Persistent Appreciations and Overshooting: A Normative Analysis

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Abstract

Most economies experience episodes of persistent real exchange rate appreciation, when the question arises whether there is a need for intervention to protect the export sector. In this paper we present a model of irreversible export destruction where exchange rate intervention may be justified if the export sector is financially constrained. However the criterion for intervention is not whether there are bankruptcies or not, but whether these can cause a large exchange rate overshooting once the factors behind the appreciation subside. The optimal policy includes ex-ante and ex-post interventions. Ex-ante (i.e., during the appreciation phase) intervention are limited while financial resources in the export sector are relatively abundant. In this case the bulk of the intervention takes place ex-post, and is concentrated in the first period of the depreciation phase. In constrast, if the financial constraint in the export sector is tight, the policy is reallocated toward ex-ante intervention and it is optimal to lean against the appreciation. On the methodological front, the solution approach should be useful in other optimal dynamic intervention problems with financially constrained agents.

JEL Codes: E0, E2, F0, F4, H2.
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1 Introduction

Most economies experience episodes of large real exchange rate appreciations. There are many factors with the potential to fuel these appreciations. For example, they can stem from domestic policies aimed at taming a stubborn inflationary episode, from the absorption of large capital inflows caused by domestic and external factors, from exchange rate interventions in trading partners, from domestic consumption booms, from a sharp rise in terms of trade in commodity producing economies or, in its most extreme form, from the discovery of large natural resources wealth (the so-called Dutch disease).

While there are idiosyncrasies in each of these instances, the common policy element is that, when the appreciation is persistent enough, the question arises whether there is a need for intervention to protect the export sector (often referred as "competitiveness" policies). This widespread concern goes beyond the purely distributional aspects associated to real appreciations. The fear is that somehow the medium and long run health of the economy is compromised by these episodes. If this concern is justified, should policymakers intervene and stabilize the exchange rate before it is too late? More generally, how does the optimal policy look like?

In this paper we propose a framework to address this common policy element. We present a dynamic model of entry and exit in the export sector where entrepreneurs face financial constraints and exchange rate stabilization may be justified. In our model, when financial constraints damage the export sector's ability to recover, the economy experiences a large exchange rate overshooting once the factors behind the appreciation subside and nontradable demand contracts. Although not always present, overshooting are pervasive, especially when financial frictions are widespread. Figure 1 illustrates three recent examples: The Finnish, Mexican and Asian episodes of the early 1990s, mid 1990s, and late 1990s, respectively. In each of them, the pattern is one where the appreciation is followed by a depreciation of the real exchange rate that overshoots its new medium term level.

The overshooting results from the export sector's inability to absorb the resources (labor) freed from the contraction in nontradables demand. This inability leads to an amplified fall in real wages, which is costly to consumer-workers.[1] There is scope for policy intervention because there is a connection between the severity of the overshooting and the extent of the contraction in the export sector during the preceding appreciation phase. If consumers were to reduce their demand for nontradables in this phase, then there would be less destruction ex-ante and a faster recovery ex-post. However, rational atomistic consumers ignore the effect of their individual decisions during the appreciation phase on the extent of the overshooting during the depreciation phase. It is this

[1] In practice, the drop in the relative price of non-tradables and real wages often takes the form of a sharp nominal depreciation which is not matched by a rise in the nominal price of nontradables and wages. See, e.g., Goldfajn and Valdes (1999) and Burnstein et al (2005).

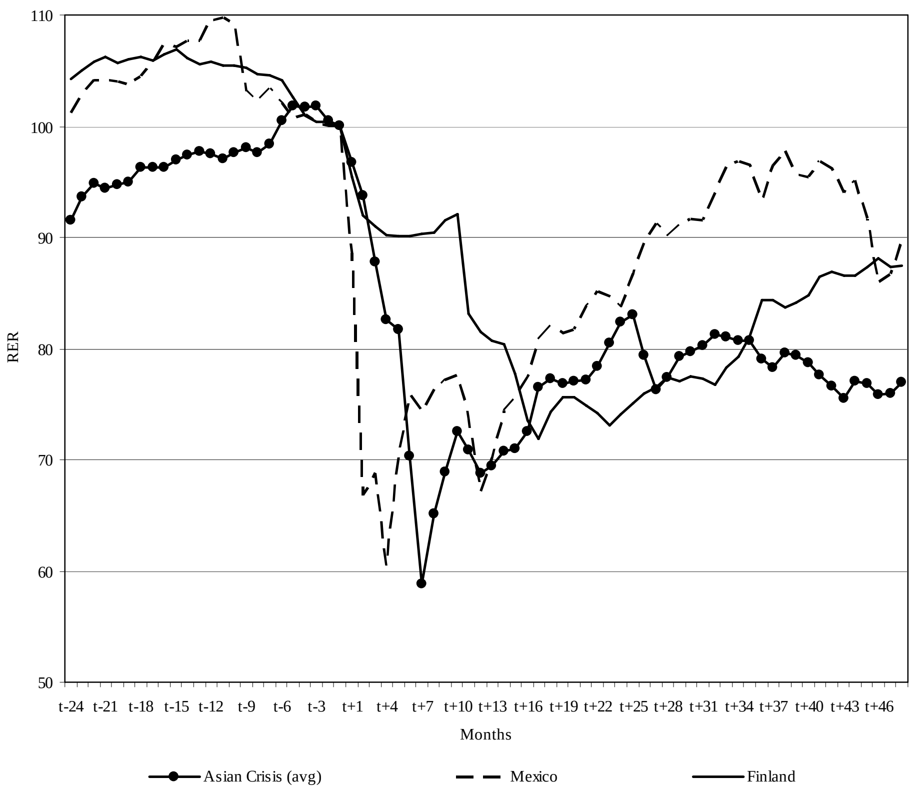

Figure 1: 1990's Overshooting in Finland, Mexico and Asia

pecuniary externality that justifies and informs policy intervention in our framework.

Our analysis has two parts, one positive and one normative. The former consists of a dynamic model of factor reallocation in the presence of financial constraints. Our model economy starts by transiting into an appreciation phase, which it exits into a depreciation phase with a Poisson probability. There are several regions of interest, indexed by the financial resources of export sector entrepreneurs at the onset of the appreciation. When financial resources are plentiful, the economy reaches the first best as real exchange rates (and real wages) are pinned down by purely technological free entry and exit conditions, and hence are orthogonal to consumers' actions. At lower levels of financial resources, financial constraints may become binding during the appreciation phase, the depreciation phase, or both. If they are only binding during the appreciation phase, then the economy experiences bankruptcies but the recovery of the export sector is swift once the depreciation phase starts and the exchange rate is again pinned down by purely technological factors. In contrast, if the financial constraint is binding during the depreciation phase, the recovery of the export is slow and the real exchange rate depreciation overshoots the long run depreciation.

For the optimal policy analysis we adopt the perspective of a social planner that seeks to maximize consumers' welfare, subject to not worsening entrepreneur' welfare and to their financial constraints. We rule out direct transfers across groups, as these are limited by a series of informational factors in practice. Instead, we focus on market-mediated transfers implemented through interventions that influence the real exchange rate. That is, interventions that affect consumers' choices and the entrepreneurial sector through their effect on equilibrium prices. From this perspective, we show that consumers gain from stabilizing the appreciation whenever this leads to a faster recovery of the export sector once the appreciation subsides. The gain derives from the increase in real wages associated to a faster reconstruction of the export sector.

Importantly, even when overshooting is expected, intertemporal consumption allocation considerations put limits on how much intervention is desirable during the appreciation phase. This connects our analysis to a central consideration for policymakers when dealing with an asset appreciation, be it the currency, real estate or any other asset with potential macroeconomic implications: If there is a need for intervention, how much should be done as prevention (ex-ante) and how much should be left for after "the crash" (ex-post)? In our framework, the answer to this question depends primarily on the extent of the financial constraint in the export sector. On one end, when the financial constraint is severe, ex-ante intervention is most effective. On the other end, when the constraint is loose, ex-post intervention is most desirable and effective. In general, the optimal policy has elements of both, ex-ante and ex-post intervention.

Our paper belongs to an extensive literature on consumption and investment booms in open economies, as well as on the role of financial factors in generating inefficiencies in these booms (see, e.g. Gourinchas et al 2001, Caballero and Krishnamurthy 2001, Aghion et al 2003). The pecuniary externality that justifies intervention in our framework is related to those identified in Geanakoplos and Polemarchakis (1996), Caballero and Krishnamurthy (2001, 2004), Lorenzoni (2006) and Farhi et al (2006). Aside from its specific context, the main novelty of our paper is to embed this externality in a tractable model of optimal policy, which allows us to fully characterize the economy's dynamics and to analyze the trade-off between ex ante and ex post interventions.

In terms of its mechanism, the paper also belongs to the literature on Dutch disease. There, intervention is justified by the presence of dynamic technological externalities through learning-by-doing (see, e.g. van Wijnbergen 1984, Corden 1984, and Krugman 1987). In contrast, our paper highlights financial frictions and the pecuniary externalities that stem from these. The policy implications of these two approaches are different: While learning-by-doing offers a justification for industrial policies as a development strategy, the financial frictions we highlight have intertemporal reallocation implications of the sort that matter for business cycle policies.

The approach to optimal policy proposed in this paper resembles that of the literature on dynamic optimal taxation. In this dimension, the main innovation of the paper is to apply this methodology to an environment where a subset of agents are financially constrained, imposing restrictions on the ability of policy to reallocate resources between these agents and the rest of the economy. This approach and the solution method we developed should be useful outside our particular application.

Section 2 presents a stylized model of creative destruction over appreciation and depreciation cycles. Section 3 characterizes optimal exchange rate intervention in such setup. Section 4 discusses different realistic extensions and considerations, and their impact on the optimal policy. Section 5 concludes and is followed by an extensive appendix.

2 A Simple Model of a Destructive Appreciation and Overshooting

In this section we present a model of an economy experiencing a temporary, but persistent, real appreciation. The export sector faces large sunk costs of investment, which limit the extent of its desired contraction, in order to keep capital operational and preserve the option to produce once the appreciation is over. However, this waiting strategy generates losses that require financing. If this financing is limited, the export sector experiences a larger contraction than desired. From the point of view of the economy as a whole, these excessive contractions may compromise the recovery of the export sector once the appreciation is over, leading to a prolonged period of deep real depreciation and low wages.

2.1 The Environment

There is a unit mass of each of two groups of agents within the domestic economy: consumers and entrepreneurs (exporters). There are two consumption goods: a tradable and a nontradable good. The consumer supplies inelastically one unit of labor each period. Labor can be used as an input for the production of tradables or nontradables. In both cases one unit of labor is needed to produce one unit of output. In addition, the production of tradables requires one unit of capital, or an "export unit." Creating an export unit requires investing f units of tradable goods. After an export unit has been set up, it needs to be maintained in operation, otherwise it is irreversibly shut down.[2] Entrepreneurs are the owners and only agents that have access to the technology to run and maintain export units. At date 0 they begin with n_{-1} open export units. The markets for tradables, nontradables and labor are competitive.

Entrepreneurs are risk neutral and only consume tradable goods. Their preferences are given by the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t c_t^{T,e},$$

where $c_t^{T,e} \geq 0$ is consumption of tradable goods. Consumers have log-separable instantaneous utility on the consumption of tradables and nontradables, c_t^T and c_t^N. Their preferences are given by the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t \theta_t \left(u\left(c_t^T\right) + u\left(c_t^N\right)\right),$$

where $u(c) = \log c$ and θ_t is a taste shock.

The taste shock is the only source of uncertainty and is determined by the state s_t, which can take two values in $S = \{A, D\}$ and follows a Markov process with transition probability $\pi(s_{t+1}|s_t)$. The economy begins with a transition into the "appreciation" state $s_t = A$, with $\theta_t = \theta_A$. Each period, with probability $\pi(D|A) = \delta$, the economy switches to the "depreciation" state $s_t = D$, with $\theta_t = \theta_D$. Once the latter transition takes place, D is an absorbing state, i.e., $\pi(A|D) = 0$. We assume that:

$$\theta_A > \theta_D = 1.$$

In the appreciation state the taste shock drives up consumers' demand for both tradable and nontradables, putting upward pressure on the real exchange rate (since the supply of tradables is fully elastic while that of tradables is not – see below). In reality, the main sources of appreciations are sharp improvements in terms of trade and capital inflows. The taste shock is a convenient

[2] These assumptions capture the fact that export oriented firms often have more specific (sunk) capital and operations than firms producing primarily for domestic markets. Of course there are important exceptions to this generalization. Later in the paper we discuss the effect of introducing adjustment costs in the nontradables sector.

device to capture the increase in consumption demand that derives from these primitive shocks, without having to add additional frictions. We will return to this issue later in the paper, once we have developed our main points.

Both groups have access to the international capital market, where they can trade a full set of state contingent securities. On each date t, agents trade one-period state-contingent securities that pay one unit of tradable good in period $t+1$ if state s_{t+1} is realized. The entrepreneurs holdings of securities are denoted by $a\left(s^t\right)$ where $s^t = \langle s_{0,} s_1, ..., s_t \rangle$ denotes the history of the economy up to date t. Note that our simple Markov chain yields histories that are limited to a block of periods in A, followed by D's (there are no alternations).

All entrepreneurs begin with an initial financial positions equal to a_0. For consumers, we set it to zero without loss of generality. Consumers face no financial constraints, while entrepreneurs face the financial constraint

$$a\left(s^t\right) \geq 0. \tag{1}$$

That is, entrepreneurs cannot commit to make any positive repayment at future dates. This is a simple form of financial markets imperfection, which captures the idea that entrepreneurs have limited access to external finance. This is the only friction we introduce in the model.

The rest of the world is captured by a representative consumer with linear preferences represented by $\mathrm{E}\sum_{t=0}^{\infty} \beta^t c_t^{T,*}$. We assume that the rest of the world has a large endowment of tradable goods. Therefore asset pricing is risk neutral: at date t, the price of a security paying one unit of tradable in state s_{t+1} is $\beta\pi\left(s_{t+1}|s_t\right)$.

2.2 Decisions and Equilibrium

Let $p\left(s^t\right)$ denote the price of the nontradable good in terms of units of tradable (the numeraire), or the real exchange rate (defined à la IMF). Given the linear technology in the nontradable sector, the equilibrium wage in terms of tradables must be equal to this price. Consumers and entrepreneurs take the real exchange rate as given. Equilibrium prices and quantities are functions of the whole history s^t. To save on notation, whenever confusion is not possible we only use the time subindex t, e.g., p_t is shorthand for $p\left(s^t\right)$.

2.2.1 Consumers

Notice that wages are equal to p_t, markets are complete, and intertemporal prices are pinned down by the world capital market. Therefore, consumers make their consumption decisions facing the single intertemporal budget constraint

$$\sum_{t,s^t} \beta^t \pi\left(s^t\right)\left(c^T\left(s^t\right) + p\left(s^t\right) c^N\left(s^t\right)\right) \leq \sum_{t,s^t} \beta^t \pi\left(s^t\right) p\left(s^t\right), \tag{2}$$

where $\pi\left(s^t\right)$ denotes the ex ante probability of history s^t. Then, consumers' demand for tradables and nontradables take the simple form:

$$\begin{aligned} c_t^T &= \kappa\theta_t, \\ c_t^N &= \frac{\kappa\theta_t}{p_t}, \end{aligned}$$

where the constant κ is

$$\kappa = \frac{1}{2}\frac{\sum_{t,s^t}\beta^t\pi\left(s^t\right)p\left(s^t\right)}{\sum_{t,s^t}\beta^t\pi\left(s^t\right)\theta\left(s_t\right)}. \tag{3}$$

There are two important features from the consumption block. First, during A periods the demand curve for nontradables shifts upward. This is the source of the appreciation. Second, κ is endogenous and is increasing in the value of the exchange rate at any future date. The latter feature will be essential in the analysis of optimal policy.

