# Fiscal Discretion Destroys Monetary Commitment

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#### Abstract

We consider the interaction between the monetary policy of a common central bank and the fiscal policies of the countries in a monetary union. We develop a model of the Barro-Gordon type extended to many countries and to allow for fiscal as well as monetary policy; the monetary and fiscal authorities may have different output-inflation ideal outcomes and tradeoffs. Each country's fiscal policy affects its own output and inflicts positive or negative externalities on the other countries. Monetary policy has its own time-consistency problem. We find the general nonlinear optimal monetary rule that allows for general nonadditive stochastic shocks. Then, we show that, as long as fiscal policy is discretionary, commitment to the optimal monetary rule by the central bank and monetary leadership when monetary policy is discretionary deliver exactly the same equilibrium. Hence, commitment to a monetary rule does not reduce inflation more than monetary leadership with discretion already does. We also show that, when the central bank is more conservative than the fiscal authorities, the Nash equilibrium is characterized by an excessive race between expansionary fiscal and contractionary monetary policies.

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## 1 Introduction

The Economic and Monetary Union of Europe (EMU) has a common central bank and decentralized fiscal authorities that decide fiscal policy in each country of the union. It is generally believed that a monetary union with decentralized and uncoordinated fiscal decisions produces high inflation and excessive deficits.

When a fiscal authority decides its own policy, it does not take into account the effects of its policy on the other members of the union. In other words, it ignores the externalities, which are the effects its own fiscal policy inflicts on others. For instance, excessive budget deficits in one country could lead to a solvency crisis and force either the other members into a fiscal bailout or the common central bank to finance the deficits by printing money. Even in the absence of a fiscal crisis, a large deficit in a country, for instance Italy, may raise the interest rate in the EMU area and the common inflation rate (perhaps because of monetary accommodation by the common central bank); it may also affect output in the rest of the union via its effect on the Italian demand for goods produced in the EMU. Because countries ignore the externalities of their own fiscal policies on others, there will be more deficits than optimal in a monetary union if such externalities are negative (as for interest rate and inflation) and too little deficits if such externalities are positive (as for demand). This would provide the case for fiscal coordination at the monetary union level.

Monetary policy has its time-consistency problem. In a world where prices do not adjust instantaneously (perhaps because nominal contracts are set before they come into effect), monetary expansions can raise output and the monetary authority is tempted to raise employment above its natural rate. But price setters anticipate the central bank's incentives, thereby eliminating any effect of anticipated monetary policy on output. With several countries facing asymmetric shocks, this problem can be aggravated in a monetary union as at any time, one country or another may suffer a bad supply shock and therefore want the common central bank to expand employment using surprise inflation.

To guard against such problems, fiscal limits have been introduced in the EMU. Since it is believed that the negative fiscal externalities dominate the positive ones, the Maastricht Treaty and the subsequent Pact for Stability and Growth stipulate that each country's fiscal deficit in each year should not exceed 3 percent of its GDP, unless the country is in a recession.

As for monetary policy, the Maastricht Treaty has gone a long way to ensure that the European Central Bank's (ECB's) primary objective is the maintenance of price stability and it is politically independent from the member governments. In fact, the treaty forbids the common central bank from purchasing any member country's debt instruments, even in a fiscal crisis. Increasingly, the use of a monetary rule has been suggested to avoid any accommodation of the members' fiscal profligacy. The new feature of the EMU is the interaction between a centralized monetary policy and decentralized and possibly non-cooperative fiscal policies. Dixit and Lambertini (1999) develop a model of the Barro-Gordon type extended to allow for fiscal as well as monetary policy and for many countries. They study the interaction between a common central bank and decentralized fiscal policy when the monetary and fiscal authorities have the same ideal outcome for output and inflation. In this case, the ideal outcome or first best can be achieved despite the inevitability of some ex post monetary accommodation to fiscal expansion, without the need for fiscal coordination, without the need for monetary commitment, without the need for a conservative central bank and irrespective of which authority moves first. Hence, if the monetary and fiscal authorities agree on the output-inflation goals, fiscal limits are simply unnecessary.

