crises will sometimes be mentioned. I define two different kinds of rational expectation equilibrium paths depending on the way agents coordinate on the possible within-period equilibria. First, I define a *safe* equilibrium path along which crises never occur.

Definition 1 (Safe equilibrium path). *A safe equilibrium path is a succession of within-period equilibria of type H where*

$$\begin{aligned} \mathrm{E}_t[\rho_{t+1}] &= \rho_{t+1} = 1\,, \\ \mathrm{E}_t[p_{t+1} | p_{t+1} \geq p_{t+1}^D] &= p_{t+1}^H\,. \end{aligned}$$

In the safe equilibrium path the economy is always in the tranquil time equilibrium H and the dynamics is deterministic.

Next, I define a *sunspot-driven* equilibrium path along which crises may occur. Such an equilibrium path is a succession of stochastic within-period sunspot equilibria where agents use the sunspot variable S_t to coordinate on one of the two deterministic equilibria H or L. Along this equilibrium

[11] Since households' savings are invested in riskless bonds denominated in tradable goods, their second period income does not depend on the real exchange rate.

path, foreign lenders rationally anticipate in period $t-1$ the probability $1-\rho_t$ that N firms default in period t and incorporate the risk of default in the interest rate charged to sector N.

The coordination rule of this within-period sunspot equilibrium has to be defined in a careful way. Indeed, when agents expect a crisis in period $t-1$ with a strictly positive probability ($\mathrm{E}_{t-1}[\rho_t] < 1$), the interest rate $R^N_{t-1} = R^D / \mathrm{E}_{t-1}[\rho_t]$ is higher than when no crisis is expected and entrepreneurs from sector N take a lower loan. As a result, the promised debt repayment B^N_t is lower.[12] Then, it can happen that condition (12) is only satisfied in period t if no crisis is expected in period $t-1$. To handle this I introduce an indicator of *expected* financial fragility F_t defined by:

$$F_t = \begin{cases} 1 & \text{if } p_{t-1} > p^B_{t-1} \text{ and} \\ & \frac{R^D}{\omega}\left[A^N_t\left[\frac{\delta p^H_t}{p^{\eta\delta}_{t-1}\frac{R^D}{\omega}}\right]^{\frac{1}{1-\delta}} - W^N_{t-1}\right] > \frac{\mu}{1+s\frac{1-\alpha}{\alpha}}(\gamma\Pi^T_t + R^D s w_{t-1} L) + \eta(1-\gamma)\lambda\Pi^T_t + \eta Z_t\,, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

The indicator F_t takes the value 1 when condition (12) is satisfied in period t even if foreign lenders' expectations are given by $\mathrm{E}_{t-1}[\rho_t] = \omega$ in period $t-1$.[13]

In a within-period sunspot equilibrium, agents perceive financial fragility in period t when $F_t = 1$. In this case, they use the sunspot S_t to coordinate on the within-period equilibrium. When $F_t = 0$ they coordinate on the tranquil time equilibrium H. The coordination rule is described by table 1. Given the definition (13) of F_t, this coordination rule can be consistent with past expectations.

Table 1: Coordination rule in the within-period sunspot equilibrium

	Financial fragility indicator	
Sunspot	$F_t = 0$	$F_t = 1$
$S_t = 0$	H	L
$S_t = 1$	H	H

The sunspot-driven equilibrium path can now be formally defined.

Definition 2 (Sunspot-driven equilibrium path). *A sunspot-driven equilibrium path is a succession of within-period sunspot equilibria where the type of within-period equilibrium (H or L) is given*

[12] From equation (5d) and from the fact that $B^N_t = R^N_{t-1}(\bar{I}^N_{t-1} - W^N_{t-1})$ when the economy is in equilibrium H in period $t-1$, the promised debt repayment B^N_t is a decreasing function of the interest rate R^N_{t-1}.

[13] This definition of F_t is slightly conservative since condition (12) is only a sufficient condition for the existence of equilibrium L. This tends to limit the scope for crises along a sunspot-driven equilibrium path.

by table 1 and where

$$\mathrm{E}_t[\rho_{t+1}] = \rho_{t+1} = 1 - (1-\omega)F_{t+1}\,,$$
$$\mathrm{E}_t[p_{t+1}|p_{t+1} \geq p^D_{t+1}] = p^H_{t+1}\,.$$

Along a sunspot-driven equilibrium path expected self-fulfilling crises may happen and the dynamics can be either deterministic or stochastic.

The remaining of this section studies the long-run dynamics along a safe equilibrium path. All quantities are normalized by the productivity trend and the reduced variables are denoted by lower-case letters (*e.g.* $y^N_t = Y^N_t/(1+g)^t$). To get lighter notations, total savings $\Sigma_t = sw_tL$ are used instead of the wage rate w_t. Accordingly, define $\sigma_t = \Sigma_t/(1+g)^t$. Using these notations, the equations describing the intermediate sectors in a safe equilibrium path are the following.

$$y^N_t = (a^N)^{1-\delta}\left[\frac{i^N_{t-1}}{(1+g)p^\eta_{t-1}}\right]^\delta \qquad y^T_t = (a^T)^{1-\delta}\left[\frac{i^T_{t-1}}{(1+g)p^\eta_{t-1}}\right]^\delta \tag{14a}$$
$$w^N_t = (1-\gamma)(p^H_t y^N_t - b^N_t) \qquad w^T_t = (1-\gamma)(y^T_t - b^T_t) \tag{14b}$$

$$i^N_t = (1+g)a^N\left[\frac{\delta p^H_{t+1}}{p^{\eta\delta}_t R^D}\right]^{\frac{1}{1-\delta}} \qquad i^T_t = \min\left[\lambda w^T_t, (1+g)a^T\left[\frac{\delta}{p^{\eta\delta}_t R^D}\right]^{\frac{1}{1-\delta}}\right] \tag{14c}$$
$$b^N_{t+1} = (i^N_t - w^N_t)\frac{R^D}{1+g} \qquad b^T_{t+1} = (i^T_t - w^T_t)\frac{R^D}{1+g} \tag{14d}$$

To complete the description of the safe equilibrium path, I also restate equation (10) that gives the expression of the real wage, and the market equilibrium for N goods (NN).

$$\sigma_t = \frac{s(1-\alpha)}{\alpha\mu}\left[p^H_t y^N_t - \eta(i^N_t + i^T_t)\right] \tag{14e}$$
$$p^H_t y^N_t = \frac{\mu}{1+\frac{s(1-\alpha)}{\alpha}}\left[\frac{\gamma}{1-\gamma}(w^N_t + w^T_t) + \sigma_{t-1}\frac{R^D}{1+g}\right] + \eta(i^N_t + i^T_t) \tag{14f}$$

Let us start by computing the stationary state before describing the transitory dynamics.

4.2 The safe stationary state

I compute the stationary state of equations (14) under the assumption that the borrowing constraint does not bind in the tradable sector. I will show later that this is indeed the case. I introduce a reduced parameter $\psi = \frac{1+g}{R^*(1+\tau)}$. The parameter ψ increases with financial openness (*i.e.* it decreases with τ), technological progress (g), and the supply of international liquidity (*i.e.* it decreases with R^*).[14] To derive the equations determining the safe stationary state, I use the fact that $\delta\psi p^H y^N = i^N$ and $\delta\psi y^T = i^T$. The financial structure of the intermediate sectors is given by

$$w^N = \frac{(1-\gamma)(\frac{1}{\delta}-1)}{\psi-(1-\gamma)} i^N\,, \qquad w^T = \frac{(1-\gamma)(\frac{1}{\delta}-1)}{\psi-(1-\gamma)} i^T\,, \tag{15a}$$

$$b^N = \frac{\psi - \frac{1-\gamma}{\delta}}{\psi-(1-\gamma)} \frac{i^N}{\psi}\,, \qquad b^T = \frac{\psi - \frac{1-\gamma}{\delta}}{\psi-(1-\gamma)} \frac{i^T}{\psi}\,. \tag{15b}$$

Then, the investment expenditures of sector N, the real exchange rate, and the savings of households are given by

$$i^N = \frac{a^N}{(1+g)^{\frac{\delta}{1-\delta}}} (\delta\psi)^{\frac{1}{1-\delta}} \left[\frac{a^T}{a^N}\frac{i^N}{i^T}\right]^{1-\eta\delta}\,, \tag{15c}$$

$$p^H = \left[\frac{a^T}{a^N}\frac{i^N}{i^T}\right]^{1-\delta}\,, \tag{15d}$$

$$\sigma = \frac{\frac{s(1-\alpha)}{\alpha}}{1+\frac{s(1-\alpha)}{\alpha}(1-\frac{1}{\psi})} \frac{\gamma}{1-\gamma} (w^N + w^T)\,. \tag{15e}$$

To complete the description, the relative investment level in both sectors follows from

$$\frac{i^N}{i^N + i^T} = h(\psi) \tag{15f}$$

where

$$h(\psi) = \delta\eta\psi + (1-\delta)\frac{\mu\gamma\psi}{\psi-(1-\gamma)} \frac{1}{1+\frac{s(1-\alpha)}{\alpha}(1-\frac{1}{\psi})}\,. \tag{15g}$$

The important equation is (15f), which is the long-run version of the market clearing condition for N goods. It determines the relative size of the non-tradable sector $\frac{I^N}{I^T} = \frac{i^N}{i^T}$, that is, the allocation of capital between the non-tradable and the tradable sectors. Once known, this ratio completely determines the stationary state and the value of all variables can be easily deduced from it.[15] Note that, from equations (15a), W^N/W^T is equal to I^N/I^T in the safe stationary state, and that Y^N/Y^T is increasing with I^N/I^T (with the elasticity δ) so that I^N/I^T can be referred to without ambiguity as the sectoral structure of the economy.