2.2.2 Exporters and Equilibrium

Even though consumption volatility is not the result of any friction, it may create problems for *both* firms and consumers, if the export sector has limited financial resources. Before discussing this issue in detail, we need to understand exporters' decisions.

It is useful to separate the entrepreneurs' decisions regarding consumption and investment from the problem of creating new units. To this end, we assume that there is a competitive adjustment sector, that creates and destroys export units and makes zero profits. Let q_t denote the price of an export unit. Equilibrium in the adjustment sector requires that

$$q_t \in [0, f], \tag{4}$$

$$q_t = f \text{ if } n_t > n_{t-1}, \tag{5}$$

$$q_t = 0 \text{ if } n_t < n_{t-1}. \tag{6}$$

That is, if units are being created their price must be equal to the creation cost f, while if they are being destroyed their price must be zero.

The entrepreneur's flow of funds constraint can then written as

$$c^{T,e}\left(s^t\right) + q\left(s^t\right)\left(n\left(s^t\right) - n\left(s^{t-1}\right)\right) + \beta\sum_{s_{t+1}\in S}\pi(s_{t+1}|s_t)a\left(\left\langle s^t, s_{t+1},\right\rangle\right) \leq \left(1 - p\left(s^t\right)\right)n\left(s^t\right) + a\left(s^t\right), \tag{7}$$

Each period, the entrepreneur uses his current profits, $(1-p_t)\,n_t$, and his financial wealth, a_t, to finance consumption, investment in new export units, and investment in state contingent securities. Notice that our timing assumption is that production units created at date t are immediately productive, i.e., they immediately generate unitary profits of $1 - p_t$.

The entrepreneur's problem is to choose a sequence $\left\{c^{T,e}\left(s^t\right), n\left(s^t\right), a\left(s^t\right)\right\}$ that maximizes his expected utility, subject to the flow of funds constraints (7) and the financial constraints (1), taking as given the price sequences $\left\{p\left(s^t\right), q\left(s^t\right)\right\}$. Notice that the entrepreneur problem is linear, given that both the objective function and the constraints are linear. In the Appendix we setup the problem in recursive form and argue that the entrepreneur's value function can be written as

$$\psi\left(s^t\right) + \phi\left(s^t\right) \cdot \left(a\left(s^t\right) + q\left(s^t\right) n\left(s^{t-1}\right)\right), \tag{8}$$

where $\phi\left(s^t\right)$ represents the marginal return on entrepreneurs' wealth (see Lemma 1 in the Appendix). The entrepreneur's wealth is given by the sum of his financial wealth, $a\left(s^t\right)$, plus the market value of the units held, $q\left(s^t\right) n\left(s^{t-1}\right)$. Here we proceed directly to discuss the optimality conditions of the entrepreneur. For the moment, we suppose that the prices $\left\{p\left(s^t\right), q\left(s^t\right)\right\}$ are such that the entrepreneur's expected utility is finite. We will check later that this condition is satisfied in equilibrium.

In each period, the entrepreneur chooses how many production units to operate, how much to consume, and how many contingent claims to purchase for states A and D in the following period. The first order conditions (and their complementary slackness conditions) with respect to these choice variables are

$$-\left(q\left(s^t\right) - \left(1 - p\left(s^t\right)\right)\right) \phi\left(s^t\right) + \beta \sum_{s_{t+1} \in S} \pi\left(s_{t+1} | s_t\right) \phi\left(\left\langle s^t, s_{t+1}\right\rangle\right) q\left(\left\langle s^t, s_{t+1}\right\rangle\right) \leq 0, \; n\left(s^t\right) \geq 0, \tag{9}$$

$$\begin{aligned} 1 - \phi\left(s^t\right) &\leq 0, \qquad c^{T,e}\left(s^t\right) \geq 0, & (10) \\ -\phi\left(s^t\right) + \phi\left(\left\langle s^t, s_{t+1}\right\rangle\right) &\leq 0, \qquad a\left(\left\langle s^t, s_{t+1}\right\rangle\right) \geq 0, \text{ for all } s_{t+1} \in S. & (11) \end{aligned}$$

The first condition states that the opportunity cost of the resources used in keeping the marginal unit in operation must be equal to the expected value of that unit tomorrow. The cost of keeping a unit in operation corresponds to the price of acquiring that unit, q_t, minus the current profits, $1 - p_t$. Remember that an open unit must remain active, so if $p_t > 1$ the firm is making current losses and these losses add to the cost of keeping the unit open. The second condition states that if the entrepreneur's consumption is positive, the marginal value of wealth must be equal to one. The third condition says that the marginal value of wealth must be non-increasing between two consecutive histories. Holdings of financial assets can only be positive between two histories where the marginal value of wealth is constant.

Finally, we are in a position to define a competitive equilibrium.

Definition 1 *A competitive equilibrium is given by a sequence of prices* $\left\{p\left(s^t\right), q\left(s^t\right)\right\}$ *and quantities* $\left\{c^T\left(s^t\right), c^N\left(s^t\right), c^{T,e}\left(s^t\right), n\left(s^t\right), a\left(s^t\right)\right\}$ *such that that: (i) the consumer's decisions* $\left\{c^T\left(s^t\right), c^N\left(s^t\right)\right\}$

are optimal under the constraint (2); (ii) the entrepreneur's decisions $\left\{c^{T,e}\left(s^t\right), n\left(s^t\right), a\left(s^t\right)\right\}$ *are optimal under the constraints (1) and (7) for each* s^t*; (iii) the sequences* $\left\{n\left(s^t\right)\right\}$ *and* $\left\{q\left(s^t\right)\right\}$ *satisfy (4)-(6); (iv) the labor market clears for each* s^t*,*

$$n\left(s^t\right) + c^N\left(s^t\right) = 1.$$

To characterize an equilibrium it will be sufficient to analyze the entrepreneur's optimality conditions for a given price sequence $\left\{p\left(s^t\right), q\left(s^t\right)\right\}$ and see what restrictions it imposes on the dynamics of the export units $\left\{n\left(s^t\right)\right\}$. The consumers' side is fully characterized by the constant κ, given in (3), and we only need to check that the real exchange rates, p_t, satisfy the labor market clearing conditions

$$n_t + \frac{\kappa\theta_t}{p_t} = 1,$$

and that the prices of export units, q_t, are consistent with equilibrium in the adjustment sector.

2.3 The Appreciation and Depreciation Phases

Recall that our economy starts with a stock of export units, n_{-1}, and has just entered state A. The situation that concerns us is one in which n_{-1} exceeds the units that the export sector wants to keep in operation during the appreciation, and where the latter units are less than the units the export sector wants to operate during the following depreciation phase. As a result, there is destruction of units during the appreciation, and creation during the depreciation. Moreover, we also wish to focus on a scenario where the option to wait is sufficiently positive that it is not optimal to destroy all export units during the appreciation. The export sector has financial resources a_0 to finance the losses during the appreciation phase.

2.3.1 An Efficient Benchmark

As a benchmark, let us first study a case where a_0 is sufficiently large that financial constraints are never binding. In this benchmark, there is no need to keep track of the history s^t except for the current state s_t, since, as we will see, equilibrium prices and quantities are constant both in the A and in the D phase.[3] Therefore, with a slight abuse of notation, we will simply index variables using A or D.

In the absence of financial constraints, $\phi\left(s^t\right)$ is constant and equal to one in both phases. We show later that in equilibrium there is destruction when the economy enters phase A, and creation when it switches to D. Correspondingly, $q_A = 0$ and $q_D = f$. It follows that the first order

[3] By "phase A" we mean all the histories of the form $s^t = \langle A, ..., A\rangle$. By "phase D" all those of the form $s^t = \langle A, ..., A, D, ..., D\rangle$.

conditions for n in the A and D phases, respectively, reduce to:

$$\begin{aligned}(1-p_A)+\delta\beta f &= 0,\\ -f+(1-p_D)+\beta f &= 0,\end{aligned}$$

which fully determine the real exchange rate in each phase:

$$p_A^{fb} = 1+\delta\beta f, \tag{12}$$

$$p_D^{fb} = 1-(1-\beta)f. \tag{13}$$

We assume that

$$(1-\beta)\, f < 1, \tag{A1}$$

ensuring that creation is profitable in the D phase and that $p_D^{fb} > 0$.

Given these prices we can find the consumption of tradables and nontradables in each state:

$$\begin{aligned} c_A^{T,fb} &= \kappa^{fb}\theta_A, \qquad c_A^{N,fb} = \frac{\kappa^{fb}\theta_A}{1+\delta\beta f},\\ c_D^{T,fb} &= \kappa^{fb}, \qquad c_D^{N,fb} = \frac{\kappa^{fb}}{1-(1-\beta)f},\end{aligned}$$

where κ^{fb} is equal to:

$$\kappa^{fb} = \frac{1-\beta(1-\delta)}{2\left((1-\beta)\theta_A+\delta\beta\right)}.$$

Market clearing yields the number of units open in each state:

$$n_A^{fb} = 1-\frac{\kappa^{fb}\theta_A}{1+\delta\beta f}, \qquad n_D^{fb} = 1-\frac{\kappa^{fb}}{1-(1-\beta)f}. \tag{14}$$

It is now easy to see that the following two assumptions guarantee that there is destruction when the economy enters state A at date 0, and that there is positive creation when the economy shifts from A to D:

$$n_{-1} > 1-\frac{\kappa^{fb}\theta_A}{1+\delta\beta f}, \tag{A2}$$

$$\theta_A > \frac{1+\delta\beta f}{1-(1-\beta)f}. \tag{A3}$$

Notice that, as long as the preference shock θ_A is sufficiently large, the equilibrium prices are fully determined on the supply side of the model, by (12) and (13). In particular, the price in the appreciation phase is set so as to make the current losses equal to the opportunity cost of creating a unit when the switch to the D phase occurs ($\delta\beta f$.).

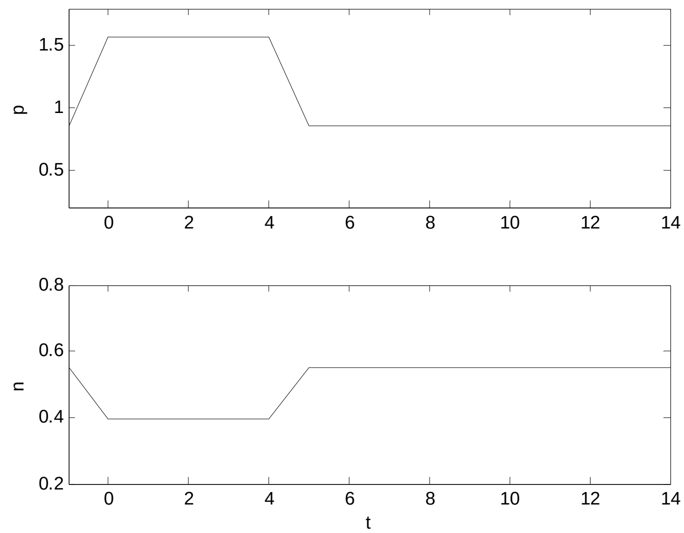

Figure 2: First Best

Figure 2 summarizes the benchmark economy, assuming that $n_{-1} = n_D^{fb}$ and $p_{-1} = p_D^{fb}$.[4] The figure illustrates the equilibrium dynamics of the real exchange rate and of the number of firms in the event the appreciation phase lasts five periods.[5] During the A phase, it is optimal for the economy to accommodate the increased demand for nontradables by contracting the export sector temporarily. However, since shutting down units wastes creation costs, it is also optimal for the export sector to keep $n_A^{fb} > 0$ units in operation, with each of them incurring flow losses of $\beta\delta f$ due to the appreciated exchange rate.

The following Proposition summarizes the case of high entrepreneurial wealth. The explicit expression for the cutoff $\hat{a}^{fb}$ is in the Appendix.

Proposition 1 *(First best) There is a cutoff $\hat{a}^{fb}$ such that if the entrepreneurs' initial wealth satisfies*

$$a_0 \geq \hat{a}^{fb},$$

then the equilibrium real exchange rate and the number of firms are constant within the A and D phases, and are given by (12), (13), and (14). The marginal value of entrepreneurial wealth, $\phi\left(s^t\right)$, is constant and equal to 1.

[4] The parameters to generate this figure are $\beta = 0.95$, $\delta = 0.2$, $f = 3$, $\theta_A = 2.5$.We set $n_{-1} = n_D^{fb}$ and $p_{-1} = p_D^{fb}$ as conventional initial conditions. These initial conditions arise if the economy makes an unexpected transition from the D state to the A state in period 0.

[5] Five periods corresponds to the expected duration of the appreciation episode since $\delta = 0.2$.

2.3.2 The Constrained Economy and Overshooting

Suppose now that a_0 is not large enough to implement the first best path (i.e., $a_0 < \hat{a}^{fb}$). There are two margins through which this deficit can materialize. First, the export sector may not have enough resources to finance the flow of losses $\left(p_A^{fb} - 1\right) n_A^{fb}$ during the appreciation. Second, even if it can, financial resources may be so depleted by the end of the appreciation phase that the financial constraint binds during the recovery in D, slowing down the reconstruction of the export sector. Either way, a constrained exports sector lowers real wages and hence consumers' income.

Relative to the benchmark case, we now need to keep track not only of the current exogenous state s_t, but also of the number of periods since the D phase started. The reason for this is that in this case there is a gradual transition in the D phase where the export sector rebuilds and is constrained by limited financial resources. At the same time, due to complete markets the A phase is still stationary, with constant prices and quantities. The next proposition summarizes the properties of equilibrium prices and quantities that will be useful in the following characterization.

Proposition 2 *Suppose $a_0 < \hat{a}^{fb}$. Then equilibrium $p\left(s^t\right), n\left(s^t\right),$ and $a\left(s^t\right)$ are constant in the A phase. In the D phase, $p\left(s^t\right), n\left(s^t\right),$ and $a\left(s^t\right)$ only depend on the number of periods that the economy has spent in D. The price $q\left(s^t\right)$ is equal to zero in the A phase and to f in the D phase.*

This proposition allows us to write $p\left(s^t\right) = p_{D,j}$ in the D phase, where j is the number of periods the economy has spent in D. We use the same notation for $n\left(s^t\right)$, $a\left(s^t\right)$ and $\phi\left(s^t\right)$.

Let us focus on the A phase. The two relevant first order conditions for the entrepreneur are

$$(1 - p_A)\,\phi_A + \delta\beta f \phi_{D,0} = 0, \tag{15}$$

and

$$\phi_A \geq \phi_{D,0} \qquad a_{D,0} \geq 0. \tag{16}$$

The stationarity of entrepreneurial wealth in A implies that $a_A = a_0$ and the budget constraint can be written as

$$(1 - (1-\delta)\beta)a_0 = (p_A - 1)n_A + \beta\delta a_{D,0}.$$

The flow generated by the initial resources a_0 can be used to finance the operational losses of the export units that remain open during the appreciation, and to transfer financial resources to the recovery phase in D. Going back to the first order conditions, equation (16) distinguishes between the case where the financial resources are used for both purposes and the case where they are only used to cover operational losses in A, In the former, $\phi_A = \phi_{D,0}$ and $a_{D,0} > 0$, while in the latter $\phi_A > \phi_{D,0}$ and $a_{D,0} = 0$.