This paper generalizes the model of Dixit and Lambertini (1999) to allow for the monetary and fiscal authorities to have different ideal outcomes. We have in mind a situation where the central bank is more conservative than the fiscal authorities in the sense that it prefers lower inflation and lower output. However, the algebra of the model goes through perfectly well without this assumption. We find the general nonlinear optimal monetary rule that allows for general nonadditive stochastic shocks. Then, we show that fiscal discretion makes monetary commitment unnecessary, provided the monetary authority has the first-mover advantage. As long as fiscal policy is discretionary, commitment even to the optimal monetary rule by the central bank and monetary leadership when monetary policy is discretionary deliver exactly the same equilibrium. Hence, commitment to a monetary rule does not reduce inflation more than monetary leadership with discretion already does.

We also show that, when the central bank is more conservative than the fiscal authorities, the Nash equilibrium may be characterized by low inflation, lower than desired by the central bank, and high output, higher than desired by the fiscal authorities. In fact, the authorities engage in an excessive race between expansionary fiscal and contractionary monetary policies in order to obtain their desired goals.

The paper is organized as follows. Section 2 reviews the literature. Section 3 presents the one-country model; Section 4, 5 and 6 analyze the Nash, monetary leadership and fiscal leadership equilibria, respectively, under discretion. Section 7 studies commitment to a monetary rule and shows the equivalence with the monetary leadership equilibrium. Section 8 extends the model to n countries in a monetary union. Section 9 concludes.

## 2 Literature Review

There have been several studies of monetary-fiscal interaction in a monetary union. Most of them consider the purpose of fiscal policy to be the provision of public goods; for example Sibert (1992), Levine and Brociner (1994), Beetsma and Bovenberg (1998). However, much of the debate about the fear of excessive fiscal expansion has been about the countercyclical role of fiscal policy; that is of special interest in the EMU since unemployment has been the most pressing problem in many of its member countries for the last few years and is likely to remain so for the next few. Therefore in this paper we focus on this aspect.

Other works have studied the desirability of fiscal constraints within a monetary union. The Pact for Stability and Growth sets limits on the debt and on general government deficit ratios for the EMU members and it provides for penalties for the countries that exceed such limits. On one hand, Chari and Kehoe (1998) and Dornbusch (1997) argue that fiscal constraints are not necessary, and possibly harmful, when the monetary authority can commit its policies; on the other hand, Beetsma and Bovenberg (1995) and Beetsma and Uhlig (1997) argue that fiscal constraints improve welfare because they correct the debt bias stemming from government myopia. We offer a theoretical argument why fiscal constraints may be redundant independently of whether the monetary authority is ultraconservative or can commit its policies ex ante.

The existing studies on the welfare effects of fiscal coordination within a monetary union generate conflicting results. Fiscal coordination is beneficial when there is a freerider problem that results in too much debt being issued, as in Chari and Kehoe (1998) and Huber (1998). In the model of Beetsma and Bovenberg (1998), however, fiscal cooperation harms welfare when it is set before monetary policy because it enhances the strategic position of fiscal authorities vis-a-vis the monetary authority. Our paper contributes to this policy debate on the need to coordinate fiscal policies in the EMU by showing that non-cooperative fiscal stabilization policy is not an obstacle to the achievement of the desired policy goals.

Canzoneri (1985) presents a Barro-Gordon stochastic setting (without fiscal policy) where the central bank has private information in the sense that private agents cannot observe, or cannot reconstruct, the actual stochastic shocks; Cukierman and Meltzer (1986) have a model where the central bank's preferences on inflation and output shift stochastically through time and are not known to private agents. With asymmetric information, the central bank has an incentive to misrepresent its information in an effort to expand output, thereby generating inflation. Our model is one of symmetric information: the realization of the stochastic shocks is not observed by private agents before inflationary expectations are set, but it is perfectly observed afterward. Full transparency of the central bank's intentions delivers the first-best in our model and is therefore beneficial.

To illustrate the interaction between monetary and fiscal policy in a monetary union, we consider a simple model of the Barro-Gordon type with an expectationsaugmented Phillips curve. Woodford (1998) shows that an expectation-augmented Phillips curve can be derived as log-linear approximation to the equilibrium conditions of a general equilibrium model with sticky prices. Along those lines, the model in the appendix derives a log-linear approximation to the the conditions used in section 3.

## 3 The Model

We consider a reduced-form model of the Barro-Gordon (1983) type, enlarged to allow for fiscal as well as monetary policy. We start considering the one-country case. Then, we enlarge it n countries belonging to a monetary union.