[14] When g and $R^D - 1$ are small, we have $\psi \approx 1 + g - (R^D - 1)$. Therefore, $\psi - 1$ is approximately equal to the difference between the growth rate and the domestic interest rate.

[15] The stationary real exchange rate and the investment expenditures i^N are deduced from $\frac{i^N}{i^T}$ by (15d) and (15c). Then, internal funds and debt levels are given by equations (15a) and (15b) and savings from households by (15e).

Sectoral structure in the long run

How does the economy adjust to an exogenous permanent shock? In the short run productive capacities are predetermined in the intermediate sectors and the market equilibrium has to be achieved by a change in the real exchange rate alone (see the within-period equilibrium in section 3). In the long run, on the contrary, the sectoral structure itself can change and thus accommodate a permanent shock. Part of the adjustment still comes from the real exchange rate whose value in the stationary state depends on the sectoral structure I^N/I^T.[16] Here, I am mainly interested in the way the economy adjusts to a change in the external financing conditions, *i.e.* to a shock in the domestic interest rate R^D. Such a shock can reflect both a change in the supply of available international liquidity (*i.e.* a change in R^*) or a change in the degree of financial openness (*i.e.* a change in τ).

Let us study I^N/I^T as a function of ψ. The function $h(\psi)$, defined by equation (15g), is continuous, positive, and U-shaped on the interval $(\psi_0, +\infty)$, where ψ_0 is the maximum value of $(1-\gamma)$ and $\left(1+\frac{\alpha}{s(1-\alpha)}\right)^{-1}$. Therefore, as the left-hand side of equation (15f) is strictly increasing with I^N/I^T, the sectoral structure I^N/I^T is also a U-shaped function of ψ on the relevant interval. It is plotted in figure 3, the parameters being calibrated with data from Argentina in the nineteen-nineties (see appendix A.6 for details on the calibration). The ψ-axis starts with $\psi = \psi^A = \frac{1+g}{R^A}$, where R^A is the autarky interest rate (see appendix A.4 for its derivation).

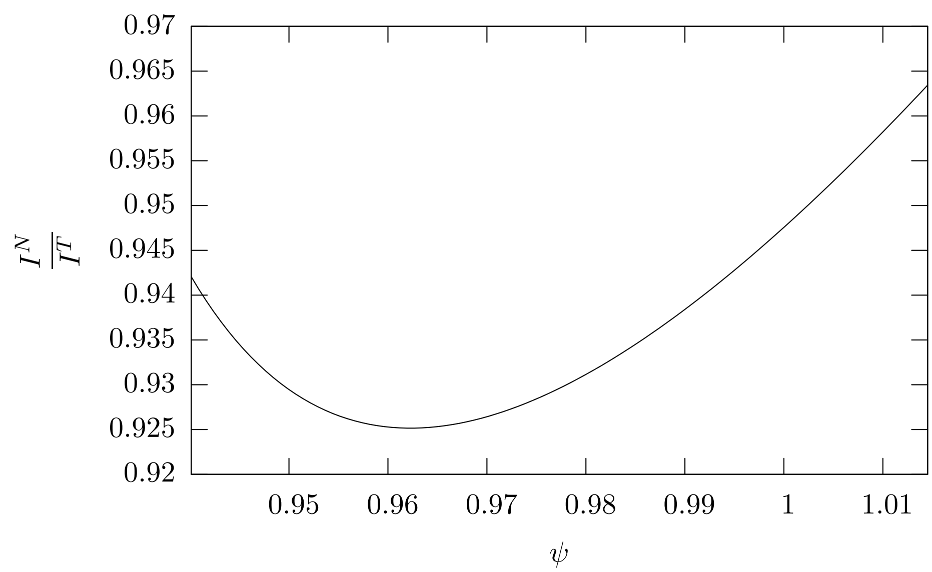


Figure 3: Sectoral structure in the safe stationary state: $\frac{I^N}{I^T}$ is plotted against ψ on the interval $\left[\psi^A, (1-\gamma)[1+\lambda(\frac{1}{\delta}-1)]\right]$, for $\alpha = 48\%$, $\mu = 46.22\%$, $\eta = 49\%$, $\gamma = 11\%$, $\delta = 0.947$, $\beta = 0.053$, and $\lambda = 2.5$.

Suppose that the economy is initially in the autarky stationary state $\psi = \psi^A$ and the capital

[16]Note that the long-run real exchange rate also depends on the ratio of sectoral productivities, a usual Balassa-Samuelson effect. Cf. equation (15d).

account is liberalized so that τ diminishes and ψ increases. If the financial opening is mild and ψ does not increase too much, the relative size of the non-tradable sector is smaller in the new stationary state and there is a real depreciation in the long run. If on the contrary it is large enough, capital is reallocated toward the non-tradable sector in the long run and the stationary real exchange rate appreciates.

To get the intuition behind this it is useful to discuss the case of zero household savings ($\beta = 0$). Then, in the closed economy ($\psi = \psi^A = \frac{1-\gamma}{\delta}$) firms have zero debt and finance all their investment expenditures by using their internal funds. When the economy opens to capital inflows, entrepreneurs issue debt abroad provided that $R^D < R^A$. This has two opposite effects on the demand for non-tradable goods. On the one hand, it allows domestic entrepreneurs to invest more, increasing the demand for N goods. On the other hand, entrepreneurs have to pay their debt back, which diminishes both internal funds and dividends (in relative terms) and leads to a decrease in investment and consumption. The net effect on the demand for N goods in the stationary state is given by

$$\underbrace{\eta\psi(b^N + b^T)}_{\substack{\text{higher investment}\\ \text{from new debt}}} - \underbrace{\eta(1-\gamma)(b^N + b^T)}_{\substack{\text{lower investment}\\ \text{from debt repayment}}} - \underbrace{\mu\gamma(b^N + b^T)}_{\substack{\text{lower consumption}\\ \text{from debt repayment}}} .$$

When $\psi < \frac{\mu\gamma+\eta(1-\gamma)}{\eta}$ capital outflows that pay back the external debt reduce more the demand for N goods than new capital inflows increase it. This induces a shift of resources from the non-tradable sector to the tradable sector. On the contrary when $\psi > \frac{\mu\gamma+\eta(1-\gamma)}{\eta}$ the net effect on the demand for N goods is positive and capital is reallocated to sector N in the long run.[17] When $\psi = \frac{\mu\gamma+\eta(1-\gamma)}{\eta}$ the ratio I^N/I^T is exactly equal to its stationary value in the closed economy.

Financial structure in the long run

The financial structure of firms, described by the ratio B^i/Π^i, depends on ψ in an unambiguous way. From equations (15a) and (15b) we have

$$\frac{B^i}{\Pi^i} = \frac{\delta}{1-\delta}\left(1 - \frac{1-\gamma}{\delta\psi}\right), \qquad i = N, T.$$

The lower the domestic interest rate R^D, the higher ψ, and the higher the ratio $\frac{B^i}{\Pi^i}$. A decrease in the domestic interest rate always leads to a more leveraged financial structure in the long run.

Restrictions on ψ

Some constraints have to be imposed on ψ to insure the existence of a safe stationary state with good properties.

[17] The mechanism described here is similar to the so-called "Dutch disease" phenomenon. See Kalantzis (2004, 2005) for a model specifically relying on this effect.

To begin with, equations (15a) and (15b) imply that internal funds and debt levels only reach a stationary state when $\psi > (1-\gamma)$.

Then, I want entrepreneurs to be net debtors in the stationary state. Therefore, I need $\psi \geq \frac{1-\gamma}{\delta}$.

Another constraint is that the definition of a safe equilibrium path requires the economy to be in the high equilibrium H in each period. The high equilibrium is characterized by the fact that the borrowing constraint does not bind for N firms. Therefore, the safe stationary state only exists if $i^N \leq \lambda w^N$. This is the case as long as $\psi \leq \psi^+ = (1-\gamma)[1+\lambda(\frac{1}{\delta}-1)]$. Under this assumption sector T is not constrained in the steady state either.

Lastly, the ratio I^N/I^T does not converge to a positive value when $h(\psi) \geq 1$. We know that $h(\psi) \longrightarrow +\infty$ when ψ tends to ψ_0 from above and to $+\infty$ and it can be easily checked that $h(1) < 1$. Therefore, there exists $\psi^{\min}$ and $\psi^{\max}$, with $\psi_0 < \psi^{\min} < 1 < \psi^{\max}$, such that $h(\psi) < 1$ for all $\psi \in (\psi^{\min}, \psi^{\max})$. More precisely, $\psi^{\min}$ and $\psi^{\max}$ are zeros of the denominator of I^N/I^T and W^N/W^T. Let us make a mild assumption on the saving rate s.

Assumption 3.
$$\frac{s(1-\alpha)}{\alpha}\left[\frac{\delta}{1-\gamma}-1\right] < \frac{1-\mu\gamma-\eta(1-\gamma)}{1-\eta(1-\gamma)}$$

This assumption is always satisfied when $\delta < 1-\gamma$. When not, it sets an upper limit on the saving rate $s = \frac{\beta}{1+\beta}$. Under this assumption, it can be shown that $\frac{1-\gamma}{\delta} > \psi_0$ and $h(\frac{1-\gamma}{\delta}) < 1$. Therefore, we have $\psi^{\min} < \frac{1-\gamma}{\delta}$.