The first order condition for n_A (equation (15)) yields an expression for the real exchange rate in the A region:

$$p_A = 1 + \beta \delta f \frac{\phi_{D,0}}{\phi_A} \leq p_A^{fb},$$

where the inequality comes from (16). As in the benchmark case, the appreciation is such that production units incur losses ($p_A > 1$). When $a_{D,0} > 0$ the real exchange rate is equal to that in the first-best, and entrepreneurs are indifferent between holding state contingent securities or holding production units. This indifference means that these two assets have the same expected returns, which pins down the equilibrium exchange rate as in the unconstrained economy. Instead, when $a_{D,0} = 0$, the expected return on export units is larger than that on state contingent securities. This wedge is possible because only entrepreneurs can purchase export units, and they are financially constrained. This constraint depressed the exchange rate to a p_A smaller than p_A^{fb}. Far from being good news, this smaller appreciation reflects the fact that financially constrained firms are unable to keep open as many production units as they would like and hence are forced to reduce production and labor demand.

For given parameters, we can show that the initial level of a_0 determines which of the two cases discussed above arises in equilibrium.

Proposition 3 *(Constrained appreciation phase) There is a cutoff $\hat{a}^A < \hat{a}^{fb}$ such that if $a_0 > \hat{a}^A$ the real exchange rate in the A phase is p_A^{fb} and $a_{D,0} > 0$, while if $a_0 < \hat{a}^A$ the real exchange rate in the A phase is $p_A < p_A^{fb}$ and $a_{D,0} = 0$.*

Let us focus now on the case where $a_0 < \hat{a}^A$. To determine n_A, note that from the consumption side and labor market equilibrium, we have that

$$p_A = \frac{c_A^T}{1 - n_A}.$$

Replacing this expression back into the budget constraint pins down the number of production units that are kept during the appreciation:

$$a_0(1 - (1 - \delta)\beta) = \left(\frac{c_A^T}{1 - n_A} - 1 \right) n_A$$

As is to be expected, for a given consumption level c_A^T, lower financial resources a_0 lower the number of production units that are kept open during the appreciation. In general equilibrium, c_A^T falls as well and hence the final effect on n_A is ambiguous. What is unambiguous (see the Appendix), is that $n_A/\overline{n}_D$ drops as a_0 declines, where $\overline{n}_D$ represents the stationary size of the export sector after the recovery phase of D is completed. This is important, because a lower $n_A/\overline{n}_D$ means that the reconstruction effort needed during the D phase rises with the tightening of a_0. We turn to the D region next.

Starting backwards, once the recovery phase is completed, entrepreneurs consume and $\phi_{\overline{D}} = 1$. Thus, from the first order conditions, we have that in the stationary phase of D:

$$p_{\overline{D}} = 1 - (1-\beta) f = p_D^{fb}$$

Eventually, the real exchange rate converges to the benchmark level.

If follows from the equilibrium condition in the labor market and the fact that the level of consumption is lower in the constrained than in the benchmark case (which we formally show later), that:

$$n_{\overline{D}} = 1 - \frac{c_D^T}{1-(1-\beta)f} \quad > \quad 1 - \frac{c_D^{T,fb}}{1-(1-\beta)f} = n_D^{fb}.$$

Since in the constrained economy not only entrepreneurs but also consumers are poorer than in the benchmark economy, demand is depressed and hence the export sector eventually expands to absorb the labor freed by the smaller nontradable sector. However, unlike the benchmark case, this stationary state is not reached instantly since financial constraints also hamper the recovery phase. The first order conditions for this transition phase are:

$$\begin{aligned}
(-f + 1 - p_{D,j})\, \phi_{D,j} + \beta f \phi_{D,j+1} &= 0, & (17)\\
\phi_{D,j} &> 1, & (18)\\
\phi_{D,j} &> \phi_{D,j+1}, & (19)\\
f(n_{D,j} - n_{D,j-1}) &= (1 - p_{D,j}) n_{D,j} & (20)
\end{aligned}$$

for $j = 0, ..J$, where J is the last period of the transition phase in D and, with abuse of notation, $n_{D,-1} = n_A$.

Starting from the bottom, equation (20) states that during the recovery phase, firms use all their profits to rebuild the sector. Condition (19) reflects that financial constraints are tightest early on in the recovery and gradually decline, and hence there is no reason to accumulate "cash" or to consume (condition (18)). Reorganizing equation (17), we obtain an expression for the real exchange rate during the transition:

$$p_{D,j} = 1 - f\left(1 - \beta \frac{\phi_{D,j+1}}{\phi_{D,j}}\right) \quad < 1 - f\,(1-\beta) = p_{\overline{D}} = p_D^{fb}$$

That is, during the recovery phase the depreciation is deeper when the economy is constrained. We refer to this deeper depreciation as the *overshooting* implication of financial constraints.

The presence of overshooting means that wages are not only lower than in the benchmark case during the appreciation phase, but also during the transition phase of D. This observation closes our argument, as it explains why the consumption level is lower in the constrained case. Recall that the consumption level is indexed by κ:

$$\kappa = \frac{\mathrm{E}\sum_t \beta^t p_t}{2 \cdot \mathrm{E}\sum_t \beta^t \theta_t}.$$

and that, history by history, p_t is greater (with some strictly greater) in the benchmark than in the constrained case.

Figure 3 depicts the constrained economy, assuming for simplicity that $n_{-1} = n_{\overline{D}}$ and $p_{-1} = p_{\overline{D}}$.[6] As in the benchmark economy (which is represented with dashes in each panel), the exchange rate appreciates in the A phase and it experiences a large and protracted overshooting in the depreciation phase. The export sector contracts during the A phase and, unlike the benchmark economy, the recovery is only gradual during the D phase. The bottom panel shows the path of the marginal value of a unit of wealth, which is highest in the A region, drops sharply upon the transition into D, and gradually declines within the D region.

Let us conclude with a summary proposition:

Proposition 4 *(Constrained depreciation phase and overshooting)* *There is a cutoff $\hat{a}^D < \hat{a}^{fb}$ such that if $a_0 > \hat{a}^D$ the real exchange rate throughout the D phase is p_D^{fb}, while if $a_0 < \hat{a}^D$ the real exchange rate depreciation in the D phase overshoots its long run value early on in the transition. That is $p_{D,j} < p_D^{fb}$ for $j = 0, 1...J$. (Note that the cutoff $\hat{a}^D$ may be greater or smaller than $\hat{a}^A$, depending on the model's parameters).*

2.4 General Equilibrium Feedback

Our discussion above highlights the export firms' problem given a consumption demand. However, firms' actions affect households' income through labor demand. The tighter is the financial constraint on firms, the lower is labor demand and income. This feedback is captured by the relation between a_0 and κ. Figure 4 plots this relation and shows that κ is increasing in a_0 until it reaches its maximum for $a_0 \geq \hat{a}^{fb}$.

Note that this general equilibrium feedback generates some counterintuitive results. For example, the model has a sort of sclerosis as a_0 declines. Even though export firms are more financially constrained when financial resources are low, in the long run they absorb a larger share of n. To see this, recall that

$$n_{\overline{D}} = 1 - \frac{\kappa}{1 - (1-\beta)f}.$$

[6] The parameters are the same as those used for Figure 2, plus $a_0 = 0.2$.

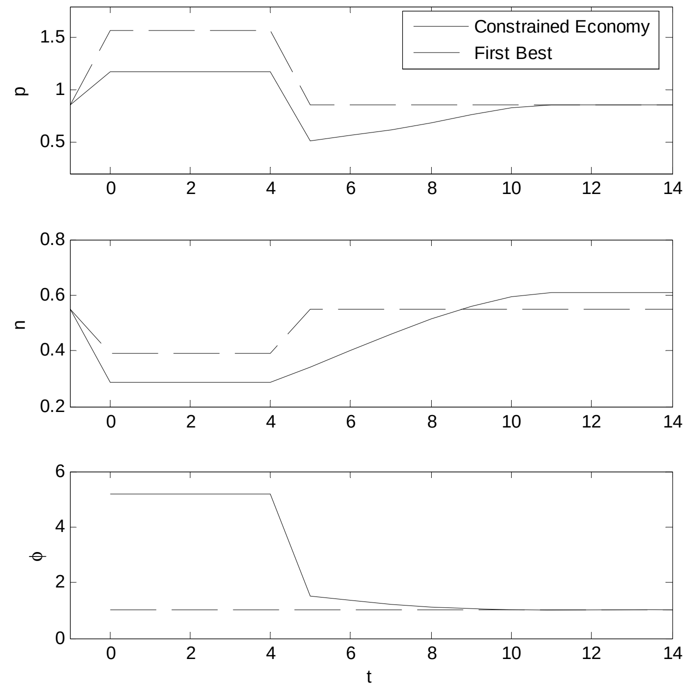

Figure 3: Constrained Economy

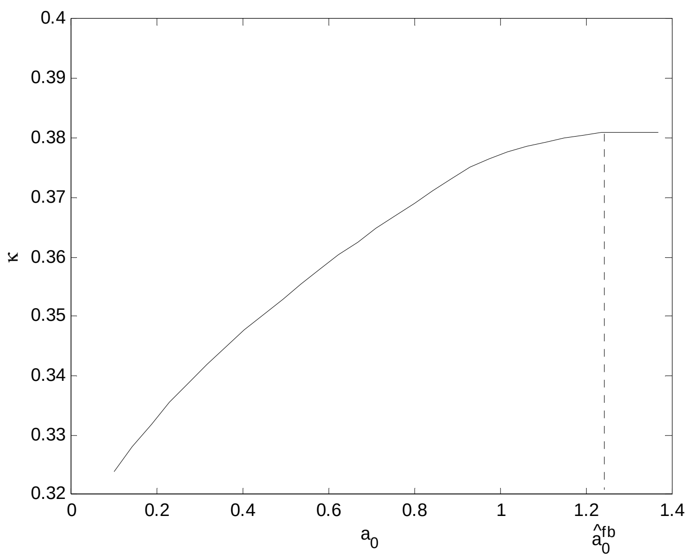

Figure 4: The Income Effect of the Financial Constraint

which rises as κ drops. This simply says that an economy with poorer consumers allocates a larger share of its resources to satisfy foreign than domestic demand.

Up to now, we have developed a model of creative-destruction and overshooting. The next section turns to the other main concern in this paper. In particular, it shows that when $a_0 < \overline{a}^{fb}$, the social planner may be able to raise κ by inducing consumers to choose a different path for c_t^N (and hence n_t).

3 Optimal Ex-ante and Ex-post Intervention

In the previous section we showed that when the export sector has limited financial resources, the depreciation phase following a persistent appreciation may come with a protracted exchange rate overshooting (a sharp real wage decline) while the export sector rebuilds. Either explicitly or implicitly, in practice it is this overshooting phase that primarily concerns policymakers and leads to a debate on whether intervention should take place during the appreciation phase to protect the export sector. In particular, the concern is whether by overly stressing the export sector during the appreciation, the economy may be exposing itself to a costly recovery phase once the factors behind the appreciation subside. In this section we study this policy problem and conclude that if an overshooting is expected, there is indeed scope for policy intervention. The reason for such intervention is that the competitive equilibrium is not constrained efficient, as consumers ignore the effect of their individual decisions on the severity and duration of the overshooting during the depreciation phase.

The optimal policy includes *ex-ante* and *ex-post* interventions. There are instances when the focus of intervention is ex-ante, and the bulk of it consists of stabilizing the exchange rate during the appreciation phase. There are others where the scope for appreciation stabilization is limited and the policy intervention is concentrated in the first period of the depreciation phase (ex-post intervention).

3.1 A Fiscal Intervention

We consider a government that uses a set of fiscal instruments to affect the time profile of consumers' demand. In particular, the government can impose a sequence of linear tax rates $\left\{\tau^T\left(s^t\right), \tau^N\left(s^t\right)\right\}$ on consumers' spending on tradables and non-tradables. Any tax revenue is returned to the consumers as a lump-sum transfer at date 0, T_0, so consumers face the budget constraint

$$\sum_{t,s^t} \beta^t \pi\left(s^t\right)\left(\left(1+\tau^T\left(s^t\right)\right) c^T\left(s^t\right)+\left(1+\tau^N\left(s^t\right)\right) p\left(s^t\right) c^N\left(s^t\right)\right) \leq \sum_{t,s^t} \beta^t \pi\left(s^t\right) p\left(s^t\right)+T_0. \quad (21)$$

For a given tax sequence $\left\{\tau^T\left(s^t\right), \tau^N\left(s^t\right)\right\}$ we can define a competitive equilibrium as we did in the economy with no taxes (see Definition 1), replacing the consumers' budget constraint with (21) and adding the condition that the lump-sum transfer T_0 satisfies the government budget balance condition

$$\sum_{t,s^t} \beta^t \pi\left(s^t\right)\left(\tau^T\left(s^t\right) c^T\left(s^t\right)+\tau^N\left(s^t\right) p\left(s^t\right) c^N\left(s^t\right)\right)=T_0.$$

We study a benevolent government that chooses $\left\{\tau^T\left(s^t\right), \tau^N\left(s^t\right)\right\}$ so as to maximize the utility of the representative consumer subject to the constraint of not making entrepreneurs worse off:

$$\sum_t \sum_{s^t} \beta^t \pi\left(s^t\right) c^{T,e}\left(s^t\right) \geq U,$$

where U is the entrepreneurs' expected utility in the competitive equilibrium.

We choose to focus on this exercise for two reasons. First, the policy described does not allow the government to make direct transfers between consumers and entrepreneurs. If the government could do that, it would be simple in this economy to undo the effects of the financial constraint (1). The government would transfer resources to the entrepreneurs in the A phase and in the early stage of the D phase, and then transfer resources back to consumers later in the D phase, hence replicating the equilibrium of a frictionless economy. However in practice, targeted transfers of this type take place but are limited by a host of informational and institutional impediments which we do not model here. In this sense, the set of policies that we consider respect the spirit of the financial constraint (1).[7] Second, the policy described is more flexible than the specific forms of macroeconomic interventions that are contemplated in the policy debate. Namely, most proposed interventions are geared towards increasing domestic savings, thus reducing the pressure on the real exchange rate by reducing the demand of *both* tradables and nontradables. As we will see, our policy-maker will choose not to distort tradable consumption decisions and will only intervene on non-tradable consumption.

In summary, we consider a general form of intervention but stay within the boundaries of a constrained efficiency exercise, by ruling out direct transfers between consumers and entrepreneurs. This choice allows us to identify a simple pecuniary externality, which should play a relevant role in all practical policy proposals that attempt to curb persistent appreciations.

3.2 Policy Perturbation and Pecuniary Externality

Before characterizing the optimal policy, let us identify the pecuniary externality by studying the impact of small policy interventions around the competitive equilibrium.

[7] A "fundamental" view of constraint (1) is that it is impossible to extract payments from entrepreneurs, whether in the form of financial payments or in the forms of taxes.