There are two authorities in the country: the central bank and the fiscal authority. The central bank chooses a policy variable  $\pi_0$  and the fiscal authority chooses a policy variable x. These policies affect the GDP level y and the inflation level  $\pi$  in the country. Let  $\pi^e$  denote the private sector's rational expectation of  $\pi$ . There are stochastic shocks in the economy, whose state vector of realization is

$$z = (\overline{y}, a, b, c).$$

The GDP level in the country is given by

$$y(z) = \overline{y} + a \ x(z) + b \ (\pi(z) - \pi^e), \tag{1}$$

where  $\overline{y}$  is the (stochastic) natural rate of output. The scalar a is the effect of fiscal policy on GDP. We think that there is some slack in the economy so that a fiscal expansion (an increase in x) has an expansionary effect on GDP. Hence a > 0. But the algebra of the model works perfectly well without this assumption, thus permitting positive or negative externalities. Positive externalities may arise because a fiscal expansion raises the demand for the good produced in the country; negative externalities may arise because, behind the facade of our reduced-form model, high interest rates crowd out investment and raise interest payments on the outstanding stock of debt. All we require is that  $a \neq 0$ . The last term on the right-hand side of equation (1) is the usual supply effect of surprise inflation.

Inflation is given by

$$\pi(z) = \pi_0(z) + c \ x(z).$$
(2)

The interpretation of equation (2) is that the central bank chooses  $\pi_0$ , which is the controlled part of monetary policy or its initial stance. Fiscal policy affects inflation or, alternatively, some ex-post accommodation of fiscal policy must be made; the impact of fiscal policy on inflation is summarized by the parameter c. We only require the parameter  $c \neq 0$ . Appendix A presents a micro-founded model whose log-linearization delivers equations (1) and (2). Since (1) and (2) are linearized functional forms, our analysis and its predictions are valid as long as the economy is in the neighborhood of the steady state.

The natural rate of output  $\overline{y}$ , the scalar parameter *a* summarizing the fiscal policy effect on GDP, the scalar parameter *b* for the supply effect of surprise inflation and the scalar parameter *c* of the effect of fiscal policy on inflation, are all stochastic shocks. The private sector's expectations are formed before any of these shocks are realized and before the policy variables are chosen, and are rational, that is  $\pi^e = E[\pi(z)]$ . To simplify the notation, we drop the dependence of output, inflation and the policy variables on *z* whenever it does not create confusion.

The fiscal authority in the country wants to minimize her loss function, which is given by

$$L_F = \frac{1}{2} \left[ \theta_F (y - y_F)^2 + (\pi - \pi_F)^2 \right].$$
(3)

We have in mind a situation where the GDP goal for the fiscal authority is such that  $y_F \geq \overline{y}$ , and extra output is desirable. For example, in a monopolistic competitive model, output is inefficiently low because of the monopoly power over produced goods.  $\pi_F$  is the inflation goal for the fiscal authority and  $\theta_F$  parametrizes the authority's preference for higher output relative to its dislike of inflation. This is a reduced form model, and inflation can be costly for any underlying structural reason. We think that  $\theta_F > 0$ , but the results go through for any value.

The monetary authority minimizes her loss function, which is given by

$$L_M = \frac{1}{2} \left[ \theta_M (y - y_M)^2 + (\pi - \pi_M)^2 \right].$$
(4)

In words, the central bank may share the objectives of the fiscal authority, in which case  $\theta_M = \theta_F$  and  $\pi_M = \pi_F$ , but does not need to. In fact, the loss function (4) is general enough to accommodate different output and inflation goals as well as different tradeoffs among the two goals. We have in mind a situation where the central bank is more conservative than the fiscal authority in the sense that she has a greater concern for inflation as in Rogoff (1985); this implies that  $\theta_M \leq \theta_F$  and/or  $\pi_M \leq \pi_F$ . The central bank may be ultraconservative, i.e. it may not care at all about output in which case  $\theta_M = 0$ , and its inflation goal may be zero, i.e.  $\pi_M = 0$ ; our results are valid for arbitrary values of the inflation and output goals and tradeoff levels.