To sum it up the following restriction is imposed on ψ.

$$\frac{1-\gamma}{\delta} \leq \psi \leq \min(\psi^+, \psi^{\max})$$

4.3 The safe transitory dynamics

Let us now turn to the transitory dynamics. I simulate the safe equilibrium path followed by an economy after a permanent exogenous shock. The economy is initially in a safe stationary state (at $t=0$). At $t=1$, the domestic interest rate R^D decreases unexpectedly and permanently (as a result of a larger financial openness or of an increase in the supply of international liquidity). The initial stationary state is chosen on the upward-sloping part of the curve $(I^N/I^T)(\psi)$ so that the long-run effect of the shock is to increase p and I^N/I^T. The parameters used for the simulation are again calibrated with Argentinian data (see appendix A.6). In addition, I set $g = 5.72\%$ (the geometric average of the Argentine growth rate between 1991 and 1998). The permanent shock is a decrease of one percentage point in the interest rate (from 6% to 5%). The resulting transition phase is displayed in figure 4.[18] To be sure that this is a safe equilibrium path I check that equilibrium H

[18]The simulation is performed using the software Dynare. The total number of periods is set to 200. Only the first 10 periods are displayed in figure 4. I check that the initial and final stationary states are locally determinate (*i.e.* the Jacobian matrix has as many eigenvalues outside the unit circle as there are forward looking variables in the dynamics).

exists in each period, *i.e.* that $i_t^N < \lambda w_t^N$ for each t.

The dynamics is essentially driven by the variables i_t^N, p_t^H, and y_t^N. Suppose for a moment that all other variables were constant. From equations (14c) and (14f) the dynamics would then be described by the equation $i_t^N = \delta\psi p_{t+1}^H y_{t+1}^N = \delta\psi(\mathrm{C}^{\mathrm{st}} + \eta i_{t+1}^N)$, where C^{st} denotes a constant. This equation can be solved forward in time and determines a unique i_t^N. As ψ increases at $t = 1$ (because R^D decreases), i_1^N would instantaneously jump to its new higher stationary value. This would permanently shift upward the demand for N goods. Because the supply y_1^N is predetermined, the adjustment in the first period can only come from a real appreciation so that p_1^H would also jump to a higher value. Then, for $t \geq 2$, y_t^N would slowly increase along dynamics of the kind $y_{t+1}^N \propto (y_t^N)^{\eta\delta}$ while p_t^H would decrease so as to keep $p_t^H y_t^N$ constant.

Consider now the way i_t^T alters this simple dynamics. At $t = 1$, a high price p_1^H means a high cost of capital. While it is offset in sector N by the expectation of high future proceeds, it leads to a decrease of investment expenditures in the tradable sector, *i.e.* i_t^T decreases at $t = 1$. Then, as the real exchange rate gradually depreciates for $t \geq 2$, i_t^T slowly increases up to its new stationary value. This has two consequences on the transitory dynamics. First, the value of the demand for N goods—the right-hand side of equation (14f)—has to be increasing with time. Therefore, i_t^N does not adjust in one period. After jumping to a higher value at $t = 1$, i_t^N goes on increasing for $t \geq 2$. Then, from equation (14c) the dynamics of the real exchange rate is given by $p_{t+1}^H \propto p_t^H (i_t^N)^{1-\delta}$. As i_t^N is now increasing with time, p_t^H may decrease below its stationary value and increase henceforth as it can be seen in figure 4. Note that the initial decrease of i_t^T explains the overshooting in the evolution of the ratio I^N/I^T.

The evolution of the financial structure B_t^i/Π_t^i of N and T firms comes from the dynamics of w_t^i and b_t^i. At $t = 1$, the internal funds w_1^N increase because of the real appreciation while w_1^T, a predetermined variable, stays constant. The debt repayments b_1^N and b_1^T are predetermined and do not react in the first period. At $t = 2$, the debt repayment b_2^N increases because N firms have issued a lot of debt to finance the higher investment expenditures at $t = 1$. This has an adverse effect on w_2^N which slightly decreases. In the meanwhile, both w_2^T and b_2^T diminish because of the lower scale of investment in the tradable sector at $t = 1$. For $t > 2$, these four variables follow the dynamics determined by equations (14b) and (14d). In particular, w_t^N and w_t^T increase with time, which reinforces the evolution of i_t^N. Furthermore, if w_t^T increases too slowly, the borrowing constraint may bind in the tradable sector during the adjustment process, slowing down the convergence. Because of the initial increase in w_t^N the ratio W^N/W^T also displays some overshooting. However, its evolution is much smoother than I^N/I^T, due to the fact that it is more dependent on lagged variables while I^N/I^T strongly depends on forward variables.

The equations (14e) and (14f) show that the dynamics of the savings from household σ_t are governed by the evolution of $w_t^N + w_t^T$. They increase in the first period because of the real appreciation, decrease in the second period and gradually increase after that.

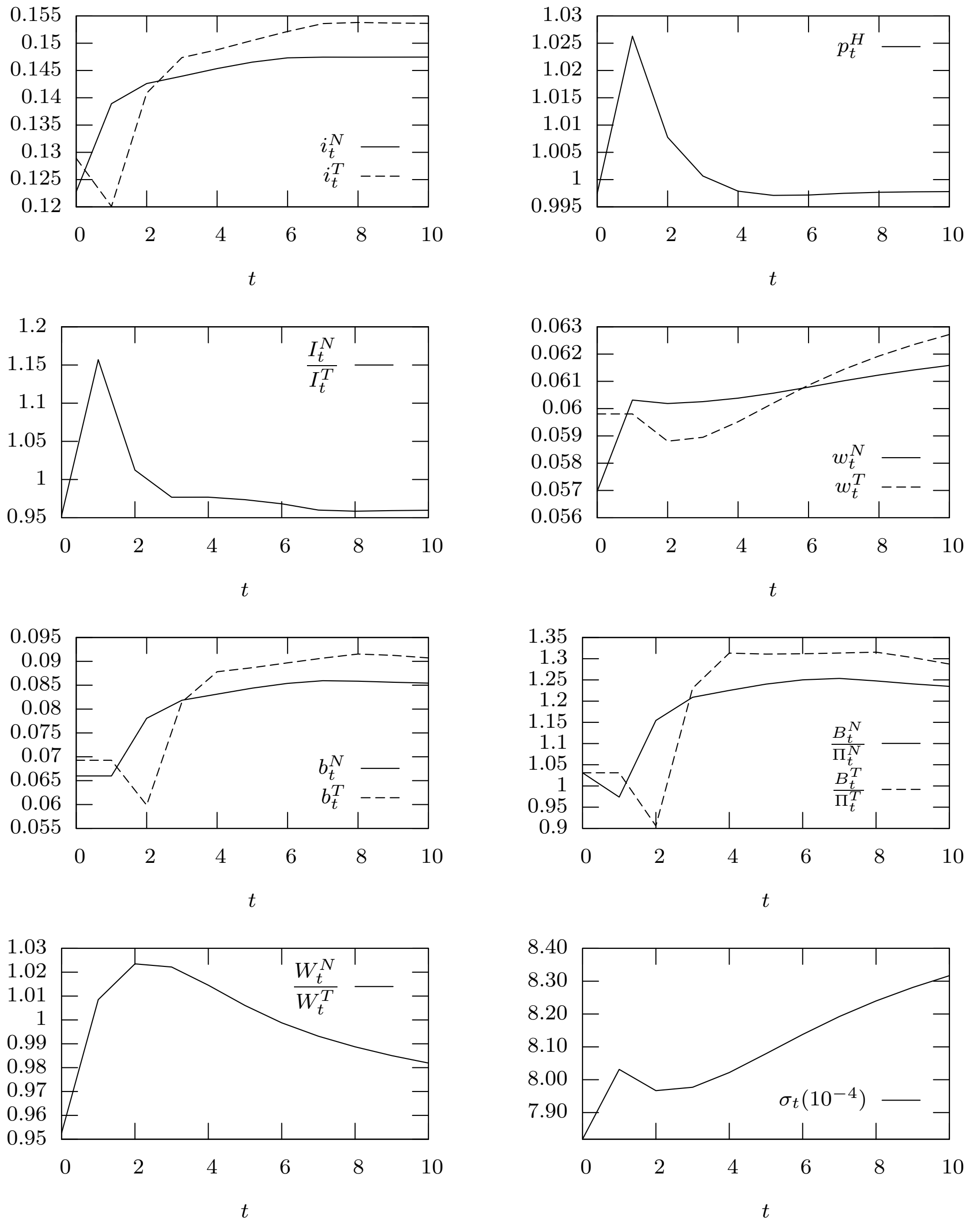


Figure 4: Safe equilibrium path following a permanent decrease in R^D. Parameters: $\alpha = 48\%$, $\mu = 46.22\%$, $\eta = 49\%$, $\gamma = 11\%$, $\delta = 0.947$, $\beta = 0.053$, $\lambda = 2.5$, and $g = 5.72\%$. At $t = 0$, the economy is in the stationary state corresponding to $R^D = 1.06$. At $t = 1$, it is hit by a permanent shock on the interest rate: $R^D = 1.05$ for $t \geq 1$.

5 Financial fragility in the long run

5.1 General results

This section addresses the issue of financial fragility and the possibility of balance-of-payments crises along an equilibrium path. Studying crises triggered by unanticipated expectational shocks is straightforward. If the evolution of W_t^N/W_t^T and B_t^N/Π_t^N along the safe equilibrium path described in the previous section is such that inequality (12) is satisfied in some period t, an unexpected self-fulfilling crisis can occur during this period.[19] But is it possible for crises to occur in a way consistent with past expectations? Along a sunspot-driven equilibrium path the financial fragility of an economy in period t must have been anticipated in period $t-1$, *i.e.* the indicator F_t must be equal to 1, and the investment decisions taken by entrepreneurs of N firms in period $t-1$, given the expected probability of default $1-\rho_t = 1-\omega$, must be such that the crisis equilibrium L exists in period t.

The analysis of a sunspot-driven equilibrium path is not an easy task. In particular, when such a path displays financial fragility, it never converges to a stationary state: in that case the dynamics of the kind studied in section 4.3 are indeed interrupted by crises each time the sunspot takes the value 1 when $F_t = 1$. To study a sunspot-driven equilibrium path, my strategy is to define a fictitious stationary state where no crisis occurs and which I dub, using the terminology of Schneider & Tornell (2004), the *lucky* stationary state. The lucky stationary state is the stationary state of the sunspot-driven equilibrium path when the sunspot always takes the no-crisis value $S = 1$, while agents still expect it to take the value 0 with probability $1-\omega$. A sunspot-driven equilibrium path eventually converges to the lucky stationary state if $S_t = 1$ for a large enough number of successive time periods.