The planner's objective is to maximize the consumer's utility, which can be written as:

$$\theta_A \left(u\left(c_A^T\right) + u\left(c_A^N\right)\right) + \delta\beta\left(\frac{1}{1-\beta}u\left(c_D^T\right) + \sum_{j=0}^{\infty}\beta^j u\left(c_{D,j}^N\right)\right),$$

where we have normalized expected utility by the factor $(1-\beta(1-\delta))$.[8] As usual in optimal taxation problems, it is simpler to characterize the problem directly in terms of equilibrium quantities rather than in terms of the underlying tax rates. Thus, we let the planner choose directly the consumption paths for tradables and nontradables.

The consumers' budget constraint is (also multiplying through by $(1-\beta(1-\delta))$):

$$c_A^T + p_A c_A^N + \delta\beta\left(\frac{1}{1-\beta}c_D^T + \sum_{j=0}^{\infty}\beta^j p_{D,j} c_{D,j}^N\right) \le p_A + \delta\beta\sum_{j=0}^{\infty}\beta^j p_{D,j}. \tag{22}$$

Relative to that of individual consumers, the social planner's problem is different in that it takes into account the effect of consumers' decisions on p_A and $\{p_{D,j}\}$. Consumers' decisions affect the equilibrium prices by changing the demand for nontradables and, thus, equilibrium wages. Using the entrepreneurs' optimality condition and market clearing in the labor market we can derive a relation between the quantities chosen by the planner and the prices p_A and $\{p_{D,j}\}$. In the appendix we show that entrepreneurs' optimality defines a mapping between the labor allocations $n_A, \{n_{D,j}\}$ and the price sequences $p_A, \{p_{D,j}\}$. Therefore, the planner chooses the consumption of nontradables, market clearing gives the labor allocations $n_A = 1 - c_A^N$ and $n_{D,j} = 1 - c_{D,j}^N$, and the mapping above yields the associated equilibrium prices.

Finally, the constraint that entrepreneurs cannot be made worse off is (also multiplying through by $(1-\beta(1-\delta))$):

$$c_A^{T,e} + \delta\beta\sum_{j=0}^{\infty}\beta^j c_{D,j}^{T,e} \ge (1-\beta(1-\delta))\,\bar{U}^e, \tag{23}$$

where $\bar{U}^e$ denotes the entrepreneurs' welfare in the competitive equilibrium.

Let us study the effect of stabilizing the appreciation phase, starting from the competitive equilibrium studied in Section 2. Specifically, consider the effect of reducing c_A^N or, equivalently, increasing n_A, while keeping the $n_{D,j}$'s unchanged.

[8] In the Appendix we show that the second best allocation shares the following features with the competitive equilibrium: the consumption of tradables is constant and equal to c_A^T and c_D^T, respectively, in the A phase and in the D phase, and the consumption of non-tradables is constant and equal to c_A^N in the A phase. Therefore, also in our perturbation argument we will limit attention to allocations with these features.

The following expression captures the marginal effect of a change in n_A in the planner's problem:

$$
\begin{aligned}
&-\theta_A u'(1-n_A)+p_A\lambda\\
&+\lambda\left(\frac{\partial p_A}{\partial n_A}n_A+\beta\delta\frac{\partial p_{D,0}}{\partial n_A}n_{D,0}\right)+\\
&+\mu\beta\frac{\partial c_{D,0}^{T,e}}{\partial n_A},
\end{aligned}
\tag{24}
$$

where λ represents the Lagrange multiplier of the consumers' budget constraint (22) and μ that of the entrepreneurs' participation constraint (23).

The first row of (24) captures the direct effect of the policy and is equivalent to the consumers' first order condition in the competitive economy. The second row captures the impact of the policy on consumers' net (of consumption) income.[9] Since we keep all the $n_{D,j}$'s constant, this policy only affects the prices p_A and $p_{D,0}$, and the entrepreneurs' consumption at date $t_{D,0}$.

We consider two cases. First, suppose the competitive equilibrium displays $p_A < p_A^{fb}$ and $p_{D,0} < p_D^{fb}$ (overshooting).

Let us start with the effect of a unit increase in n_A on p_A. If the planner wants entrepreneurs to carry an extra unit of n_A, then p_A must drop for the firm to be able to finance the extra losses from that unit. Recall that the firm's budget constraint in phase A is

$$(1-(1-\delta)\beta)\,a_0=(p_A-1)\,n_A,$$

from which we obtain:

$$\frac{\partial p_A}{\partial n_A}=-\frac{p_A-1}{n_A}.$$

We now turn to the effects of n_A on $p_{D,0}$. Since $p_{D,0} < p_D^{fb}$, i.e., there is equilibrium overshooting, the entrepreneur budget constraint at date $t_{D,0}$ is

$$f\,(n_{D,0}-n_A)=(1-p_{D,0})\,n_{D,0}.$$

The entrepreneur's financial constraint is binding and he uses all his current profits to invest in new units. In this case, a unit increase in n_A affects $p_{D,0}$ since it reduces by f the investment required to rebuild to $n_{D,0}$. Wages must rise to compensate for this fall in investment expenditure, so as to keep the financial constraint exactly binding at $n_{D,0}$. Thus,

$$\frac{\partial p_{D,0}}{\partial n_A}=\frac{f}{n_{D,0}}.$$

Finally, in this case $c_{D,0}^{T,e}=0$ so the third line of (24) is zero.

[9]Note that the consumer's labor income is p_A while its expenditure on nontradables is $p_A(1-n_A)$. Thus net income is p_An_A.

Consumers are hurt by the decline in their wage (real exchange rate) during the A phase, but gain from the rise in their wage in the first period of the D phase. Which effect dominates? Replacing the price derivatives in the expression in the second row of the first order condition we have:

$$\frac{\partial p_A}{\partial n_A} n_A + \delta\beta \frac{\partial p_{D,0}}{\partial n_A} n_{D,0} = 1 - p_A + \beta\delta f > 0.$$

The inequality follows from $p_A < p_A^{fb} = 1 + \beta\delta f$. That is, in the planner's problem there is an extra term capturing the marginal benefit of increasing n_A on the expected present value of wages. The planner has an incentive to reduce nontradables consumption, so as to reduce the appreciation (i.e. reduce p_A) and allow firms to keep a larger number of units open, which in turn raises wages at $t_{D,0}$. Because $\phi_A > \phi_{D,0}$, reducing wages in A generates an excess return in export firms that is transferred back to workers in the form of higher wages at $t_{D,0}$. Summing up, when $p_A < p_A^{fb}$ and $p_{D,0} < p_D^{fb}$ the expression in (24), computed at the competitive equilibrium, is positive.

The planner is choosing the consumption of nontradables c_A^N *as if* the real exchange rate was higher (i.e., more appreciated) than the market price. How much higher? It is easy to show from the marginal benefit of increasing n_A that the planner would like to pick c_A^N *as if* the price was at its first best level:

$$\begin{aligned} &-\theta_A u'(1 - n_A) + \lambda p_A + \lambda(1 - p_A + \beta\delta f) = \\ &-\theta_A u'(1 - n_A) + \lambda p_A^{fb}. \end{aligned}$$

In a competitive equilibrium, consumers increase their demand for nontradables in response to the taste shock, which leads to an appreciation of the real exchange rate. However, due to the firms' financial constraint, the appreciation is smaller than it would be in the first best. This price gap implies that consumers further increase their consumption of nontradables, at the expense of export units. The planner taxes consumption of nontradables enough to offset this additional effect, and in so doing lowers the real exchange rate and allows firms to maintain a larger number of production units open.

Consider now a second case, where $p_A < p_A^{fb}$ and $p_{D,0} = p_D^{fb}$ (no overshooting). In this case, entrepreneurs are unconstrained at $t_{D,0}$ and hence $c_{D,0}^{T,e} > 0$, which implies

$$\frac{\partial p_{D,0}}{\partial n_A} = 0$$

and

$$\frac{\partial c_{D,0}^{T,e}}{\partial n_A} = f.$$

Replacing these terms in (24) we obtain

$$-\theta_A u'(1 - n_A) + \lambda p_A + \lambda(1 - p_A) + \mu\beta f.$$

Suppose initially that the participation constraint was slack ($\mu = 0$), then $\theta_A u'(1 - n_A) = \lambda < \lambda p_A$. The planner would want to increase c_A^N and *reduce* n_A. The reason is that it makes no sense for consumers to cut their wage today if this action does not raise wages in the future, which it will not when there is no overshooting to remedy and $p_{D,0}$ is pinned down by the technological free entry condition. Instead, the planner, representing the consumers, would like to exercise its "monopoly" power during the appreciation phase and raise wages by increasing their demand for nontradables. However this increase would reduce the consumption of entrepreneurs and violate their participation constraint. Therefore $\mu = 0$ is not possible. In fact, when there is no expected overshooting, it is optimal to not intervene and μ can be chosen so that

$$-\theta_A u'(1 - n_A) + \lambda p_A + \lambda (1 - p_A) + \mu \beta f = 0,$$

where n_A and p_A are at their competitive equilibrium values. This discussion gives us an important reference result:

Proposition 5 *(Constrained efficiency) If $a_0 > \hat{a}^D$ (no overshooting), then the competitive equilibrium is constrained efficient and it is optimal not to stabilize the appreciation, even if firms are financially constrained and the export sector contracts more than in the first best (i.e., even if $a_0 < \hat{a}^A$).*

Put differently, if there is no overshooting, there is no intertemporal pecuniary externality for consumers, so they cannot trade-off a wage reduction today for a wage increase in the recovery phase. The flip side of this argument is that it is the presence of overshooting that makes individual consumers underestimate the social cost of their increased demand during the appreciation phase.

It is useful to show that the argument just made for n_A can also be made for $n_{D,j}$ during the depreciation phase. That is, suppose the planner can only intervene in period j of the D phase and change $n_{D,j}$ by a small amount. Suppose the entrepreneurial sector has not fully recovered in periods $t_{D,j}$ and $t_{D,j+1}$, i.e., we are in the middle of the overshooting phase with $p_{D,j} < p_{D,j+1} < p_D^{fb}$. Then, it is optimal to reduce $c_{D,j}^N$ further and *exacerbate* the depreciation in period j. By doing so, the consumers accelerate the recovery of the export sector and of real wages. The planner first order condition is similar to that derived for n_A. In particular, since these derivatives take future n's as given, a change in $n_{D,j}$ only affects the current and next period's prices. As before, the financial constraint is still binding after a small intervention, so $c_{D,j+1}^{T,e} = c_{D,j}^{T,e} = 0$. Therefore, the marginal effect of an increase in $n_{D,j}$ is given by

$$-u'(1 - n_{D,j}) + p_{D,j}\lambda + \lambda \left(\frac{\partial p_{D,j}}{\partial n_{D,j}} n_{D,j} + \beta \frac{\partial p_{D,j+1}}{\partial n_{D,j}} n_{D,j+1} \right)$$

and

$$\frac{\partial p_{D,j}}{\partial n_{D,j}} n_{D,j} + \beta \frac{\partial p_{D,j+1}}{\partial n_{D,j}} n_{D,j+1} = -\left(f - (1 - p_{D,j})\right) + \beta f > 0,$$

where the inequality follows from the fact that $p_{D,j} < p_D^{fb} = 1 - (1 - \beta) f$. This, together with consumers' optimality, implies that a reduction in $p_{D,j}$ leads to a marginal welfare gain.

In the competitive equilibrium, the firms' financial constraints depresses labor demand, making non-tradables cheaper and inducing consumers to demand more of them. The social planner offsets the consumers' reaction to the overshooting and reduces $c_{D,j}^N$ (it taxes nontradable consumption). Note that as a result of this reduction in $c_{D,j}^N$ the overshooting is exacerbated, but this is precisely what increases profits and allows financially constrained firms to accelerate investment. The trade-off is between a deeper overshooting and lower wages today in exchange for a faster recovery in wages. Because $\phi_{D,j} > \phi_{D,j+1}$, reducing wages at $t_{D,j}$ generates an excess return in export firms that is transferred back to workers in the form of substantially higher wages at $t_{D,j+1}$.

Once we allow the planner to set an optimal policy in both phases, A and D, some of the incentive to exacerbate the overshooting in the D phase goes away, because the planner would have already achieved higher levels of investment by protecting entrepreneurial wealth in the appreciation phase. This interaction between preventive intervention and intervention in the depreciation phase is a central aspect of the optimal policy discussion that follows.

3.3 Optimal Policy

We learned from the perturbation argument above that if there is an expected overshooting, then the competitive equilibrium is constrained inefficient and there is scope for policy. We now turn to characterizing the economy's dynamics under the optimal policy.

Figure 5 plots the real exchange rate and the number of export units in the competitive equilibrium and in the "second best" allocation, in a baseline scenario.[10] The first feature of the optimal policy is that the planner attenuates the exchange rate appreciation. This attenuation has the effect of keeping more export units open in the appreciation phase, and makes the recovery of the export sector faster during the depreciation phase. In turn, the faster recovery increases the demand of non-tradables by entrepreneurs and reduces the extent and duration of the real exchange rate overshooting during the depreciation phase. The final effect of the intervention is a form of real exchange rate stabilization.

Two robust features of the optimal price path are that p_A is lower than the competitive equilibrium level, and that the increase in the expected present value of the $p_{D,j}$'s more than offsets the decline in p_A. The price path depicted in Figure 5 also displays a reduction in the initial

[10] The parameters are the same as those used for Figures 2 and 3. As in those figures, we plot the realized path when the appreciation lasts five periods.

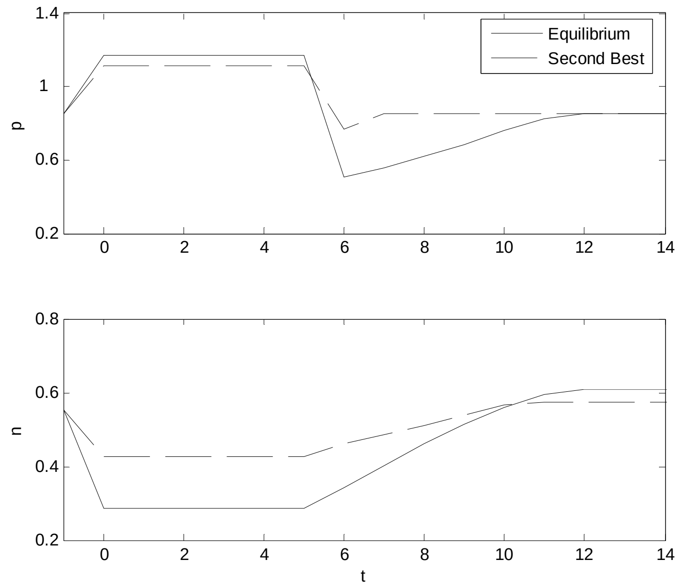

Figure 5: Optimal Policy

overshooting at $t_{D,0}$. However, this feature is less robust and depends on the economy's initial conditions. We will discuss other possible cases below. Notice also that, in the example presented, the level of second best destruction of export units in the A phase is not only lower than that of the competitive equilibrium, but it is also lower than destruction in the first best (see Figure 2). The reason is that in the second best creation is anticipated to be constrained during the recovery phase.