Monetary policy can be committed or discretionary. Hence, the timing of events is as follows:

- 1. If the monetary policy regime is one of commitment, the central bank chooses its policy rule  $\pi_0 = \pi_0(\overline{y}, a, b, c)$ , which specifies how it will respond to the stochastic shocks. If the monetary regime is one of discretion, nothing happens at this step.
- 2. The private sector forms expectations  $\pi^e$ .
- 3. The stochastic shocks  $\overline{y}, a, b, c$  are realized.

- 4. (a) If the monetary policy regime is one of discretion, the central bank chooses  $\pi_0$ . If the monetary regime is one of commitment, the central bank simply implements the monetary rule  $\pi_0$  that was chosen at step 1.
  - (b) The fiscal authority chooses fiscal policy x.

When monetary policy is discretionary, the relative timing of step 4 (a) and 4 (b) raises some questions. In fact, monetary and fiscal policy may be chosen simultaneously or their order may be reversed. We are going to consider the three possible cases below.

### 4 Monetary Discretion, Nash equilibrium

The fiscal authority chooses x, taking  $\pi_0$  as given, so as to minimize the loss function  $L_F$ ; the monetary authority chooses  $\pi_0$ , taking x as given, so as to minimize her loss function  $L_M$ . The two authorities act non-cooperatively and simultaneously; however, when their choices are made, the private sector's expectations  $\pi^e$  are fixed.

The first-order condition for fiscal policy is obtained by differentiating (3) with respect to x, which gives

$$\theta_F(y - y_F)(a + b c) + (\pi - \pi_F) c = 0,$$

or

$$\pi = \pi_F - \theta_F \left(\frac{a}{c} + b\right) (y - y_F).$$
(5)

This defines the reaction function of the fiscal authority (FRF) in the  $(y,\pi)$  space. Substituting y and  $\pi$  into (5) using (1) and (2), one can obtain the reaction function in terms of the policy variables  $(\pi_0, x)$ . The FRF is positively or negatively sloped in the  $(y,\pi)$  space depending on the realizations of the stochastic shocks  $\overline{y}, a, b, c$ . We have in mind a situation where a, b are positive. In the model of appendix A, expansionary fiscal policy 1. has a positive impact on inflation, hence c is positive, while expansionary fiscal policy 2. has a negative effect on inflation, hence c is negative. Here we consider both cases. If c > 0 or c < -(a/b), the FRF is negatively sloped; for  $c \in (-a/b, 0)$ , the FRF is positively sloped.

The first-order condition for monetary policy is obtained by differentiating (4) with respect to  $\pi_0$ , which gives

 $\theta_M(y - y_M)b + (\pi - \pi_M) = 0$ 

or

$$\pi = \pi_M - \theta_M b \ (y - y_M). \tag{6}$$

This defines the reaction function for the monetary authority (MRF) in the  $(y, \pi)$  space. Again, substituting y and  $\pi$  into (6) using (1) and (2), one can obtain the monetary



Figure 1: Nash Equilibrium

reaction function in terms of the policy variables  $(\pi_0, x)$ . Since b > 0, the MRF is always negatively sloped.

Let

$$\Omega \equiv \theta_F(\frac{a}{c} + b) - \theta_M b.$$

Then, the solution exists as long as  $\Omega$  is different from zero, which is a probability zero event, and it is given by

$$y = \frac{1}{\Omega} \left[ \pi_F - \pi_M + \theta_F (\frac{a}{c} + b) y_F - \theta_M b \ y_M \right], \tag{7}$$

and

$$\pi = \left(1 + \frac{\theta_M b}{\Omega}\right) \left(\theta_M b \ y_M + \pi_M\right) - \frac{\theta_M b}{\Omega} \pi_F + \frac{\theta_M b \ \theta_F \left(\frac{a}{c} + b\right)}{\Omega} y_F.$$
(8)

Rational expectations imply  $\pi^e = E[\pi]$  over the distribution of  $(\overline{y}, a, b, c)$ , which is the expected value of (8). From (7) and (8), making use of (1) and (2) and  $\pi^e = E[\pi]$ , one can back out the policy variables x and  $\pi_0$  that emerge in equilibrium and deliver output and inflation as in (7) and (8).