If the lucky stationary state is not financially fragile in a way consistent with expectations (*i.e.* if $F = 0$), it is the actual stationary state of the sunspot-driven equilibrium path. Then, anticipated crises are impossible in the long run although they might possibly occur during the transition phase. On the contrary, if the lucky stationary state is financially fragile (*i.e.* if $F = 1$), the actual sunspot-driven equilibrium path might eventually reach the lucky state, but only to be driven off it when the sunspot takes the value 0 and a crisis occurs.[20] In this case, anticipated crises are possible in the long run and any sunspot-driven equilibrium path then experiences recurrent crises so that the economy never converges to a stationary state. The important point is therefore to determine whether and under what condition the lucky stationary state is financially fragile.

[19] Of course, if it does, the trajectory is not a rational expectation equilibrium path any more.

[20] Note that the economy needs not even reach the lucky stationary state since crises are likely to be possible before.

The lucky stationary state

To determine whether the lucky stationary state is financially fragile the sufficient condition for the existence of the crisis equilibrium (12) has to be restated in the lucky stationary state. When it is not financially fragile, the lucky stationary state is similar to the safe stationary state and is determined by equations (15). When it is financially fragile, the interest rate in sector N is equal to R^D/ω, which is higher than the riskless rate faced by sector T, and the lucky state slightly differs from the safe stationary state.

First, the constraints on ψ are different in a lucky stationary state subject to financial fragility. The threshold ψ^+ above which equilibrium H does not exist now depends on the value taken by ρ. Denote it $\psi^+_\rho = \frac{1}{\rho}(1-\gamma)\big[1+\lambda\big(\frac{1}{\delta}-1\big)\big]$. Thus, cases where $\psi^+_1 < \psi \leq \psi^+_\omega$ are *a priori* possible. Note that when ψ is in this interval, the borrowing constraint is binding in the tradable sector.[21] Likewise, the threshold $\psi^{\max}$ above which I^N/I^T and W^N/W^T do not converge to a finite stationary value, *i.e.* above which their denominator becomes negative, also depends on the value taken by ρ. Denote it $\psi^{\max}_\rho$.

Then, the variables involving the non-tradable sector have different stationary values because of the higher interest rate. They are now given by

$$w^N = \frac{(1-\gamma)(\frac{1}{\delta}-1)}{\psi\rho-(1-\gamma)} i^N\,, \tag{16a}$$

$$b^N = \frac{\psi\rho - \frac{1-\gamma}{\delta}}{\psi\rho-(1-\gamma)}\,\frac{i^N}{\psi\rho}\,, \tag{16b}$$

$$\frac{B^N}{\Pi^N} = \frac{\delta}{1-\delta}\left(1-\frac{1-\gamma}{\delta\psi\rho}\right)\,. \tag{16c}$$

As W^N/I^N differs from W^T/I^T when $\rho=\omega$, the stationary value of W^N/W^T is not necessarily equal to that of I^N/I^T. It is determined by the following equation.

$$\frac{W^N}{W^T} = \frac{\dfrac{\mu\gamma}{1+\frac{s(1-\alpha)}{\alpha}(1-\frac{1}{\psi})} + \eta(1-\gamma)\dfrac{I^T}{W^T}}{\dfrac{1-\gamma}{\delta\psi\rho}\dfrac{I^N}{W^N} - \dfrac{\mu\gamma}{1+\frac{s(1-\alpha)}{\alpha}(1-\frac{1}{\psi})} - \eta(1-\gamma)\dfrac{I^N}{W^N}} \tag{16d}$$

[21] The other end of the spectrum is not modified. If $\frac{1-\gamma}{\delta} \leq \psi \leq \frac{1}{\rho}\frac{1-\gamma}{\delta}$, we have $\rho=1$ since equilibrium L cannot exist with a strictly negative debt in sector N.

Can crises occur in the long run?

By using the condition (12) and the expression of stationary savings (15e) the sufficient condition for financial fragility in the lucky stationary state can now be stated:

$$\frac{W^N}{W^T}\left[\frac{B^N}{\Pi^N}+\frac{\mu\gamma}{1+\frac{s(1-\alpha)}{\alpha}}-\frac{\mu\gamma}{1+\frac{s(1-\alpha)}{\alpha}(1-\frac{1}{\psi})}\right]$$
$$> \frac{\mu\gamma}{1+\frac{s(1-\alpha)}{\alpha}(1-\frac{1}{\psi})}+\eta(1-\gamma)\lambda+\frac{\eta(1-\gamma)z}{w^T}. \quad (17)$$

Denote $Q(\rho,\psi) = \text{LHS} - \text{RHS}$ where LHS and RHS are the left- and right-hand sides of this inequality. If ψ is such that $Q(\omega,\psi) > 0$, the lucky stationary state is financially fragile and is characterized by $F = 1$ and $\rho = \omega$. Then, all sunspot-driven equilibrium paths corresponding to this ψ are subject to recurrent self-fulfilling crises. When $Q(1,\psi) > 0$, the safe stationary state is subject to unexpected financial fragility and only non anticipated self-fulfilling crises can occur. The following lemma and proposition show that financial fragility is indeed possible for a small rescue package z and for high values of ψ.

Lemma 2. *If z is small enough, the function $\rho \mapsto Q(\rho,\psi_\rho^+)$ is strictly positive on the set $\{\rho\in[0,1]|\psi_\rho^+<\psi_\rho^{max}\}$.*

Proof. See appendix A.5. □

Proposition 3. *Assume that $\frac{B^N}{\Pi^N} > \frac{\mu\gamma}{1+\frac{s(1-\alpha)}{\alpha}(1-\frac{1}{\psi})} - \frac{\mu\gamma}{1+\frac{s(1-\alpha)}{\alpha}}$ when $\psi=\psi_\omega^{max}$. Then, if z is small enough there exists $\psi_{\text{exp}} \in \left(\frac{1-\gamma}{\delta}, \min(\psi_\omega^+,\psi_\omega^{max})\right)$ and $\psi_{\text{unexp}} \in \left(\frac{1-\gamma}{\delta}, \min(\psi_1^+,\psi_1^{max})\right)$ such that:*

- *The lucky stationary state is financially fragile when $\psi_{\text{exp}} < \psi < \min(\psi_\omega^+,\psi_\omega^{max})$. In that case any sunspot-driven equilibrium path necessarily goes through periods of balance-of-payments crisis.*
- *Unexpected balance-of-payments crises can happen in the safe stationary state when $\psi_{\text{unexp}} < \psi < \min(\psi_1^+,\psi_1^{max})$.*
- *In addition, if ω is sufficiently close to 1, $\psi_{\text{exp}} < \psi_1^+$.*

Proof. To prove the first part of the proposition we just have to show that $Q(\omega,\psi) > 0$ when $\psi \xrightarrow{\leq} \min(\psi_\omega^+,\psi_\omega^{\max})$ and use the fact that $\psi \mapsto Q(\omega,\psi)$ is continuous on the left of $\min(\psi_\omega^+,\psi_\omega^{\max})$. If $\psi_\omega^+ < \psi_\omega^{\max}$ we know from lemma 2 that $Q(\omega,\psi_\omega^+) > 0$. Consider now the case $\psi_\omega^{\max} \leq \psi_\omega^+$. When $\psi \xrightarrow{\leq} \psi_\omega^{\max}$, $\frac{W^N}{W^T} \longrightarrow +\infty$ by definition and $Q(\omega,\psi) \longrightarrow +\infty$ from the assumption on $\frac{B^N}{\Pi^T}(\psi_\omega^{\max})$.

The second part of the proposition is just a special case of the previous result when $\omega = 1$. The last part of the proposition comes from the fact that $\rho \mapsto Q(\rho,\psi)$ is continuous at $\rho = 1$.

The assumption on $\frac{B^N}{\Pi^T}(\psi_\omega^{\max})$ is not a very strong one. It is always satisfied when the saving rate s is small enough. Numerical simulations show that it is also satisfied for larger values of s. □

This proposition establishes that the steady state can always be financially fragile provided that the conditional endowment is small enough and ψ is large enough, which is the case when:

1. the economy is very opened to international capital flows (the iceberg cost to international transactions τ is small),
2. there is a large supply of international liquidity (the world interest rate R^* is low),
3. the growth rate is high (because of large productivity gains).[22]

In general, it is possible to have $\psi_{\text{exp}} \geq \psi_1^+$. When this is the case, the firms of the tradable sector are credit-constrained in financially fragile lucky stationary states with $\psi > \psi_{\text{exp}}$. The second part of the proposition, however, shows that if the probability ω is large enough, *i.e.* if the probability of default is sufficiently small, the threshold value ψ_{exp} can be strictly lower than ψ_1^+ so that a stationary state where T firms do not face a binding borrowing constraint can also be financially fragile.[23]

5.2 The example of Argentina

Is this story reasonable? What kind of values do the model's parameters have to take so that financial fragility arise in the long run? To illustrate the preceding results the model is now calibrated using data from Argentina in the nineteen-nineties.[24] This exercise is not meant to reproduce precisely the actual evolution of the Argentine economy. It simply intends to show with a specific example that the model is roughly consistent with the empirical evidence and that its predictions have an acceptable order of magnitude. The calibrated parameters are given in table 2. The calibration procedure is described in details in appendix A.6.

Figure 5 displays $Q(1,\psi)$ and $Q(\omega,\psi)$ as a function of ψ for this set of parameters and in the limit of a zero rescue package. Three cases are possible:[25]

[22] This confirms the result by Rancière et al. (2003) stating that there might be a trade-off between high growth and financial stability.