Let us explore the optimal paths in more detail, by analyzing first the intervention in the A phase (i.e., the choice of n_A) and then the intervention in the D phase (i.e., the $n_{D,j}$'s). First, there is an optimal degree of exchange rate stabilization during the A phase. Given that $p_A < p_A^{fb}$ and $p_{D,0} < p_D^{fb}$ we can proceed as we did above and write the planner's first order condition for n_A as:

$$\theta_A u'(1 - n_A) = \lambda p_A^{fb} = \lambda (1 + \beta \delta f). \tag{25}$$

The social planner allocates n_A *as if* prices were at first best. Since firms are financially constrained, this allocation can be achieved only by depressing consumption of nontradables, which attenuates the appreciation in equilibrium. However, this first order condition also reveals that the optimal depreciation during the A phase is not unlimited. Once quantities correspond to those implied by the consumers' first order condition when facing first-best prices, the intervention stops.

Turning to the D phase, notice that along the recovery path the entrepreneurs' financial constraint is exactly binding:

$$f\left(n_{D,j}-n_{D,j-1}\right)=\left(1-p_{D,j}\right)n_{D,j},$$

until the point where $n_{D,j}$ reaches its "nondistorted" level, i.e., the value $\bar{n}_D$ that satisfies

$$u'\left(1-\bar{n}_D\right)=\lambda p_D^{fb}.$$

Entrepreneurs use all their profits for investment and delay their consumption until they have reached $\bar{n}_D$. The latter happens in period eleven in Figure 5. Notice also that some amount of overshooting is still present in the second best, i.e., $p_{D,0}<p_D^{fb}$. Recall the argument made above on the social benefits of the overshooting, which makes the recovery faster and increases future values of $p_{D,j}$. At the optimum, the argument is subtler since $p_{D,1}$ is equal to p_D^{fb}. If the planner were to increase $p_{D,0}$ he would have to reduce $n_{D,0}$. But then, since the entrepreneurs are exactly constrained at $t_{D,1}$, this would imply a wage loss at that date. For consumers, the net effect of this reduction in $n_{D,0}$ would be

$$\begin{aligned} u'\left(1-n_{D,0}\right)-\lambda p_{D,0}-\lambda\left(1-f-p_{D,0}+\beta f\right) &= \\ u'\left(1-n_{D,0}\right)-\lambda p_D^{fb} &< 0. \end{aligned}$$

On the other hand, if the planner tried to increase $n_{D,0}$, the current wage would drop but there would be no gain in terms of future wages, given that the entrepreneurs' would be unconstrained and would employ the extra funds for consumption. In this case, there would be a benefit in terms of relaxing the entrepreneur's participation constraint and the marginal effect would be

$$-u'\left(1-n_{D,0}\right)+\lambda p_{D,0}+\lambda\left(1-f-p_{D,0}\right)+\mu\beta f\geq 0$$

(recall that μ is the Lagrange multiplier on the entrepreneur's participation constraint. The argument showing that this expression is positive is in the appendix.[11]

The two conditions just derived show that the planner is at a 'kink,' and state the optimality of the given value of $p_{D,0}$. Notice that the reasoning behind these conditions applies only because $p_{D,j}=p_D^{fb}$ for all the periods following $t_{D,0}$. This is key since it shows that under the optimal policy, the overshooting can only happen in the first period of the recovery. If we had $p_{D,j}<p_D^{fb}$ in some other period, then in the previous period it would be optimal to increase $n_{D,j-1}$ and accelerate the adjustment toward $\bar{n}_D$. Essentially, the optimal path requires that if the planner wants to allow for some depreciation in the D phase to speed up the recovery, it completely frontloads this depreciation.[12]

[11] See the proof of Proposition 6.

[12] If the real exchange rate reaches 0 in the first period, then the optimal depreciation will continue for a second period, and so on.

In terms of consumption of non-tradables, a distortion is also concentrated in the early periods of the D phase (although, not only in the first period). In these periods the following inequality holds,

$$u'(1-n_{D,j}) < \lambda p_D^{fb},$$

and the planner would like to decrease the consumption of nontradables (i.e., increase $n_{D,j}$) so as to smooth nontradable consumption. However, since the entrepreneurs financial constraint is exactly binding, increasing $n_{D,j}$ in any of these periods would reduce the current wages, $p_{D,j}$, below their first best level. This has no advantages in terms of future wages, given that $p_{D,j+1}$ is already at its maximum level p_D^{fb}. The potential cost in terms of current wages (plus the shadow cost of the entrepreneurs' participation) exactly compensates for the distortion in nontradables consumption.

Let us summarize the main results of this section with a proposition:

Proposition 6 *(Optimal policy) If $a_0 < \hat{a}^D$, then the competitive equilibrium is constrained inefficient. Depending on parameters, the optimal policy involves some depreciation (relative to the competitive equilibrium) of the exchange rate in A, some further depreciation in the first period of the D phase, or some combination of both. In all of the above the overshooting phase in the second best lasts at most one period.*

For completeness, note that in terms of quantities the unambiguous results are that the optimal policy reduces fluctuations in n and the long run size of the export sector. The reason for the former result is the counterpart of the exchange rate stabilization. The reason for latter result is that consumers are richer in the "second best," and hence use a larger share of their labor resources to produce nontradables. To illustrate this wealth effect, Figure 6 compares the value of κ in the competitive equilibrium and the "second best," for different levels of financial resources in the export sector, a_0. As expected, for high values of a_0 the competitive equilibrium is close to the second best (and they coincide for $a_0 \geq \hat{a}^D$), which in turn is closer to the first best. However, for low levels of a_0, the pecuniary externality is significant and the second best income is substantially higher than that of the competitive equilibrium. Of course, if the government had effective direct transfer instruments, then by increasing a_0 it would be able not only to narrow the wedge between the competitive equilibrium and the second best, but also that between the latter and the first best.

3.4 Ex-ante versus Ex-post Intervention

In our discussion of the optimal policy, we touched on a pervasive policy concern in the presence of an appreciation in the value of the currency or of any other asset with potential macroeconomic consequences (e.g., real estate or stocks). Should the intervention take place *ex-ante* (i.e., during the appreciation phase) or *ex-post* (i.e., after the "crash" or depreciation takes place)?

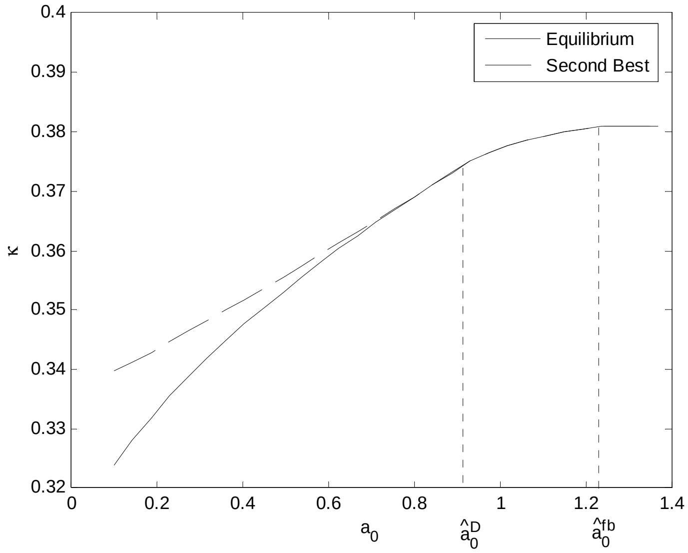

Figure 6: Income Effects in Equilibrium and Second Best

In the example we used in Figure 5, the optimal policy involved a combination of both. The planner stabilized the exchange during the appreciation, but preserved some of the overshooting in the first period of the depreciation phase as well, both helping export companies accelerate the recovery.

In contrast, panel (a) of Figure 7 represents a case in which the intervention to offset the pecuniary externality is entirely done ex-ante.[13] An attenuation of the appreciation in A, by increasing n_A, increases $p_{D,0}$. Therefore, it is possible that before reaching the level of n_A that satisfies (25), $p_{D,0}$ reaches its first best level p_D^{fb}. At this point there is no gain for the consumer from cutting wages further during the A phase, since this has no effect on wages in the D phase. Remember that (25) was derived under the assumption that $p_A < p_A^{fb}$ and $p_{D,0} < p_D^{fb}$. Once n_A reaches the level such that $p_{D,0} = p_D^{fb}$, the Lagrangian for the planner problem has a kink and the first order condition for n_A takes the form

$$\theta_A u' (1 - n_A) = \lambda p_A + \lambda [1 - p_A + \xi \beta \delta f] \tag{26}$$

where $\xi \in [0, 1]$.[14] In this case, the optimal n_A is such that it makes the entrepreneurs financial constraint at date $t_{D,0}$ exactly binding and ξ adjusts so as to ensure that (26) is satisfied.

[13] Parameters are the same as those used for Figures 5 except for $a_0 = .03$ in panel (a), $a_0 = .2$ in panel (b), $a_0 = 1$ in panel (c).

[14] At this point the map from $n_A, \{n_{D,j}\}$ to $p_A, \{p_{D,j}\}$ has a kink, and, in terms of generalized derivatives we have

$$\frac{\partial p_{D,0}}{\partial n_A} = \xi \frac{\delta f}{n_{D,0}},$$

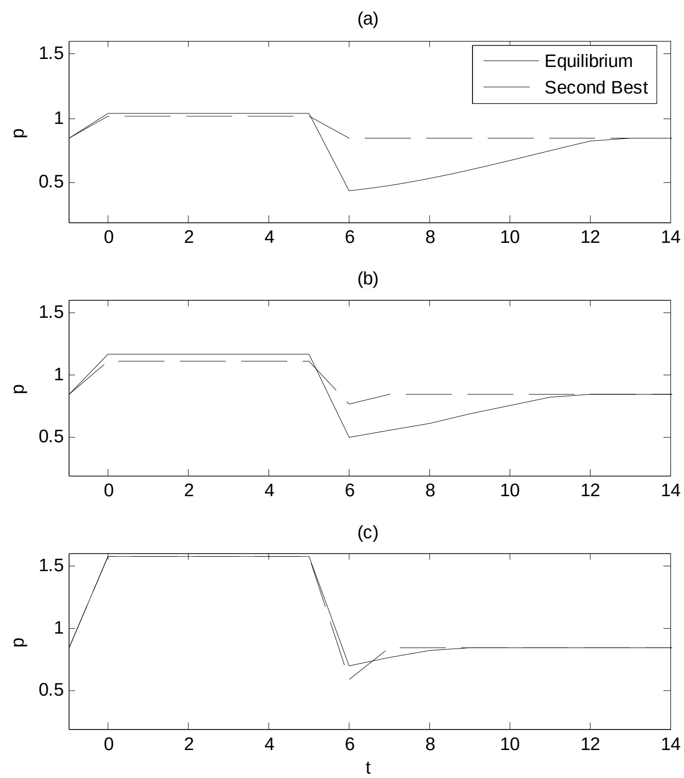

Figure 7: Entrepreneurial Wealth and Optimal Intervention

The polar opposite is in panel (c) of Figure 7 (note that panel (b) simply reproduces Figure 5), where the policy is all ex-post. This takes place if the competitive equilibrium price during the appreciation is equal to p_A^{fb}, for in such case there is no scope for intervention during this phase. The only difference between this scenario and that in panel (a), is the level of financial resources a_0, which is relatively low in panel (a) and high in (c) (while it is at an intermediate level in panel b). When financial resources are relatively abundant, the price-distortion during the appreciation phase is small and hence the cost of distorting intertemporal consumption by taxing consumption of nontradables is high. Thus the social planner opts for postponing intervention.[15]

Figure 8 generalizes the message of the previous figure and shows the level of p_A and $p_{D,0}$ in the competitive equilibrium (solid) and optimal policy (dashes) for a wide range of financial resources a_0. At low levels of a_0 the intervention during the appreciation phase has a large impact on allocations and there is no need to exacerbate the initial overshooting in the depreciation phase. In fact, the latter is significantly reduced in this case. However, as a_0 rises there is less scope for ex-ante intervention and a larger share of the adjustment is deferred to the depreciation phase. Eventually, when a_0 is sufficiently large, the policy is mostly concentrated ex-post, even to the point of causing an over-overshooting (the region where the dashed line is below the solid line in the bottom panel). Finally, as a_0 is sufficiently high that there is no overshooting in the competitive equilibrium, there is no longer scope for policy.

In terms of implementation of the optimal policy in each of these scenarios, Figure 9 reports the paths of nontradable consumption taxes corresponding to the panels in Figure 7. In particular, the government imposes a proportional ad-valorem tax τ_t on the purchases of non-tradables, τ_t may be negative, corresponding to a subsidy. The pure ex-ante policy in panel (a) requires strictly positive taxes during the appreciation phase and a subsidy during the depreciation phase. That is, the ex-ante aspect of the policy refers to the fact that the pecuniary externality is entirely resolved during the appreciation phase. The subsidy component of the policy is simply a mechanism by which, through higher wages (recall that all taxes and subsidies are returned back to consumers as a lump sum), consumers take back any surplus transferred to the entrepreneurs during the intervention in the appreciation phase. Panel (b) shows the intermediate case, where the exchange rate is allowed to depreciate in the first period of the D phase. In this case the path for the subsidy is kept lower in $t_{D,0}$, increases in $t_{D,1}$ and then converges to zero. Panel (c), the pure ex-post policy case has no taxes during the appreciation phase, and instead the tax is concentrated in the first period of the depreciation phase.

with $\xi \in [0, 1]$.

[15] Note also that when export firms have abundant financial resources, there is a sort of Ricardian equivalence, in that any (at least small) intervention can be undone by the private sector (this is an exact result whenever $\phi_A = \phi_{D,0}$).

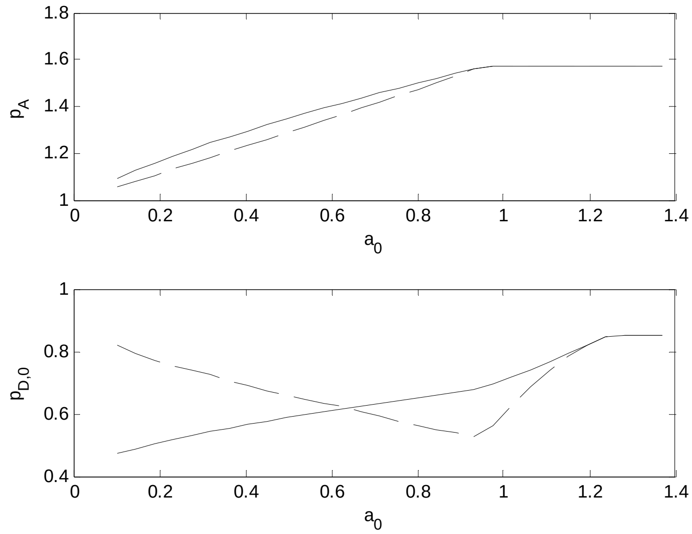

Figure 8: Financial Constraint and Second Best Intervention

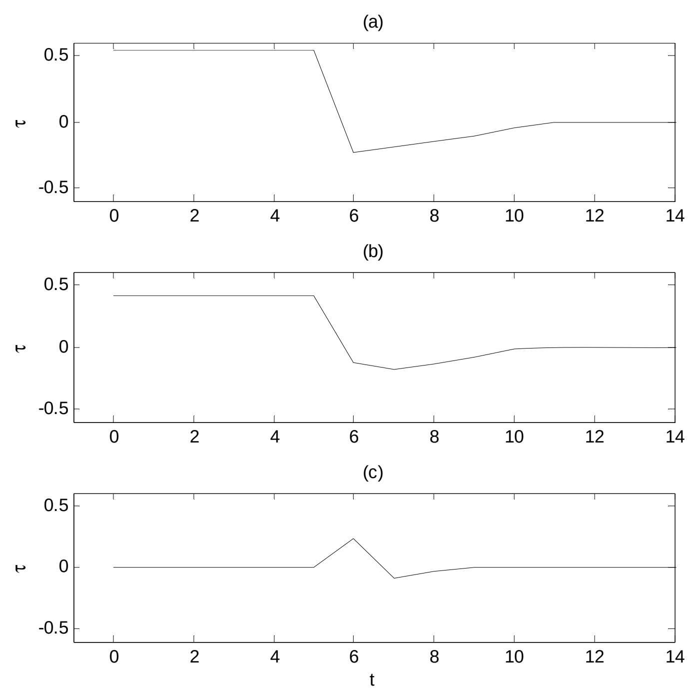

Figure 9: Implementation: Proportional Tax on Non-tradables

4 Further Considerations for Intervention

In this section we briefly discuss three important considerations for intervention: The persistence of the appreciation, distortions in consumers' perceptions, and frictions in the nontradables sector.