We think of a situation where the central bank is more conservative and cares more about inflation than the fiscal authority; hence, in the remainder of the paper we are going to assume that  $\theta_M < \theta_F$ ,  $y_M < y_F$  and  $\pi_M < \pi_F$ ; we are also assuming that a, b > 0, while c can be positive or negative. Under these assumptions, the Nash equilibrium exists and is stable. Figure 1 depicts the MRF (6) and the FRF (5) in the  $(y,\pi)$  space; point F is the bliss point for the fiscal authority, whereas M is the bliss point for the monetary authority. When c > 0 or c < -(a/b), the FRF is downward sloping and, under the assumptions above, it is steeper than the MRF. When  $c \in (-a/b, 0)$ , the FRF is upward sloping. The Nash equilibrium occurs at the intersection of the two reaction functions and it is labelled N.

When the FRF is downward sloping, inflation in the Nash equilibrium is such that

$$\pi \leq \pi_M < \pi_F,$$

where the equality sign holds if  $\theta_M = 0$ , i.e. if the central bank is ultraconservative. Output is such that

$$y \ge y_F > y_M,$$

and the equality sign holds when  $\theta_F = +\infty$ . When the FRF is upward sloping, the ranking of inflation and output with respect to the authorities' ideal outcomes depends on whether the FRF intersects the MRF to the right or to the left of the monetary authority's bliss point M. When the intersection is on the right of the central bank's ideal outcome, as depicted in figure 1, the ranking is as follows

$$\pi < \pi_M < \pi_F$$
, and  $y_F > y > y_M$ .

When the intersection is on the left of the central bank's ideal outcome, the ranking is

$$\pi_M < \pi < \pi_F$$
 and  $y_F > y_M > y$ .

This result shows that the race between expansionary fiscal and contractionary monetary policies is carried out to excessive levels in the Nash equilibrium. Inflation is at least as low as the central bank desires; output is always above the central bank's goal and, when c > 0 or c < -(a/b) so that the FRF is negatively sloped, it is even above the fiscal authority's goal. When the FRF is negatively sloped, the further apart the goals of the two authorities (i.e. as point F moves North-East and point M moves South-West), the higher output and the lower inflation in equilibrium. A fiscal limit in the form of an upper bound on the fiscal variable x reduces output if it is binding in equilibrium; in case of bad shocks, however, such as a low realization of the natural rate of output  $\bar{y}$ , the fiscal limit is likely to be binding in spite of output being well below its unrestricted equilibrium level (7).

Dixit and Lambertini (1999) consider the interaction between monetary policy of a common central bank in a monetary union and the separate fiscal policies of the member countries under the case where the monetary and the fiscal authority have the same ideal outcome or first best, namely  $y_F = y_M$  and  $\pi_F = \pi_M$ . It is shown there that the first best can be achieved despite the need for monetary commitment, despite the inflationary effect of fiscal profligacy (c > 0), without the need for fiscal coordination, without the need for a conservative central bank and irrespectively of which authority moves first. This result can be easily understood in the one-country setting developed so far. When the fiscal and monetary authority have the same ideal outcome, the point F and M in figure 1 coincide; even if the authorities have different tradeoffs between output and inflation goals (different  $\theta$ s), the FRF and the MRF intersect exactly at the first best. When the authorities agree on the ideal outcome, there are two policy tools to achieve two goals.

#### 5 Monetary Discretion and Leadership

It is not clear whether the timing of actions 4 (a) and 4 (b) should be as described or the other way round. The current policy debate seems to assume that the central bank moves first and the fiscal authority follows; hence, the central bank may fear that subsequent fiscal expansions will bring inflation well above her goal. The literature, for example Beetsma and Bovemberg (1998), has argued that it takes a long time to change tax rates whereas monetary policy can be adjusted quite quickly; hence the timing of 4 (a) and (b) should be actually reversed.

Here we consider the case of monetary leadership. Monetary policy is open to discretionary choice at step 4 (a); when fiscal policy is chosen at step 4 (b),  $\pi_0$  is known. Private sector's expectations  $\pi^e$  are set before and known when  $\pi_0$  and x are chosen.

Fiscal policy is exactly as described in the previous section. The fiscal authority minimizes (3) with respect to x taking  $\pi_0$  and  $\pi^e$  as given. Hence, the fiscal authority's reaction function is still described by (5).