[23] A disturbing characteristic of the case $\psi_{\text{exp}} \geq \psi_1^+$ is that the lucky stationary state may not exist for all $\psi \in (\psi_1^+, \psi_{\text{exp}})$. On this interval we have $F = 0$ and $\rho = 1$. When this is the case, the stationary state is not defined since $\psi > \psi_\rho^+ = \psi_1^+$. This problem disappears when ω is close enough to 1.

[24] This country has implemented a reform package, including the opening of the capital account, between 1989 and 1991. The economy has then experienced a decade of high growth (interrupted by the "Tequila" crisis of 1995) until the recession of 1999 that culminated in a banking crisis, the abandon of the hard-peg, a default on external debt, and a collapse of economic activity in 2001-2002.

[25] To simplify the exposition I do as if the sufficient condition (17) for financial fragility were also necessary. That is not always true. Therefore, the possibility of crises is slightly underestimated by these results.

Table 2: Calibration with data from Argentina in the nineteen-nineties

Parameter	Value
α	48%
μ	46.22%
η	49%
s	5%
β	0.053
γ	11%
δ	0.947
λ	2.5
ω	0.99

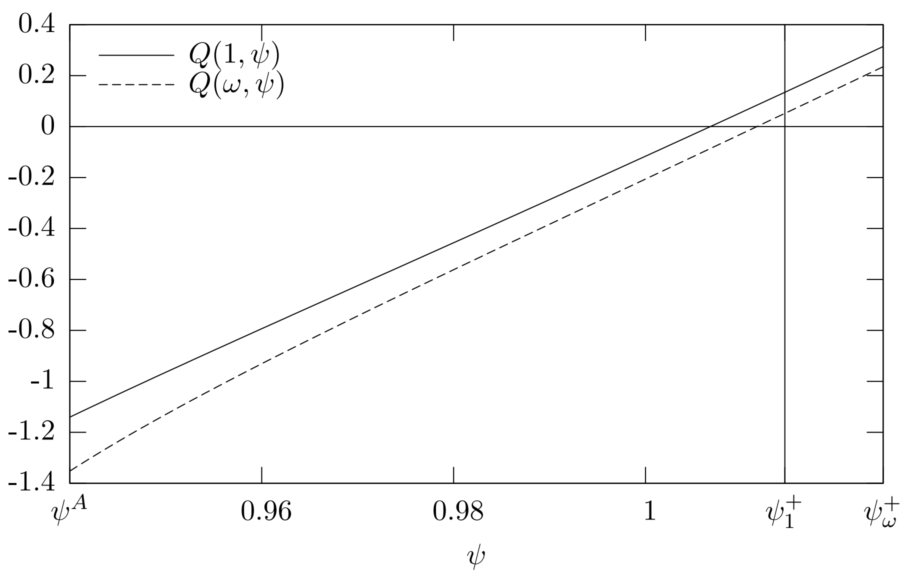


Figure 5: Financial fragility in the long run: $Q(1,\psi)$ and $Q(\omega,\psi)$ are plotted on the interval $\left(\psi^A, \psi_\omega^+\right)$ for $\alpha = 48\%$, $\mu = 46.22\%$, $\eta = 49\%$, $\gamma = 11\%$, $\delta = 0.947$, $\beta = 0.053$, $\lambda = 2.5$, $\omega = 0.99$, and $z \to 0$. The solid vertical line corresponds to $\psi = \psi_1^+$ and divides the plan in a zone where sector T is not financially constrained (on the left) and a zone where it is (on the right).

- For $\psi \lesssim 1.007$, $Q(\omega,\psi) < Q(1,\psi) < 0$ and there is no financial fragility, neither expected nor unexpected. Sunspot-driven and safe equilibrium paths are identical. With a growth rate $g = 5.72\%$ for the nineteen-nineties (see appendix A.6), $\psi \lesssim 1.007$ corresponds to a domestic riskless interest rate $(R^D - 1)$ approximately larger than 5.0%.

- For $1.007 \lesssim \psi \lesssim 1.012$, $Q(\omega,\psi) \leq 0 < Q(1,\psi)$. The lucky stationary state is not financially fragile. However, unexpected crises are possible along the safe stationary state. This interval of values for ψ is equivalent to the interval $[4.5\%, 5\%]$ for the domestic interest rate.

- For $\psi \gtrsim 1.012$, that is for a domestic interest rate lower than 4.5%, $Q(1,\psi) > Q(\omega,\psi) > 0$. The lucky stationary state is financially fragile. Crises are possible along an equilibrium path, even if agents expect them rationally. Sunspot-driven equilibrium paths are hit by recurrent crises.

The average value of ψ in 1991-1998 is approximately equal to 1.012, and probably slightly higher (appendix A.6). This corresponds to a average domestic riskless interest rate slightly lower than 4.5%. Thus, it seems plausible that Argentina was financially fragile in the nineteen-nineties, even if foreign lenders rationally took this fragility into account with a small subjective probability of crisis.

Next, I simulate the effect on a sunspot-driven equilibrium path of an unexpected and permanent decrease in the real interest rate similar to the one experienced by Argentina at the beginning of the nineteen-nineties, from 6.0% to 4.5%.[26] Details on the way these values were computed are provided in appendix A.6. Figure 6 shows the evolution of $\frac{B^N}{\Pi^T}$, the financial fragility threshold—*i.e.* the right-hand side of condition (12)—, and ρ_t along a sunspot-driven equilibrium path where $S_t = 1$ and $z = 0$.[27] The economy becomes financially fragile in the second year following the shock. It is interesting to note that $\frac{B^N}{\Pi^T}$ overshoots with respect to the threshold, an evolution driven by the overshooting of $\frac{W^N}{W^T}$ (see figure 4). This suggests that an economy should be more fragile during the transition phase following the financial opening than in the long run, a result consistent with the empirical evidence reported by Kaminsky & Schmukler (2003).

Note that all those results where derived in the limit of a zero rescue package. A high enough rescue package can always prevent self-fulfilling crises to occur.

6 Effect of exogenous shocks

The previous sections focused on balance-of-payments crises triggered by a self-fulfilling change in expectations. This kind of *pure* financial fragility is of theoretical interest since it shows the possibility of crises independently of exogenous shocks in fundamentals. However, there are exogenous

[26] The growth rate g is set to the value corresponding to the nineteen-nineties, $g = 5.72\%$.
[27] The total number of period is 200.

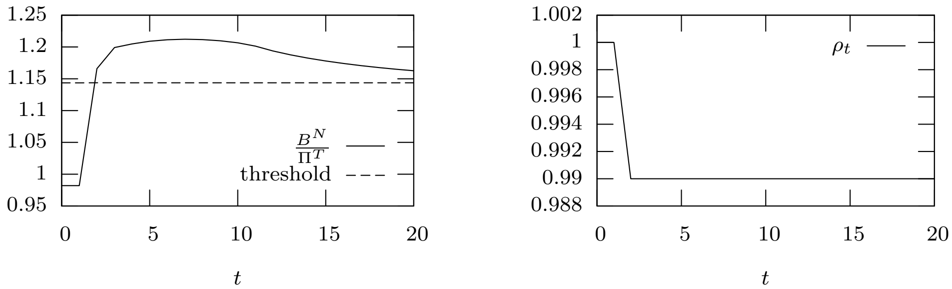


Figure 6: Sunspot-driven equilibrium path following a permanent decrease in R^D. Parameters: $\alpha = 48\%$, $\mu = 46.22\%$, $\eta = 49\%$, $\gamma = 11\%$, $\delta = 0.947$, $\beta = 0.053$, $\lambda = 2.5$, $g = 5.72\%$, $\omega = 0.99$, and $z \to 0$. At $t = 0$, the economy is in the stationary state corresponding to $R^D = 1.06$. At $t = 1$, it is hit by a permanent shock on the interest rate and $R^D = 1.045$ for $t \geq 1$. The sunspot variable always takes the value 1 in this simulation.

shocks in real economies. For example, Calvo et al. (2004) argue that the sudden stop of capital inflows that followed the Russian crisis of 1998 led to episodes of real depreciation in emerging countries. This section shows how the possibility of self-fulfilling crises can also be used to explain that small changes in fundamentals can have very large effects.

Moreover, the conditions under which a sunspot-driven equilibrium path eventually leads to a crisis might seem counter-intuitive. While I have shown that a sufficiently weak borrowing constraint[28] and a low enough domestic interest rate,[29] for example, were ingredients of financial fragility, one might expect on the contrary that crises are driven by a sudden rise in the interest rate[30] or the tightening of the borrowing constraint. This seeming paradox comes from the important difference between short-run and long-run effects. On the one hand, a given value of the interest rate affects financial fragility in the long run through its effect on the sectoral structure of the economy $\left(\frac{W^N}{W^T}\right)$ and the financial structure of the non-tradable sector $\left(\frac{B^N}{\Pi^N}\right)$, two variables that need time to change. On the other hand, an unexpected increase in the interest rate, for example, provokes an immediate adjustment of the within-period equilibrium.

Suppose the economy is in the safe stationary state studied in section 4.2 in period $t-1$. In period t an unexpected shock hits one of the model's parameters. To make things simple the shock is assumed to be known after agents have formed their expectations of future prices, so that the conditional expected relative price $\mathrm{E}_t[p_{t+1} | p_{t+1} \geq p^D_{t+1}]$ is still equal to the high equilibrium price in the stationary state, that is, to p^H. I consider three different kinds of shocks: (a) an increase in R^*, which can be thought of as a sudden stop, (b) a decrease in the productivity A^T_t of the tradable sector, which is a way of modeling a negative shock on the terms of trade, (c) a decrease in the financial multiplier λ, which could be the result of a sudden lack of trust or reflect a disruption in

[28]Cf. condition (11).
[29]Cf. proposition 3.
[30]See Frankel & Rose (1996) and Milesi-Ferretti & Razin (1998) for empirical evidence.

the domestic financial market. The shock is temporary and only concerns the period t.