4.1 Appreciation persistence

In our complete markets context, persistence matters only in an ex-ante sense and is captured by the parameter δ. On one extreme, if δ is close to one (very short lived appreciations) then the losses to be financed are not much and entrepreneurs' internal resources may suffice. On the other extreme, if δ is very close to zero (very persistent appreciations), then the option value of keeping units is low, and there is no reason to protect the export sector either. It is for intermediate δ's that policy intervention may be needed.

Figure 10 illustrates this non-monotonicity by showing the region where policy intervention is called for in the $(1/\delta, a_0)$ space. The shaded region corresponds to the case where the equilibrium is constrained inefficient and exchange rate intervention is warranted. Note that there are many general equilibrium effects hidden in this figure. For example, as δ changes, so does κ. Also, when δ rises, firms reluctance to destroy during the appreciation rises. This reluctance exacerbates the (now shorter lived) appreciation, and hence the resources required to survive each period of appreciation. However, none of these additional effects is strong enough to change the qualitative shape of the figure and the conclusion that follows from it. Medium run appreciations are most likely to justify intervention.[16]

4.2 Consumers' overoptimism and incomplete insurance

In reality, consumption binges rarely occur by themselves. In the international context, they often come as a response to a rise in national income due to a positive terms of trade shock in commodity producing countries, or due to a large increase in the supply of capital flows to the country. Adding external income shocks directly onto our complete markets, rational representative agent setup, would have no impact on consumption behavior. We need to add some "friction" on the consumption side as well.

[16] In an earlier draft we relaxed the complete markets assumption and studied the polar opposite case, where export firms only have access to a riskless bond. In this context, the export sector resources dwindle as the appreciation progresses. The main policy implication that follows from this modification is the timing of the exchange rate stabilization in the appreciation phase. Early on in the appreciation, the optimal policy is to postpone much of the intervention to the D phase. However, as the appreciation continues and the export sector's resources dwindle, the optimal policy shifts a larger share of the intervention to the appreciation phase (essentially, this amounts to a gradual leftward movement in Figure 8 .)

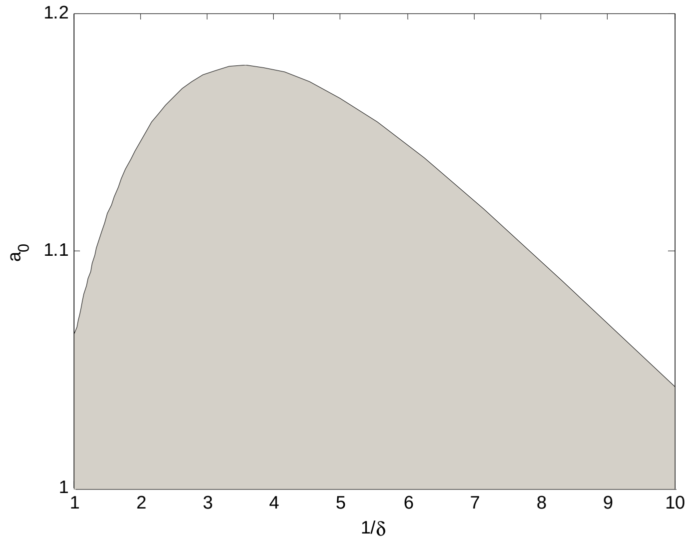

Figure 10: Expected Duration and Policy Intervention

One extension along these lines is to replace the taste shocks for income (terms of trade) shocks, but assume that either foreigners charge an insurance premium to consumers, or that the latter are overoptimistic with respect to the expected duration of the high income phase A:[17]

$$\delta^{cons} < \delta.$$

In either case, consumers find D-insurance too expensive and choose not to insure fully. The analysis is more complex in this case since incomplete insurance introduces wealth and price dynamics within the appreciation phase, however the important point for us is that the basic structure of our environment is preserved. In particular, nontradables demand drops at the time of the switch and consumers still ignore the pecuniary externality associated to their high expenditure during the appreciation. Of course, if the social planner does not share in the consumers' optimism, then it would be justified to implement some sort of saving policy, with the goal of reducing not only c_A^N but also c_A^T. More importantly, even if the planner *does share* consumers' view on the expected

[17] Alternatively, we could introduce procyclical consumption (or short horizons) through non-representative agents. The extreme version of this formulation is one where consumers live for only one period and must consume their income in that period. The social planner Pareto-weighs a generation t periods from the current one by β^t. If no intergenerational transfer mechanism other than through the real exchange rate is available, then we are again in the situation just described. The constrained goal of the social planner is to reallocate consumption away from nontradables during the appreciation phase. Relative to the pure taste shock scenario, a larger share of the adjustment is done in the A phase, in order to reduce the burden of the adjustment on the first generation in the D phase.

duration of the appreciation, there is a role for intervention to offset the pecuniary externality, as in our main case.

4.3 Rigidities in the nontradables sector

Frictions in the nontradable sector generally shift a share of the intervention toward the A phase. For example, this is typically the case in the sudden stops literature, particularly when liabilities are dollarized. The latter limits the possibility and desirability of implementing a large overshooting in D, even if short lived.

Another example is the presence of a real wage rigidity, either as the result of a distortion or of a reservation wage.[18] Yet another is that some of the inputs of production in the nontradables sector are tradables.

Let us develop the simplest of these examples and assume that workers have a reservation wage of w units of tradable goods, which is not binding except, possibly, during the overshooting phase. Suppose that this reservation wage is binding for the optimal policy but not for the competitive equilibrium. That is, in the over-overshooting scenario the social planner would like to bring $p_{D,0}^{sb}$ below w, but it cannot. What is the impact of this binding constraint on the optimal policy? In particular, how much of the intervention is reallocated to the appreciation phase? Let us return to the complete markets environment to answer the latter question. We know that in this context the social planner's first order condition in the A phase is:

$$\theta_A u'\left(1-n_A\right) = \lambda p_A^{fb} = \lambda\left(1+\beta\delta f\right). \tag{27}$$

It follows immediately that

$$p_A^{sb,w} < p_A^{sb}$$

where $p_A^{sb,w}$ and p_A^{sb} stand for the second best real exchange rate during the appreciation with and without a reservation wages w, respectively. The reason for the inequality is that the binding constraint must necessarily lower κ relative to the unconstrained case, and this implies that $\lambda = u'(\theta_A\kappa)$ rises with the constraint. In turn, the latter implies that n_A increases, which given the firms financial constraint can only be achieved with a larger intervention that drops the real exchange rate below that of the unconstrained case.

5 Final Remarks

This paper shows how financial frictions lend support to the view that persistent appreciations may justify intervention, even if agents are fully rational and forward looking. The reason for

[18] See Blanchard (2006b) for a more thorough discussion of wage rigidities and appreciations; and Blanchard (2006a) for an application to the case of Portugal.

the intervention is not to improve the health of the export sector per se, as our social planner is primarily concerned about consumers (workers), but a pecuniary externality within consumers. By putting excessive cost pressure on financially constrained export firms during the appreciation phase, consumers reduce these firms' ability to recover once the factors behind the appreciation subside. The result is a severe overshooting and real wage collapse at that stage, which hurts consumers more than they gain from the extra consumption during the appreciation.

Our normative framework sheds light on the perennial policy problem of when to intervene. We show that, other things equal, if financial constraints on the export sector are tight during the appreciation phase, then it is optimal to intervene ex-ante. Conversely, if the export sector has substantial financial resources (although not enough to fully finance the recovery), then ex-ante intervention is either costly or ineffective, and it is optimal to postpone intervention until after the "crash."

In abstract, the optimal policy can be implemented through an appropriate sequence of taxes and subsidies on nontradable consumption. In reality, the flexibility of such policies is limited, leaving to expenditure policy and central bank's reserves management most of the burden. While these are not perfect substitutes for taxes and subsidies, much of our insights still carry over to them.

Let us conclude with a few clarifying remarks and extensions. When thinking about policy, it is worth noting the distinction between an "appreciation" and an "overvaluation." The latter is an elusive concept in practice but it has a well defined meaning in ours: an overvaluation refers to a situation where the exchange rate is higher than it is socially optimal. However, this gap is not limited to an appreciation episode as it can also take place during a depreciation phase. The over-overshooting result is an example of an overvaluation during the depreciation phase. A wage rigidity is an example of when such overvaluation cannot be cured fully with intervention within the depreciation phase. The latter example also illustrates the role of early intervention in limiting *future* overvaluations.

We note that our model uses a single reason —a financial friction— for constrained production in the appreciation and depreciation phases. However, some of our conclusions extend to other scenarios as well. In particular, we could replace the financial constraint in the depreciation phase for a technological time to build assumption. In such case, the overshooting is also directly linked to excessive export destruction in the appreciation phase and there is a reason for intervention. The main difference in this instance is that the optimal policy does not prolong the intervention into the depreciation phase.

Finally, while our analysis focuses on the real exchange rate, it seems suitable for other important relative prices within an economy. For example, a real estate boom can have important cost consequences for sectors that compete with the construction sector for inputs and factors of production. More broadly, ours is a model of the optimal management of sectoral reallocation in the presence of temporary (but persistent) shocks, when some sectors have limited financial and technological flexibility.

6 Appendix

6.1 The entrepreneur's problem in recursive form

Let $V(a, n^-; s^t)$ denote the expected utility of an entrepreneur in state s^t who is holding a units of cash and n^- production units. Suppose the price sequences $\{p(s^t), q(s^t)\}$ are such that this expected utility is finite for each $a \geq 0$ and $n^- \geq 0$. Then $V(a, n^-; s^t)$ satisfies the Bellman equation:

$$\begin{aligned} V\left(a, n^-; s^t\right) &= \max_{c^{T,e}, n, \{a(s_{t+1})\}_{s_{t+1}\in S}} c^{T,e} + \beta \sum_{s_{t+1}\in S} \pi\left(s_{t+1}|s_t\right) V\left(a\left(s_{t+1}\right), n; \left\langle s^t, s_{t+1}\right\rangle\right) \quad (28) \\ s.t. \quad & c^{T,e} + q\left(s^t\right) n + \beta \sum_{s_{t+1}\in S} \pi\left(s_{t+1}|s_t\right) a(s_{t+1}) \leq (1 - p\left(s^t\right))n + a + q\left(s^t\right) n^- \\ & c^{T,e} \geq 0, \ n \geq 0, \ a\left(s_{t+1}\right) \geq 0. \end{aligned}$$

It is straightforward to setup the entrepreneur's problem in sequential form and argue directly that $V(a, n^-; s^t)$ is linear in a and n^-. Moreover, a and n^- only appear in the flow of funds constraint at history s^t, in the expression $a + q(s^t) n^-$. It follows that the value function $V(a, n^-; s^t)$ takes the form (8). The first order conditions in the text can be derived from problem (28), using the form (8) for $V(a, n^-; s^t)$ and the envelope theorem. Notice that the envelope theorem implies that the Lagrange multiplier on the budget constraint is equal to $\phi(s^t)$.

6.2 Proof of Proposition 1

The cutoff is given by

$$\hat{a}^{fb} = \frac{\left(p_A^{fb} - 1\right) n_A^{fb} + \beta\delta f \left(n_D^{fb} - n_A^{fb}\right) - \beta\delta \left(1 - p_D^{fb}\right) n_D^{fb}}{1 - \beta(1 - \delta)},$$

where $p_A^{fb}, p_D^{fb}, n_A^{fb}$, and n_D^{fb} are defined in the text. Let us conjecture and verify that if $a_0 \geq \hat{a}^{fb}$ these prices and quantities form an equilibrium. Given the conjectured prices it is possible to show that $V(a, n^-; s^t) = a + q(s^t) n^-$ (i.e., $\phi(s^t) = 1$). Then, inspecting the optimization problem (28) for each s^t shows that the entrepreneur is indifferent among all feasible choices of $c^{T,e}, n$, and $\{a(s_{t+1})\}_{s_{t+1}\in S}$. If the entrepreneur begins with a_0, he can consume the difference $a_0 - \hat{a}^{fb}$ and then adopt the following rule: set $n(s^t) = n_A^{fb}$, set $a(\langle s^t, A\rangle) = f(n_D^{fb} - n_A^{fb}) - (1 - p_D^{fb})n_D^{fb}$ and $a(\langle s^t, D\rangle) = \hat{a}^{fb}$, for each history $s^t = \{A, A, ..., A\}$; set $n(s^t) = n_D^{fb}, a(\langle s^t, D\rangle) = 0$ for each history $s^t = \{A, ...A, D, ..., D\}$. These decisions are consistent with labor market clearing. One final check, which we left aside in the main text, is that $n_A^{fb} > 0$. This follows from substituting κ^{fb} in the definition of n_A^{fb} and using the inequalities $1 - \beta(1-\delta) < 1$, $\theta_A / ((1 - \beta)\theta_A + \delta\beta) < 1$, and $1/(1 + \delta\beta f) < 1$.

6.3 Proof of Proposition 2

First, we establish a preliminary lemma.

Lemma 1 *Define the function*

$$H(n) \equiv fn - \left(1 - \frac{\kappa}{1 - n}\right) n - x,$$

the equation $H(n) = 0$ has a unique solution $n^* \in (0,1)$, for each $\kappa > 0$ and $x > 0$. Moreover, $H(n) > 0$ for each $n > n^*$. The solution n^* is increasing in x. If $x = 0$ the equation can have one or two solutions, one of which is 0. In this case, the properties above apply to the largest solution.

Proof. A solution exists because H is continuous in $[0,1)$, $H(0) = -x$ and $\lim_{n \to 1} H(n) = \infty$. Consider the case $x > 0$. Let n^* be a solution, then $f - (1 - \kappa/(1-n^*)) > 0$ must hold. If $n > n^*$, $H'(n) = f - (1 - \kappa/(1-n)) + \kappa n/(1-n)^2 > 0$ follows from $f - (1-\kappa/(1-n)) > f - (1-\kappa/(1-n^*)) > 0$. This implies that $H(n) > 0$ for each $n > n^*$, and the solution is unique. The comparative statics result with respect to x follows from the implicit function theorem. When $x = 0$ the solution $n^* = 0$ is trivial. If there is another solution $n^* > 0$, the properties stated can be proved following the steps of the case $x > 0$. ■

The proof will proceed in three steps. First, we define a map T for the coefficient κ. Second, we derive some properties of this map. Finally, we show that this map has a unique fixed point. From this fixed point we can construct an equilibrium with the desired properties.