Figure 7 graphically represents the effects of the three kinds of shocks.[31] The important point is that they can make equilibrium H disappear so that the economy has to jump on the low equilibrium L. Thus, a small unexpected shock can have very dramatic effects and trigger a crisis similar to the self-fulfilling crises of the previous sections.

When does it happen? An obvious condition is that the crisis equilibrium L has to exist; otherwise, a small shock only provokes a small shift of the equilibrium. In other words, a small exogenous shock can only result in a large crisis in financially fragile economies.[32] Furthermore, in the case of changes in R^* or A_t^T, a shock of a given size makes the economy switch to equilibrium L if the borrowing constraint is close enough to binding in the non-tradable sector before the shock, *i.e.* if p^H is close enough to p^B. This is the case when ψ is close enough to ψ_1^+.[33] Thus, a given increase in the interest rate triggers a crisis if the interest rate was low enough for a long time before the crisis. The case of a decrease in the financial multiplier λ is slightly different. Provided that the economy is financially fragile and equilibrium L exists, the shock provokes a switch to the crisis equilibrium if λ decreases sufficiently so that condition (11) is not satisfied any more.

Two economies can therefore react very differently to the same global shock. In the case of a sudden stop, for example, a financially fragile economy can jump on the crisis equilibrium, while other economies remain in the high equilibrium, simply experiencing a slight real depreciation and a low decrease in investment. This is fully consistent with the way Argentina and Chile reacted to the 1998 sudden stop, as reported by Calvo & Talvi (2005): the Argentine economy collapsed while Chile went through a mild recession.

7 Conclusion

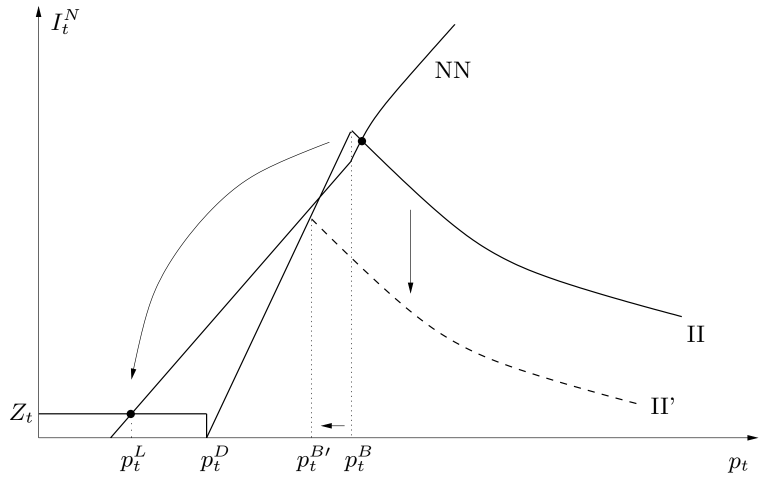
This paper has built an overlapping generation model of financial fragility in a small open economy. The model has been used to study the evolution of the sectoral structure of the economy as well as the financial structure of firms, and their interaction with the possibility of balance-of-payments crises.

After a large and permanent increase in financial openness or in the supply of available international liquidity capital is reallocated towards the non-tradable sector and firms in this sector increase their leverage. This can create financial fragility and make balance-of-payments crises pos-

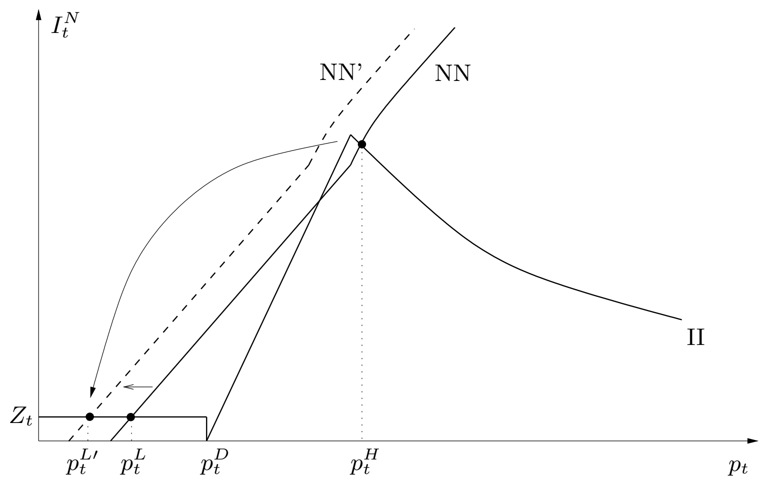
[31] Panel (a), a rise in R^* increases the opportunity cost of investment in the intermediate sectors and therefore decreases the unconstrained level of desired investment $\bar{I}_t^N$ and $\bar{I}_t^T$. The right part of the II schedule moves down and the unconstrained part of the NN schedule moves up. Panel (b), a decrease in A_t^T leads to a lower production Y_t^T in the tradable sector. Then, Π_t and W_t decrease so that the NN schedule moves to the left. Panel (c), a decrease in λ diminishes the slope of the II schedule in its constrained part.

[32] This is actually not true in the case of a decrease in A_t^T. A large enough shock can at the same time make the economy financially fragile and make the high equilibrium disappear. However, non financially fragile economies need a larger shock than financially fragile ones to switch to equilibrium L.

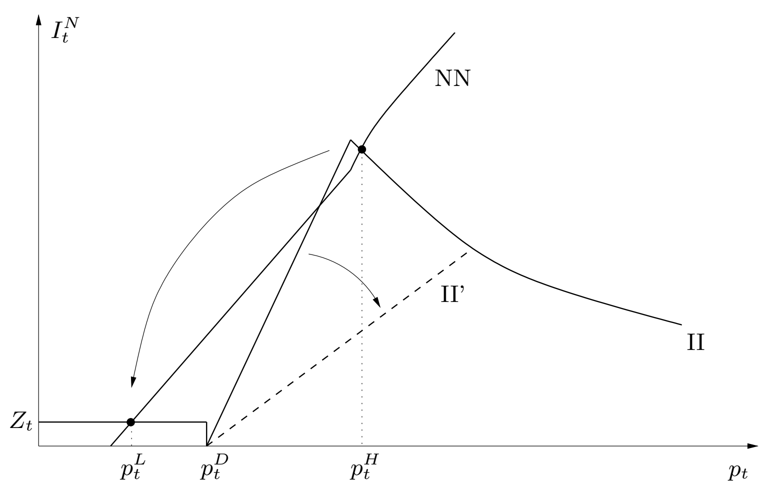
[33] Remember the economy was assumed to be in the *safe* stationary state at $t-1$, so that $\rho = 1$.



(a) Unexpected increase in R^*



(b) Unexpected negative shock on terms of trade



(c) Unexpected negative shock on the financial multiplier

Figure 7: Crises triggered by unexpected shocks.

sible. I have shown that this is also true in the long run, after the initial overshooting phase is over. Financial fragility in the long run requires a low world interest rate, a large financial openness, and a high growth rate.

Thus, this paper generalizes the results of Schneider & Tornell (2004). While they proved that crises are possible along the transition phase that followed some good news about future productivity, this paper have shown that the long-run evolution of the sectoral structure can lead to financial fragility independently of any shock.

These results apply to both unexpected crises and sunspot-triggered crises along a rational expectation equilibrium, which proves that this kind of crises is not due to an "irrational" behavior of either foreign lenders or entrepreneurs. Moreover, balance-of-payments crises can be the results of either purely expectational shocks or observed exogenous shocks on fundamentals. The latter case explains in particular why countries can react very differently to the same global shock, as was the case of Chile and Argentina after the 1998 sudden stop (Calvo & Talvi 2005).

The model could be modified in several ways. First, the intensity of the borrowing constraint could differ across sectors. While the necessary condition for financial fragility requires a large financial multiplier in sector N, the sufficient condition for the existence of the crisis equilibrium is not satisfied if the financial multiplier is very large in sector T. A tradable sector subject to a borrowing constraint weaker than the non-tradable sector would then limit the scope for financial fragility in the long run.

Another possible extension would be to allow bonds denominated in non-tradable goods to be traded among domestic agents. The economy would of course still suffer from an aggregate currency mismatch but it would be possible for households to insure part of the real exchange rate risk. It is not clear how it would affect the results. On the one hand, it might decrease the exposure of the non-tradable sector to a currency devaluation and diminish the probability of default. On the other hand, it would make the return on household's savings dependent on the real exchange rate and thus increase the volatility of the demand for non-tradable goods. Whatever the dominant mechanism, if the saving rate of household is small, the overall effect is likely to be weak.

The analysis of this paper was focused on the private sector. A government borrowing abroad in tradable goods would add another source of financial fragility. The crucial point would then be the composition of government spending in tradable and non-tradable goods.

This paper has several policy implications for an emerging country wishing to prevent balance-of-payments crises. The ideal policy would of course consist in removing the market imperfections necessary to the crises. For example, one could hope that a more developed financial market would suppress the borrowing constraint. However, as even developed economies are subject to strong borrowing constraints, this might not be a feasible objective. An opposite and rather provocative policy could be to limit the borrowing capacity of the non-tradable sector by an adequate regulation.

As has been seen, multiple equilibria are not possible if the financial multiplier is low enough. The major drawback of such a policy would be to decrease investment to a sub-optimal level in the non-tradable sector, creating a trade-off between high investment and financial stability.

Regarding the currency composition of external debt, more promising is the attempt to foster the development of international markets for bonds denominated in emerging market currencies. This policy has already been advocated in the debate on *Original Sin* (Eichengreen, Hausmann & Panizza 2005*a*). In the meanwhile, policy-makers have to pay attention to mismatches in firm balance sheets, a lesson that is now widely agreed on.[34]

But the main message of the model is that financial fragility is related to changes in the sectoral structure which are, in turns, determined by the external financing conditions. In times of high international liquidity restrictions to capital inflows set at an adequate level could limit the increase in the relative size of the non-tradable sector while allowing the economy to reap parts of the benefits of a cheaper foreign debt.

Last of all, financial fragility is only possible if the rescue package to defaulting firms is low enough. This confirms the need for an international lender of last resort, a much debated issue that goes far beyond the scope of the present work.

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[34]See for example Allen, Rosenberg, Keller, Setser & Roubini (2002).

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A Appendix

A.1 A microfoundation for the borrowing constraint

Following Schneider & Tornell (2004) and Aghion, Banerjee & Piketty (1999), it is possible to deduce the borrowing constraint from individual decisions in the context of moral hazard and imperfect monitoring.

Suppose, as in Schneider & Tornell (2004), that the entrepreneur has the possibility at the beginning of period $t+1$, if the firm is solvent,[35] to run away with the production without repaying its debt B^i_{t+1}. This requires some special effort and costs her a disutility $d\, I^i_t$ proportional to the size

[35]The firm is solvent when the expectation (at the beginning of period $t+1$) of the payment B^i_{t+1} does not exceed the value of production.

of the investment project. If she chooses to run away, the lender can try to find her and force her to repay her debt. He can choose the probability of success m, which can be thought of as the intensity of monitoring. But, as in Aghion et al. (1999), monitoring also requires some effort and costs him a disutility $C(m)\frac{B^i_{t+1}}{R^i_t}$ proportional to the size of the loan, with $C(m) = -c\log(1-m)$.[36] Therefore, if the entrepreneur has disappeared in the beginning of the period $t+1$, the lender chooses the intensity of monitoring m_{t+1} to maximize $m_{t+1}B^i_{t+1} - C(m_{t+1})\frac{B^i_{t+1}}{R^i_t}$, which yields $m_{t+1} = 1 - \frac{c}{R^i_t}$ provided that B^i_{t+1} does not exceed the value of production.

Note that the entrepreneur has incentives to repay her debt if the disutility from running away is higher than the expected debt repayment:

$$dI^i_t + \left(1 - \frac{c}{R^i_t}\right) B^i_{t+1} \geq B^i_{t+1}\,.$$

This condition can be reduced to

$$\frac{B^i_{t+1}}{R^i_t} \leq (\lambda - 1)W^i_t \tag{$*$}$$

where $\lambda - 1 = \frac{1}{\frac{c}{d}-1}$.

There are two kinds of debt contracts: secured loans which satisfy condition $(*)$, possibly by limiting the size of the loan $\frac{B^i_{t+1}}{R^i_t}$, and unsecured loans which do not satisfy $(*)$. Consider an unsecured loan. Since foreign lenders are risk neutral, an unsecured loan in the non-tradable sector must satisfy the following break-even constraint (when B^N_{t+1} is lower than the value of production):

$$\mathrm{E}_t[\rho_{t+1}][m_{t+1}R^N_t - C(m_{t+1})] = R^*(1+\tau)\,.$$

With m_{t+1} being a function of c and R^N_t, this equation implicitly defines the risky interest rate R^N_t as an increasing function of c. Likewise, in the tradable sector, unsecured debt contracts have an interest rate R^T increasing with c. Assume now that c is greater than d (this ensures that $\lambda > 1$) and is so large that the debt repayment, in the state of nature where the lender succeeds in forcing the entrepreneur to repay, always exceeds the value of production. Then, the entrepreneur never repays the lender even when the latter succeeds in finding her. Because of the bankruptcy cost the lender never receives anything out of an unsecured loan.[37] Therefore, all loans have to be secured in equilibrium. This gives rise to the borrowing constraint $(*)$.

[36] This functional form gives a financial multiplier independent of the interest rate. This is a special case whose only purpose is analytical tractability.

[37] The bankruptcy cost is not necessary. Without bankruptcy costs, a high enough value of c entails that the lender takes all the expected value of production. The net expected value of investment would then be negative for the entrepreneur, who would never invest.

A.2 The saving problem of households

The young household maximizes $U = \log\left(c_t^y\right) + \beta\,\mathrm{E}_t\left[\log\left(c_{t+1}^o\right)\right]$ subject to the budget constraint

$$p_{t+1}^C c_{t+1}^o = (w_t - p_t^c c_t^y)[\varphi^N R_t^N + \varphi^F R^*/(1+\tau) + (1-\varphi^N-\varphi^F)R^T]$$

where φ^N is the proportion of savings invested in bonds issued by sector N and φ^F the proportion of savings invested in riskless bonds abroad.

The first order condition with respect to c_t^y is

$$\begin{aligned}\frac{\partial U}{\partial c_t^y} &= \frac{1}{c_t^y} + \beta\,\mathrm{E}_t\left[\frac{-p_t^c}{p_{t+1}^c c_{t+1}^o}[\varphi^N R_t^N + \varphi^F R^*/(1+\tau) + (1-\varphi^N-\varphi^F)R^T]\right] \\ &= \frac{1}{c_t^y} - \beta\frac{p_t^c}{w_t - p_t^c c_t^y} = 0\end{aligned}$$

and yields $p_t^c c_t^y = \frac{w_t}{1+\beta}$ which corresponds to a saving rate $s = \frac{\beta}{1+\beta}$.

The partial derivative of U with respect to φ^F is

$$\frac{\partial U}{\partial \varphi^F} = \beta\,\mathrm{E}_t\left[\frac{w_t - p_t^c c_t^y}{p_{t+1}^c c_{t+1}^o}(R^*/(1+\tau) - R^T)\right] < 0$$

because $R^*/(1+\tau) < R^T = R^*(1+\tau)$.

The partial derivative of U with respect to φ^N is

$$\begin{aligned}\frac{\partial U}{\partial \varphi^N} &= \beta\,\mathrm{E}_t\left[\frac{w_t - p_t^c c_t^y}{p_{t+1}^c c_{t+1}^o}(R_t^N - R^T)\right] \\ &= \beta\,\mathrm{E}_t\left[\frac{R_t^N - R^T}{\varphi^N R_t^N + \varphi^F R^*/(1+\tau) + (1-\varphi^N-\varphi^F)R^T}\right].\end{aligned}$$

Let us suppose R_t^N is given by equation (7) and let us show that the household does not hold bonds issued by the non-tradable sector. When $R_t^N = R^T$ (*i.e.* when no default is expected and $\mathrm{E}_t[\rho_{t+1}] = 1$), $\partial U/\partial\varphi^F = 0$ and the household is indifferent between holding bonds issued by the N or T sector. For simplicity I assume it holds bonds issued by sector T. On the contrary, when $\mathrm{E}_t[\rho_{t+1}] < 1$

$$\begin{aligned}\frac{\partial U}{\partial \varphi^N} = \beta R^D(1-\mathrm{E}_t[\rho_{t+1}])\Bigg[&\frac{1}{\varphi^N \frac{R^D}{\mathrm{E}_t[\rho_{t+1}]} + \varphi^F R^*/(1+\tau) + (1-\varphi^N-\varphi^F)R^T} \\ &- \frac{1}{\varphi^F R^*/(1+\tau) + (1-\varphi^N-\varphi^F)R^T}\Bigg] < 0\,.\end{aligned}$$

Therefore, as long as the supply of bonds from the tradable sector is large enough, which is the case by assumption 1, the optimal portfolio composition is the corner solution given by $\varphi^N = \varphi^F = 0$.

The household only holds bonds issued by sector T.

A.3 Proof of proposition 1

An equilibrium real exchange rate is a zero of the function

$$f(p_t) = p_t Y_t^N - \frac{\mu}{1+s\frac{1-\alpha}{\alpha}}\left[\gamma\left(\Pi_t^T + \Pi_t^N(p_t)\right) + R^D s w_{t-1} L\right] - \eta\left(I_t^T(p_t) + I_t^N(p_t)\right).$$

Suppose the condition expressed in equation (11) is *not satisfied*. Then, when $p_t \geq p_t^D$, we have

$$\begin{aligned} f'(p_t) &= Y_t^N\left[1 - \frac{\mu\gamma}{1+s\frac{1-\alpha}{\alpha}}\right] - \eta\left[\frac{\partial I_t^T}{\partial p_t} + \frac{\partial I_t^N}{\partial p_t}\right] \\ &\geq Y_t^N\left[1 - \frac{\mu\gamma}{1+s\frac{1-\alpha}{\alpha}} - \eta(1-\gamma)\lambda\right] \\ &\geq 0 \end{aligned}$$

where we use the fact that I_t^T is either strictly decreasing with p_t or constant, so that $\frac{\partial I_t^T}{\partial p_t} \leq 0$, and that $\frac{\partial I_t^N}{\partial p_t}$ is either strictly negative (when $p_t \geq p_t^B$) or equal to $\lambda(1-\gamma)Y_t^N$ (when $p_t < p_t^B$).

The function f is continuous and increasing on the interval $[p_t^D, +\infty)$. Besides, it tends to $+\infty$ when p_t tends to $+\infty$. Therefore, it has a zero on this interval if and only if $f(p_t^D) \leq 0$.

On the interval $[0, p_t^D)$, f is also an increasing function. This implies:

$$\begin{aligned} \forall p_t \in [0, p_t^D), \quad f(p_t) &\leq \lim_{p_t \xrightarrow{<} p_t^D} f(p_t) \\ &\leq f(p_t^D) - \eta Z_t \\ &< f(p_t^D)\,. \end{aligned}$$

Therefore if there exists an equilibrium with $p_t \geq p_t^D$, $f(p_t) < f(p_t^D) \leq 0$ for all p_t in $[0, p_t^D)$ so that there cannot be another equilibrium with $p_t < p_t^D$ at the same time. With a similar argument, if there exists an equilibrium with $p_t < p_t^D$, there cannot be another equilibrium with $p_t \geq p_t^D$ at the same time.