Step 1. Fix a value for $\kappa \in \left[0, \kappa^{fb}\right]$ and construct an equilibrium as follows.

Phase A. If

$$(1 - \beta(1-\delta)) a_0 > \left(p_A^{fb} - 1\right)\left(1 - \frac{\kappa\theta_A}{p_A^{fb}}\right), \tag{29}$$

then set p_A equal to p_A^{fb}, set $n_A = \hat{n}_A = \left(1 - \frac{\kappa\theta_A}{p_A^{fb}}\right)$ and

$$a_{D,0} = \frac{1}{\beta\delta}\left[(1-\beta(1-\delta)) a_0 - \left(p_A^{fb} - 1\right) n_A\right] > 0. \tag{30}$$

Notice that $\hat{n}_A > 0$. Since $\kappa \leq \kappa^{fb}$ we have $1 - \frac{\kappa\theta_A}{p_A^{fb}} \geq 1 - \frac{\kappa^{fb}\theta_A}{p_A^{fb}} > 0$, where the last inequality follows from assumption (A1).

If (29) does not hold, then set p_A equal to the solution of

$$(1-\beta(1-\delta)) a_0 = (p_A - 1)\left(1 - \frac{\kappa\theta_A}{p_A}\right), \tag{31}$$

(which has a unique solution in $[1, p_A^{fb}]$), set $n_A = \left(1 - \frac{\kappa\theta_A}{p_A}\right)$, and set

$$a_{D,0} = 0.$$

Notice that when $p_A = \kappa\theta_A$, the RHS of (31) is zero, therefore $p_A \in \left[\kappa\theta_A, p_A^{fb}\right]$ and $n_A \geq 0$.

Phase D. Define

$$\hat{n}_D = 1 - \frac{\kappa}{p_D^{fb}}.$$

Construct the sequence $\{n_{D,j}\}$ that satisfies:

$$f(n_{D,0} - n_A) = \left(1 - \frac{\kappa}{1 - n_{D,0}}\right) n_{D,0} + a_{D,0} \tag{32}$$

$$f(n_{D,j} - n_{D,j-1}) = \left(1 - \frac{\kappa}{1 - n_{D,j}}\right) n_{D,j} \text{ for } j = 1, 2, ..., J \tag{33}$$

until $n_{D,J+1}$ is larger than $\hat{n}_D$. From then on set:

$$n_{D,j} = \hat{n}_D \text{ for all } j > J.$$

Letting $x = a_{D,0} + fn_A$, Lemma 1 ensures that (32) has a solution for $n_{D,0}$ (if $a_{D,0} + fn_A = 0$, pick the solution with the largest $n_{D,0}$). To show that $n_{D,0} \geq n_A$ consider the following: Either $H(\hat{n}_D) \leq 0$, and the solution will be larger than $\hat{n}_D$. In this case the economy converges to $\hat{n}_D$ immediately and $\hat{n}_D \geq \hat{n}_A \geq n_A$ from assumption (A2). If, instead $H(\hat{n}_D) > 0$ then $H(n_{D,0}) = 0$. Notice that

$$H(n_A) = \left(\frac{\kappa}{1 - n_A} - 1\right) n_A - a_{D,0} \leq \left(p_D^{fb} - 1\right) n_A - a_{D,0} < 0$$

where the first inequality follows from the following chain of inequalities

$$\frac{\kappa}{1 - n_A} = \frac{\kappa \theta_A}{1 - n_A} \frac{1}{\theta_A} < p_A^{fb} \frac{1}{\theta_A} < p_D^{fb} < 1,$$

(the second inequality in the chain follows from assumption (A2)). Therefore, Lemma 1 implies that $n_{D,0} > n_A$. In a similar way, it is possible to prove that (33) implies $n_{D,j} \geq n_{D,j-1}$ for each j.

From these two steps we obtain a sequence $p_A, \{p_{D,j}\}$, which can then be substituted in expression (3), to obtain κ'. This defines a map $T : \left[0, \kappa^{fb}\right] \to \left[0, \kappa^{fb}\right]$.

Step 2. It can be shown that the map T is continuous. Furthermore, let us prove that

$$T(\kappa(1 + \Delta)) < (1 + \Delta) T(\kappa).$$

In the construction in Step 1, an increase in κ leads to a (weak) reduction in n_A and $n_{D,j}$ for all j (for the initial conditions of phase D notice that if (29) is satisfied, then, using the definition of p_A^{fb}, it is possible to show that $a_{D,0} + fn_A$ is independent of κ; if (29) is not satisfied, then an increase in κ leads to a decrease in n_A). But since $n_A = 1 - \theta_A \kappa / p_A$, $n_{D,j} = 1 - \kappa / p_{D,j}$, this implies that the prices p_A and $p_{D,j}$ must increase less than proportionally than κ. Therefore, κ' increases less than proportionally.

Step 3. Define the following map for $z \equiv \log(\kappa)$:

$$z' = \tilde{T}(z) \equiv \log(T(\exp^z)).$$

Step 2 shows that this map is continuous and has slope smaller than 1. Therefore this map has a unique fixed point (uniqueness is not needed for the statement of this proposition, but will be useful for the following results). Let κ be the fixed point and consider the prices and quantities constructed in Step 1. To ensure that they are an equilibrium, it remains to check that the sequence of prices and quantities are optimal for the entrepreneur. Derive the marginal utility of money at $t_{D,0}$ from the recursion:

$$\phi_{D,j} = \beta \frac{f}{f - (1 - p_{D,j})} \phi_{D,j+1}. \tag{34}$$

By construction we have $p_{D,j} \leq p_D^{fb}$, which implies that $\phi_{D,j} \geq 1$. Moreover, entrepreneurs' consumption and cash savings are zero until the point where $\phi_{D,j} = 1$. To check optimality in phase A, notice that

$$\phi_A = \frac{\beta \delta f}{p_A - 1} \phi_{D,0}$$

and $\phi_A > \phi_{D,0}$ iff $p_A < p_A^{fb}$.

Proof of Propositions 3 and 4

The following lemma provides a useful preliminary result.

Lemma 2 *The equilibrium value of κ is non-decreasing in a_0.*

Proof. Let $T(\kappa; a_0)$ be the mapping $\left[0, \kappa^{fb}\right] \rightarrow \left[0, \kappa^{fb}\right]$ defined in the proof of Proposition 2, indexed by the initial wealth a_0. Choose two values $a_0' < a_0''$. Let κ' and κ'' be the corresponding equilibrium values of κ. Now, fixing κ' we want to show that T is monotone in a_0, i.e., $T(\kappa'; a_0'') \geq T(\kappa'; a_0')$.

If (29) holds at a_0', then an increase in a_0 leaves p_A unchanged, and it increases $a_{D,0}$ (from 30) and leaves n_A unchanged. If (29) does not hold, an increase in $a_{D,0}$ leads to an increase in p_A, and an increase in $a_{D,0} + fn_A$, since $a_{D,0}$ either remains zero or becomes positive and n_A increases. In both cases, $a_{D,0} + fn_A$ increases. This means that, in phase D, there will be a (weak) increase in $n_{D,j}$ for all j, and, thus, a (weak) increase in $p_{D,j}$ for all j. Therefore, $T(\kappa'; a_0'') \geq T(\kappa'; a_0') = \kappa'$.

This implies that $T(\kappa; a_0'')$ has a fixed point in $[\kappa', \kappa^{fb}]$. Since T has a unique fixed point and $T(\kappa''; a_0'') = \kappa''$, by construction, this implies $\kappa'' \geq \kappa'$. ■

Now we can prove the two propositions. Consider first Proposition 3. Suppose that at a_0' we have $p_A = p_A^{fb}$ in equilibrium. This means that (29) holds at a_0'. Since $\kappa'' \geq \kappa'$, (29) holds *a fortiori* for $a_{D,0}'', \kappa''$, it follows that at the new equilibrium $p_A = p_A^{fb}$ and $a_{D,0} > 0$.

Consider next Proposition 4. Suppose that at a_0' we have $p_{D,0} = p_D^{fb}$ in equilibrium. This means that the following inequality holds

$$f\hat{n}_D' \leq \left(1 - \frac{\kappa'}{1 - \hat{n}_D'}\right)\hat{n}_D' + fn_A' + a_{D,0}' \tag{35}$$

where $\hat{n}_D' = 1 - \kappa'/p_D^{fb}$.

Now we want to prove the following inequality

$$fn_A'' + a_{D,0}'' \geq fn_A' + a_{D,0}'. \tag{36}$$

If (29) holds at a_0' then some algebra (using the definition of p_A^{fb}) shows that

$$fn_A'' + a_{D,0}'' = fn_A' + a_{D,0}' = \frac{1}{\beta\delta}\left[(1 - \beta(1-\delta))\, a_0\right].$$

If (29) does not hold at a_0', then we have $a_{D,0}'' \geq 0 = a_{D,0}'$. Furthermore, we can show that $n_A'' \geq n_A'$. Notice that κ'' greater than κ' only because, on average, equilibrium prices are larger. However, $p_{D,j}' = p_D^{fb}$ at a_0', and $p_{D,j}'' \leq p_D^{fb}$. This implies that

$$\frac{p_A''}{p_A'} \geq \frac{\kappa''}{\kappa'},$$

and since $n_A = 1 - \kappa/p_A$ this implies $n_A'' \geq n_A'$. Therefore, (36) holds in all cases. This implies that (35) also holds at a_0'', and therefore $p_{D,0} = p_D^{fb}$.

Notice that if (29) holds at a_0' then, we can proceed as in the proof of Proposition 3 and show that $a_{D,0}'' > a_{D,0}'$ and $n_A'' = n_A' = \hat{n}_A$.

6.4 Setup of the optimal policy problem

The set of feasible allocations is defined as follows.

Definition 2 *(Feasibility) The allocation* $\{c^T(s^t), c^N(s^t), c^{T,e}(s^t), n(s^t)\}$ *is feasible iff there exists a sequence of tax rates* $\{\tau^T(s^t), \tau^N(s^t)\}$*, wealth levels* $\{a(s^t)\}$*, and prices* $\{p(s^t), q(s^t)\}$ *such that the prices and quantities* $\{p(s^t), q(s^t)\}$ *and* $\{c^T(s^t), c^N(s^t), c^{T,e}(s^t), n(s^t), a(s^t)\}$ *constitute a competitive equilibrium under the tax rates* $\{\tau^T(s^t), \tau^N(s^t)\}$*.*

We now derive three necessary conditions, (37) to (39) below, that any feasible allocation must satisfy. Then, we define the problem of a planner that chooses an allocation subject only to the consumer's budget constraint (21), the entrepreneur's budget constraint (7), and conditions (37)-(39). This is a relaxed version of the original planning problem, given that this set of constraints is, in general, necessary but not sufficient for feasibility. We also perform a change of variables that makes the relaxed planning problem a concave problem and we derive first-order conditions which are sufficient for an optimum. In the proof of Proposition 6 we make use of these first-order conditions to find allocations that solve the relaxed planning problem.

First, notice that the entrepreneur's optimality implies that a feasible allocation must satisfy the condition

$$\left(q\left(s^t\right) - \left(1 - p\left(s^t\right)\right)\right) n\left(s^t\right) \geq \beta \sum_{s_{t+1} \in S} \pi\left(s_{t+1}|s_t\right) q\left(\left\langle s^t, s_{t+1}\right\rangle\right) n\left(s^t\right). \tag{37}$$

To prove this inequality, multiply by $n(s^t)$ both sides of (9), and use the complementarity condition to obtain

$$\left(q\left(s^t\right) - \left(1 - p\left(s^t\right)\right)\right) \phi\left(s^t\right) n\left(s^t\right) = \beta \sum_{s_{t+1} \in S} \pi\left(s_{t+1}|s_t\right) \phi\left(\left\langle s^t, s_{t+1}\right\rangle\right) q\left(\left\langle s^t, s_{t+1}\right\rangle\right) n\left(s^t\right).$$

Moreover, the entrepreneur's optimality condition for $a(s^t)$ implies that $\phi(\langle s^t, s_{t+1}\rangle) \geq \phi(s^t)$ for all s_{t+1}. Substituting in the equation above, gives (37).

Second, recall that $q(\langle s^t, s_{t+1}\rangle) \leq f$ which implies

$$q\left(\left\langle s^t, s_{t+1}\right\rangle\right) n\left(s^t\right) \leq f n\left(s^t\right). \tag{38}$$

Finally, define the function

$$G(x) \equiv f \max\{x, 0\},$$

for a generic variable x, and notice that equilibrium in the adjustment sector implies that, for all s^t and s_{t+1},

$$q\left(\left\langle s^t, s_{t+1}\right\rangle\right)\left(n\left(\left\langle s^t, s_{t+1}\right\rangle\right) - n\left(s^t\right)\right) = G\left(n\left(\left\langle s^t, s_{t+1}\right\rangle\right) - n\left(s^t\right)\right). \tag{39}$$

We perform a change in variables, defining

$$\begin{aligned} z\left(s^t\right) &\equiv p\left(s^t\right) n\left(s^t\right) - G\left(n\left(s^t\right) - n\left(s^{t-1}\right)\right), \\ y\left(s^t\right) &\equiv q\left(s^t\right) n\left(s^{t-1}\right), \end{aligned}$$

for all consecutive histories s^t and s^{t+1}.

It is possible to write the problem in terms of the sequences $\{c^T(s^t), c^N(s^t), c^{T,e}(s^t), n(s^t), a(s^t)\}$ and $\{z(s^t), y(s^t)\}$, and show that the solution is stationary in phase A and in phase D quantities and prices only depend on the number of periods since the transition. Here, to save space, we write the problem directly in terms of variables indexed by A and (D,j). Moreover, we normalize the utility of the consumer and the budget constraint by the constant $(1-\beta(1-\delta))$. We impose market clearing by setting $c^N(s^t) = 1 - n(s^t)$. Then, the planner problem is to choose sequences for n, c^T, $c^{T,e}$, z, y that maximize the consumer's utility

$$\theta_A \left[u(1-n_A) + u\left(c_A^T\right)\right] + \delta\beta \sum_{j=0}^{\infty} \beta^j \theta_D \left[u(1-n_{D,j}) + u(c_{D,j}^T)\right],$$

subject to the consumer's budget constraints

$$\begin{aligned} c_A^T + \sum_{j=0}^{\infty} \beta^j c_{D,j}^T + \quad & G(n_A - n_{-1}) + \delta\beta G(n_{D,0} - n_A) + \delta\beta \sum_{j=1}^{\infty} \beta^j G(n_{D,j} - n_{D,j-1}) \leq \\ \leq \quad & (z_A - y_A + n_A) + \delta\beta \sum_{j=0}^{\infty} \beta^j (z_{D,j} - y_{D,j} + n_{D,j}), \qquad (\lambda) \end{aligned}$$

the entrepreneur's participation constraint

$$c_A^{T,e} + \delta\beta \sum_{j=0}^{\infty} \beta^j c_{D,j}^{T,e} \geq (1-\beta(1-\delta))U, \qquad (\mu)$$

the entrepreneur's flow of funds constraints

$$\begin{aligned} (1-\beta(1-\delta))a_0 - \beta\delta a_{D,0} + y_A - z_A - c_A^{T,e} &\geq 0, & (\nu_A) \quad (40) \\ a_{D,0} - \beta a_{D,1} + y_{D,0} - z_{D,0} - c_{D,0}^{T,e} &\geq 0, & (\delta\beta v_{D,0}) \quad (41) \\ a_{D,j} - \beta a_{D,j+1} + y_{D,j} - z_{D,j} - c_{D,j}^{T,e} &\geq 0 \quad \text{for } j \geq 1, & (\delta\beta^{j+1} v_{D,j}) \quad (42) \end{aligned}$$

and conditions (37)-(38), which take the form

$$\begin{aligned} y_A &\leq fn_A, & (\gamma_A) \\ y_{D,j} &\leq fn_{D,j} \quad \text{for all } j, & (\delta\beta^{j+1}\gamma_{D,j}) \\ z_A &\geq \delta\beta y_{D,0}, & (\eta_{D,0}) \\ z_{D,j} &\geq \beta y_{D,j+1} \quad \text{for all } j. & (\delta\beta^{j+1}\eta_{D,j+1}) \end{aligned}$$

Next to each constraint we write the respective Lagrange multiplier. Notice that the consumer's expected utility and both sides of the consumer's and of the entrepreneur's budget constraints have been normalized by the factor $(1-\beta(1-\delta))$. Condition (39) has been used to rewrite the entrepreneur's flow of funds constraints in term of z and y.

Let us take first-order conditions with respect to n

$$\begin{aligned} -\theta_A u'(1-n_A) - \delta\beta f\lambda + \lambda + f\gamma_A &= 0, \\ -\delta\beta^{j+1}\theta_D u'(1-n_{D,j}) - \delta\beta^{j+1} f\lambda + \delta\beta\beta^{j+1} f\lambda + \delta\beta^{j+1}\lambda + \delta\beta^{j+1} f\gamma_{D,j} &= 0, \end{aligned}$$

with respect to y and z

$$\begin{aligned} -\lambda + \nu_A - \gamma_A &= 0, \\ -\lambda\delta\beta^{j+1} + \delta\beta^{j+1}\nu_{D,j} - \delta\beta^{j+1}\gamma_{D,j} - \beta\delta\beta^{j}\eta_{D,j} &= 0, \\ \lambda - \nu_A + \eta_{D,0} &= 0, \\ \lambda\delta\beta^{j+1} - \delta\beta^{j+1}\nu_{D,j} + \delta\beta^{j+1}\eta_{D,j+1} &= 0, \end{aligned}$$

and with respect to a and $c^{T,e}$

$$-\nu_A + \nu_{D,0} \geq 0 \ (a_{D,0} \geq 0), \tag{43}$$

$$-\nu_{D,j} + \nu_{D,j+1} \geq 0 \ (a_{D,j} \geq 0), \tag{44}$$

$$\mu \geq \nu_A \quad (c_A^{T,e} \geq 0), \tag{45}$$

$$\mu \geq \nu_{D,j} \quad (c_{D,j}^{T,e} \geq 0). \tag{46}$$

Rearrange these conditions shows that a sufficient condition for an optimum is that there are Lagrange multipliers $\lambda \leq \nu_A \leq \nu_{D,0} \leq \nu_{D,1} \leq ... \leq \mu$, such that

$$-\theta_A u'(1 - n_A) + \lambda(1 + \delta\beta f) + f(\nu_A - \lambda) = 0, \tag{47}$$

$$-\theta_D u'(1 - n_{D,0}) + \lambda(1 - (1-\beta) f) + f(\nu_{D,0} - \nu_A) = 0, \tag{48}$$

$$-\theta_D u'(1 - n_{D,j}) + \lambda(1 - (1-\beta) f) + f(\nu_{D,j} - \nu_{D,j-1}) = 0, \tag{49}$$

and conditions (43)-(46) are satisfied.

Proof of Proposition 6

The proof is split in two steps. In the first step, we define two maps J and $\tilde{J}$. In the second step, we use these maps to construct a candidate optimal allocation and we show that this allocation is optimal.

Step 1. We define two maps J and $\tilde{J}$ that will be used in following. Define the map $J : [1+\delta\beta f/\phi_A, p_A^{fb}] \rightarrow \mathbb{R}$ as follows (notice that $1 + \delta\beta f/\phi_A < 1 + \delta\beta f = p_A^{fb}$ if $\phi_A > 1$). For any $p_A \in [1 + \delta\beta f/\phi_A, p_A^{fb}]$ find the unique ξ that solves

$$\xi = \frac{1-\beta}{(1-\beta)\theta_A + \delta\beta\theta_D}\left(p_A n_A + \delta\beta\left(p_{D,0} n_{D,0} + \sum_{j=1}^{\infty}\beta^j p_D^{fb} n_{D,j}\right)\right), \tag{50}$$

where $p_{D,0} = 1 - f + \delta\beta^2 f^2 / (\phi_A(p_A - 1))$, and the sequence $\{n_A, \{n_{D,j}\}\}$ is given by

$$n_A = \frac{1}{p_A - 1}(1 - (1-\delta)\beta) a_0, \tag{51}$$

$$n_{D,0} = \frac{1}{\delta\beta^2 f}\phi_A(1 - (1-\delta)\beta) a_0, \tag{52}$$

$$n_{D,j} = \min\left\{\beta^{-j} n_{D,0}, \bar{n}_D\right\} \text{ for } j \geq 1, \tag{53}$$

where

$$\bar{n}_D = 1 - \xi\theta_D/p_D^{fb}. \tag{54}$$

To show that such a ξ exists and is unique, notice that the right-hand side of (50) is a continuous non-increasing function of ξ, and ranges between a positive value, at $\xi = 0$, and $-\infty$ for $\xi \to \infty$. Set $J(p_A) = \xi$ (the function J is allowed to take negative values, we will see below that at the relevant values of p_A we have $J(p_A) > 0$). Combining the terms containing p_A on the right-hand side of (**??**) we obtain the expression

$$\frac{p_A}{p_A - 1}(1 - (1-\delta)\beta)a_0 + \delta\beta\left(1 - f + \frac{1}{p_A - 1}\frac{\delta\beta^2 f^2}{\overline{\phi}_A}\right)\frac{1}{\delta\beta^2 f}\phi_A(1 - (1-\delta)\beta)a_0,$$

which is monotone decreasing in p_A. Applying the implicit function theorem, it follows that $J'(p_A) < 0$.

Define the map $\tilde{J} : [0, (1-(1-\delta)\beta)a_0/(p_A^{fb}-1)]$ as follows. For any $n_A \in [0, (1-(1-\delta)\beta)a_0/(p_A^{fb}-1)]$ find the unique positive ξ that solves

$$\xi = \frac{1-\beta}{(1-\beta)\theta_A + \delta\beta\theta_D}\left(p_A^{fb} n_A + \delta\beta\left(p_{D,0}n_{D,0} + \sum_{j=1}^{\infty}\beta^j p_D^{fb} n_{D,j}\right)\right),$$

where $p_{D,0} = 1 - f + \beta f/\phi_A$, and the sequence $\{n_{D,j}\}$ is given by (52)-(53). Again, it is easy to show that such a ξ exists and is unique. Set $\tilde{J}(n_A) = \kappa$. It is immediate to show that $\tilde{J}'(n_A) > 0$.

Step 2. Define the function

$$H(p_A) \equiv 1 - \frac{\theta_A J(p_A)}{p_A^{fb}} - \frac{1}{p_A - 1}(1-(1-\delta)\beta)a_0.$$

From step 1 we know that $H(p_A)$ is an increasing function of p_A. Therefore, three mutually exclusive cases are possible. Either there exists a unique $p_A \in [1 + \delta\beta f/\phi_A, p_A^{fb}]$ that solves the equation $H(p_A) = 0$, or $H(1 + \delta\beta f/\phi_A) > 0$, or $H\left(p_A^{fb}\right) < 0$. We can construct and optimum for each of these cases. We will consider in detail the first case. Let p_A^* be such that $H(p_A^*) = 0$. Set

$$p_{D,0}^* = 1 - f + \frac{1}{\phi_A}\frac{\delta(\beta f)^2}{p_A^* - 1},$$

the assumption $\phi_A < (f-1)/\beta f$ ensures that $p_{D,0}^* > 0$, given that $p_A^* \leq p_A^{fb}$. Set $p_{D,j}^* = p_D^{fb}$ for all $j \geq 1$, $q_A^* = 0$, and $q_{D,j}^* = f$ for all $j \geq 0$. Let $\xi^* = J(p_A^*)$ and set $c_A^T = \theta_A\xi^*$ and $c_{D,j}^T = \theta_D\xi^*$. Set the sequence $\left\{n_A^*, \left\{n_{D,j}^*\right\}\right\}$ according to (51)-(53). Finally, the values for the entrepreneur's consumption are set as

$$\begin{aligned}
c_A^{T,e*} &= (1 - \beta(1-\delta))a_0 + (1 - p_A)n_A, \\
c_{D,0}^{T,e*} &= (1 - p_{D,0})n_{D,0} - f(n_{D,0} - n_A), \\
c_{D,j}^{T,e*} &= (1 - p_{D,j})n_{D,j} - f(n_{D,j} - n_{D,j-1}) \text{ for } j \geq 1.
\end{aligned}$$

Given the construction of the sequence $\left\{n_A^*, \left\{n_{D,j}^*\right\}\right\}$, entrepreneur's consumption is always non-negative.

Having defined a candidate optimal allocation, we can define the corresponding sequences for $y_A^*, \left\{y_{D,j}^*\right\}$ and $z_A^*, \left\{z_{D,j}^*\right\}$, and show that we have found an optimum for the relaxed problem defined in 6.4. To do so, we need to find Lagrange multipliers $\lambda^*, \nu_A^*, \{\nu_{D,j}^*\}$, and μ^* such that conditions (47)-(**??**) are satisfied. Set $\lambda^* = 1/\xi^*$. Notice that the condition $H(p_A^*) = 0$ can be re-arranged to give

$$\lambda^* p_A^* = \theta_A u' (1 - n_A^*) . \tag{55}$$

Moreover, by construction

$$n_{D,j}^* \leq 1 - \xi^* \theta_D / p_D^{fb} \text{ for all } j,$$

with equality for j greater or equal than some J^*. This implies that

$$\lambda^* p_D^{fb} \leq \theta_A u' \left(1 - n_{D,j}^*\right) \text{ for all } j,$$

with equality for $j \geq J^*$. Then we can set $\nu_A^* = \lambda$,

$$\begin{aligned} \nu_{D,0}^* &= \nu_A + \frac{1}{f}\left(\theta_D u' \left(1 - n_{D,0}^*\right) - \lambda^* p_D^{fb}\right), \\ \nu_{D,j}^* &= \nu_{D,j-1} + \frac{1}{f}\left(\theta_D u' \left(1 - n_{D,j}^*\right) - \lambda^* p_D^{fb}\right). \end{aligned}$$

By construction $\nu_{D,j}^*$ will be constant for $j \geq J^*$ and we can set $\mu^* = \nu_{D,J^*}^*$. This confirms that (**??**) is satisfied for all j and we can check that $c_{D,j}^{T,e*} > 0$ only for $j \geq J^*$, i.e., when $\nu_{D,j}^* = \mu^*$.

Furthermore, we can check that the proposed allocation satisfies the consumer's budget constraint and the entrepreneur's participation constraint. The consumer's budget constraint can be rewritten as

$$c_A^{T*} + \sum_{j=0}^{\infty} \beta^j c_{D,j}^{T*} \leq p_A^* n_A^* + \delta\beta \sum_{j=0}^{\infty} \beta^j p_{D,j}^* n_{D,j}^*,$$

the construction of the functions H and J (in particular equation (50)) guarantees that this condition holds as an equality. Some lengthy but straightforward algebra, using the flow of funds constraints, shows that

$$\frac{1}{1 - \beta(1-\delta)} \left(c_A^{T,e*} + \delta\beta \sum_{j=0}^{\infty} \beta^j c_{D,j}^{T,e*} \right) = \frac{\beta\delta f}{p_A^* - 1} \prod_{j=0}^{\infty} \frac{\beta f}{f - \left(1 - p_{D,j}^*\right)} a_0.$$

Given the prices p_A^* and $\left\{p_{D,j}^*\right\}$, the right-hand side of this equation is equal to $\phi_A a_0$, which is equal to the entrepreneur's expected utility U in the competitive equilibrium. This completes the argument that the candidate allocation solves the relaxed planning problem. It remains to show that this allocation is feasible. To do so, we first derive values for the ϕ_A^* and $\phi_{D,j}^*$. We set $\phi_A^* = \phi_A$,

$$\phi_{D,0}^* = \prod_{j=0}^{\infty} \frac{\beta f}{f - \left(1 - p_{D,j}^*\right)},$$

and $\phi_{D,j}^* = 1$ for all $j \geq 1$. These, can be used to check that entrepreneur's behavior is optimal, i.e. that (10) and (11) are satisfied. To check these conditions notice that $\phi_A^* \geq \phi_{D,0}^* \geq \phi_{D,1}^*$ and $\phi_{D,j}^* = 1$ for all j, while $c_A^{T,e*} = c_{D,0}^{T,e*} = 0$ and $a_{D,j}^* = 0$ for all j. Finally, the tax rates are set as follows: $\tau_A^{T*} = \tau_{D,j}^{T*} = 0$ for all j and τ_A^{N*} and $\left\{\tau_{D,j}^{N*}\right\}$ are such that

$$\begin{aligned} \theta_A u' (1 - n_A^*) &= \lambda^* p_A^* (1 - \tau_A^*), \\ \theta_D u' \left(1 - n_{D,j}^*\right) &= \lambda^* p_{D,j}^* \left(1 - \tau_{D,j}^*\right). \end{aligned}$$

Let us discuss briefly the cases where $H\left(1+\delta\beta f/\phi_A\right) > 0$ and $H\left(p_A^{fb}\right) < 0$. In the first case, we have that condition (55) now holds as an inequality, and we have $\nu_A^* > \lambda^*$. The rest of the construction is analogous to the one derived above. In the second case, we make use of the function $\tilde{J}$ to find the value of ξ^*. In particular, define the function

$$\tilde{H}\left(n_A\right) \equiv 1 - \frac{\theta_A \tilde{J}\left(n_A\right)}{p_A^{fb}} - \frac{1}{p_A^{fb}-1}\left(1-\left(1-\delta\right)\beta\right)a_0,$$

and find an n_A^* such that $\tilde{H}\left(n_A^*\right) = 0$. Then, set $\xi^* = \tilde{J}\left(n_A^*\right)$ and $\lambda^* = 1/\xi^*$. In this case we have $n_A^* < \left(1-\left(1-\delta\right)\beta\right)a_0/(p_A^{fb}-1)$, so the entrepreneurs have positive financial savings when they enter phase D,

$$a_{D,0}^* = \left(1-\left(1-\delta\right)\beta\right)a_0 - (p_A^{fb}-1)n_A^*.$$

This is consistent with feasibility, given that $p_A = p_A^{fb}$, so that $\phi_A^* = \phi_{D,0}^*$ (which implies that (11) is satisfied with $a_{D,0}^* > 0$). The rest of the proof proceeds as in the case discussed above.

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