A.4 Autarky interest rate

In the closed economy the domestic interest rate R^A is endogenously determined by the equilibrium on the domestic credit market:

$$I_t^N + I_t^T = W_t^N + W_t^T + \Sigma_t\,.$$

In the stationary state we have $i^N + i^T = w^N + w^T + \sigma$, where σ is given by equation (15e) and

$$i^j = \min\left(\lambda, \frac{\psi - (1-\gamma)}{(1-\gamma)(\frac{1}{\delta} - 1)}\right) w^j \,, \qquad j = N, T.$$

The autarky value of the parameter $\psi^A = \frac{1+g}{R^A}$ is a solution of

$$\min\left(\lambda, \frac{\psi^A - (1-\gamma)}{(1-\gamma)(\frac{1}{\delta} - 1)}\right) = 1 + \frac{\gamma}{1-\gamma} \frac{\frac{s(1-\alpha)}{\alpha}}{1 + \frac{s(1-\alpha)}{\alpha}(1 - \frac{1}{\psi^A})} \,.$$

An unconstrained solution $\psi^A < (1-\gamma)\big[1 + \lambda\big(\frac{1}{\delta} - 1\big)\big]$ has to be a root of the following quadratic polynomial:

$$P(X) = \left[1 + \frac{s(1-\alpha)}{\alpha}\right] X^2 - \left[\frac{1-\gamma}{\delta} + \frac{s(1-\alpha)}{\alpha}\left[1 - \gamma + \frac{1}{\delta}\right]\right] X + \frac{s(1-\alpha)}{\alpha}\frac{1-\gamma}{\delta} \,.$$

Note that $P\left(\big(1 + \frac{\alpha}{s(1-\alpha)}\big)^{-1}\right) < 0$ so that P has a unique root greater than $\big(1 + \frac{\alpha}{s(1-\alpha)}\big)^{-1}$. Denote X^+ this root. We obtain

$$\psi^A = \min\left((1-\gamma)\left[1 + \lambda\big(\frac{1}{\delta} - 1\big)\right], X^+\right) \,.$$

It is easy to check that $\psi^A \geq \frac{1-\gamma}{\delta}$. When households do not save ($\beta = s = 0$), we simply have $\psi^A = \frac{1-\gamma}{\delta}$. For low saving rates, $\psi^A = X^+$ and the borrowing constraint does not bind in autarky, which is the case in the calibration exercise of section 5.2. Note that X^+ increases with $s(1-\alpha)/\alpha$. Therefore, when the borrowing constraint does not bind, the autarky interest rate R^A increases with the entrepreneurs' income share α and decreases with the households' discount factor β.

A.5 Proof of lemma 2

By continuity of Q with respect to z, we just have to show that $Q(\rho, \psi_\rho^+) > 0$ when $z = 0$ and $\psi_\rho^+ < \psi^{\max}$. To simplify notations define $x = \frac{s(1-\alpha)}{\alpha}$, $a = \mu\gamma$, $b = \eta(1-\gamma)$, and $A(\psi) = \frac{a}{1 + x\left(1 - \frac{1}{\psi}\right)}$.

$$Q(\rho, \psi_\rho^+) = \frac{W^N}{W^T}(\psi_\rho^+)\left[\frac{B^N}{\Pi^N}(\psi_\rho^+) + \frac{a}{1+x} - A(\psi_\rho^+)\right] - \left[A(\psi_\rho^+) + b\lambda\right]$$

From equations (16c) and (16d) we have $\frac{B^N}{\Pi^N}(\psi_\rho^+) = \frac{\lambda - 1}{1 + \lambda(\frac{1}{\delta} - 1)}$and

$$\frac{W^N}{W^T}(\psi_\rho^+) = \frac{A(\psi_\rho^+) + b\lambda}{\frac{\lambda/\delta}{1 + \lambda(\frac{1}{\delta} - 1)} - A(\psi_\rho^+) - b\lambda}$$

where we use the fact that $\frac{I^N}{W^N}(\psi_\rho^+) = \lambda$ by definition and $\frac{I^T}{W^T}(\psi_\rho^+) = \lambda$ because $\psi_\rho^+ \geq \psi_1^+$. We can then compute $Q(\rho, \psi_\rho^+)$.

$$\begin{aligned}
Q(\rho, \psi_\rho^+) &= \frac{A(\psi_\rho^+) + b\lambda}{\frac{\lambda/\delta}{1+\lambda(\frac{1}{\delta}-1)} - A(\psi_\rho^+) - b\lambda} \left[\frac{\lambda - 1}{1 + \lambda(\frac{1}{\delta} - 1)} + \frac{a}{1+x} - A(\psi_\rho^+) \right] \\
&\quad - \left[A(\psi_\rho^+) + b\lambda \right] \\
&= \frac{A(\psi_\rho^+) + b\lambda}{\frac{\lambda/\delta}{1+\lambda(\frac{1}{\delta}-1)} - A(\psi_\rho^+) - b\lambda} \left[\frac{\lambda - 1}{1 + \lambda(\frac{1}{\delta} - 1)} + \frac{a}{1+x} - A(\psi_\rho^+) \right. \\
&\quad \left. - \frac{\lambda/\delta}{1 + \lambda(\frac{1}{\delta} - 1)} + A(\psi_\rho^+) + b\lambda \right] \\
&= \frac{A(\psi_\rho^+) + b\lambda}{\frac{\lambda/\delta}{1+\lambda(\frac{1}{\delta}-1)} - A(\psi_\rho^+) - b\lambda} \left[\frac{a}{1+x} + b\lambda - 1 \right]
\end{aligned}$$

The first factor is strictly positive because $\psi_\rho^+ < \psi_\rho^{\max}$ and the second factor is strictly positive from assumption 2.

A.6 Calibration

The time period is set to a year. Manufacturing, agriculture, and mining are classified as tradable sectors, and services, construction, water, electricity, and gas as non-tradable sectors.[38] When not specified, the data comes from the Ministerio de Economía (MECON) and the Instituto Nacional de Estadística y Censos (INDEC).

I first estimate the empirical values of ψ and ψ^A. The variable g is proxied by the growth rate of the Gross Domestic Product (GDP). The real interest rate in tradable goods $R^D - 1$ is measured as the average nominal interest rate on external debt (data from the Institute of International Finance) deflated by the US GDP price index (data from the Bureau of Economic Analysis). Then, ψ is computed as the geometric average of $(1 + g)/R^D$ over the 8-year period 1991-1998 which starts with the reform package and ends before the beginning of the collapse. Symmetrically, ψ^A is computed as the geometric average of ψ over 1983-1990, the 8-year period preceding the opening of the capital account. I get $\psi^A = 0.940$ and $\psi = 1.012$.[39] As the nominal interest rate used includes a risk premium, ψ is likely to be slightly underestimated.

The coefficient α is chosen to be equal to 0.48, which is the profit share given by the 1993 National Accounts (Maia & Nicholson 2001). The coefficient μ is proxied by the share of non-

[38] I do not have sufficiently disaggregated data to be able to distinguish water, electricity, and gas.

[39] The value $\psi^A = 0.940$ corresponds to $g = -0.34\%$ and $R^A = 1 + 6.0\%$; the value $\psi = 1.012$ to $g = 5.72\%$ and $R^D = 1 + 4.5\%$.

tradables in consumption expenditures. According to the composition of the Consumer Price Index (available for 1999), $\mu = 46.22\%$. To calibrate η I use the fact that the price of composite capital is equal to p_t^{η}. I compute the implicit price of investment (relative to tradable goods) and regress it (in logarithm) on the implicit price for non-tradable goods (relative to tradable goods) over the period 1993-2004. The estimated elasticity is $\eta = 0.49$ (the coefficient is statistically significant at the 1% level).

The coefficient β is calibrated to match the empirical saving rate s. In the model, the households' savings should be the difference between aggregate and corporate savings. Corporate savings were estimated by Bebczuk (2000) to be equal to 13% of GDP on average over 1990-1996. With data from the Penn World Table (2002) the corresponding average aggregate savings represent 15.5% of GDP. Using $1 - \alpha$ as the income share of households I get a saving rate $s = 5\%$. Accordingly, β is set to 0.053.

The coefficient γ is the dividend pay-out ratio. I use different sources to calibrate this parameter. Using Bloomberg data on 28 Argentinean non-financial firms listed on the Buenos Aires stock market for the year 2005,[40] I find an average pay-out ratio equal to 4% (measured as dividends over EBITDA). Bebczuk (2004) uses the database Economatica with data on 55 non financial Argentinean listed companies and reports an average dividend to cash-flow ratio equal to 14.2% for 1996-2000. Bebczuk (2005) works with data from still other sources on 65 non financial Argentinean listed companies. The average dividend to cash-flow ratio is this time equal to 15.5% for 1996-2000. I set $\gamma = 11\%$, the average of these three figures. There are two caveats. On the one hand, the last two figures are likely to overestimate the true aggregate pay-out ratio. A lot of Argentinean firms pay no dividends at all and the proportion of listed firms paying dividends is probably higher than in the whole economy. On the other hand, the figure computed from the Bloomberg data corresponds to a post-crisis year and is probably underestimated. In 2005, investment had almost entirely recovered to its pre-crisis level while firms still had no access to bank credit or to external finance. This implies that investment was financed from retained earnings and makes it likely that firms cut their dividend payments to meet their investment expenditures.

The model predicts the autarky ψ^A as a function of δ (appendix A.4). The coefficient δ is set to $= 0.947$ to fit the empirical value $\psi^A = 0.940$.

With this set of parameters assumption 3 is satisfied and assumption 2 is satisfied for $\lambda > 2.2$. I choose $\lambda = 2.5$. Finally, I set $\omega = 0.99$, which corresponds to a perceived probability of crisis equal to 1% per year.

[40]The total number of listed firms was 129 in 2000.