The monetary authority minimizes the loss function (4) with respect to  $\pi_0$  subject to the FRF (5). The first-order condition is

$$\theta_M b(y - y_M) - \theta_F \left(\frac{a}{c} + b\right) b(\pi - \pi_M) = 0,$$

or

$$\pi = \pi_M + \frac{\theta_M}{\theta_F \left(\frac{a}{c} + b\right)} (y - y_M).$$
(9)

Let

$$\Phi = \frac{\theta_F \left(\frac{a}{c} + b\right)}{\theta_M + \theta_F^2 \left(\frac{a}{c} + b\right)^2}$$

Substituting the FRF (5) into the first-order condition (9), we obtain the monetary



Figure 2: Monetary Leadership

leadership solution

$$y = \Phi\left[\pi_F - \pi_M + \theta_F\left(\frac{a}{c} + b\right)y_F + \frac{\theta_M}{\theta_F\left(\frac{a}{c} + b\right)}y_M\right],\tag{10}$$

and

$$\pi = \left[1 - \frac{\theta_M}{\theta_F\left(\frac{a}{c} + b\right)}\Phi\right] \left[\pi_M - y_M\frac{\theta_M}{\theta_F\left(\frac{a}{c} + b\right)}\right] + \frac{\theta_M}{\theta_F\left(\frac{a}{c} + b\right)}\Phi \left[\pi_F + \theta_F\left(\frac{a}{c} + b\right)y_F\right].$$
(11)

Consider first the case where  $\theta_F(a/c+b)$  is positive and the FRF is negatively sloped. Then, the first-order condition (9) defines an upward sloping line passing through point M in the  $(y, \pi)$  plane. This implies that, with monetary leadership, output is lower and inflation is higher than in the Nash equilibrium. Figure 2 shows that the tangency between the monetary authority's indifference curves and the FRF indeed occurs at point ML, which is further up along the fiscal reaction function.

On the other hand, when  $\theta_F(a/c + b)$  is negative so that the FRF is positively sloped in the  $(y, \pi)$  plane, then (9) slopes down and it is flatter than the MRF (6) of Section 4.<sup>1</sup> This implies higher inflation and higher output than in the corresponding

$$c < -\frac{a}{b} \ \frac{b^2 \theta_F}{1 + b^2 \theta_F}$$

<sup>&</sup>lt;sup>1</sup>More precisely, (9) defines a downward sloping locus flatter than (6) as long as

Nash equilibrium, as shown in figure 2.

As before, the private sector sets its expectations rationally as  $\pi^e = E[\pi]$ , with  $\pi$  given by (11). Using  $\pi^e$  and the solution for output (10) and for inflation (11), one could solve explicitly for the equilibrium fiscal policy x and monetary policy  $\pi_0$ .

### 6 Monetary Discretion and Fiscal Leadership

In this section we consider the case of fiscal leadership. After the private sector's expectations are set and the shocks are realized, the fiscal authority chooses x. With x fixed, the monetary authority chooses  $\pi_0$ . As usual, we solve this game by backward induction. This requires starting with the last player that, in this case, is the monetary authority.

The monetary authority minimizes the loss function (4) with x given; hence, the first-order condition with respect  $\pi_0$  is given by the MRF (6) of section 4. The fiscal authority minimizes the loss function (3) with respect to x, subject to the MRF (6). The first-order condition with respect to x is

$$\theta_F(y - y_F) - \theta_M b \ (\pi - \pi_F) = 0$$

or

$$\pi = \pi_F + \frac{\theta_F}{\theta_M b} (y - y_F).$$
(12)

Under the parameter configuration we have in mind (with b positive), (12) defines an upward sloping locus in the  $(y, \pi)$  space going thru the bliss point for the fiscal authority, F.

Let

$$\Gamma \equiv \frac{\theta_M b}{\theta_F + \theta_M^2 b^2}$$

The equilibrium under fiscal leadership is along the MRF and it is given by

$$y = \Gamma \left[ -(\pi_F - \pi_M) + \frac{\theta_F}{\theta_M b} y_F + \theta_M b \ y_M \right], \tag{13}$$

and

$$\pi = [1 - \theta_M b \Gamma] [\pi_M + \theta_M b y_M] + \Gamma [\theta_M b \pi_F - \theta_F y_F].$$
(14)

Proceeding by backward induction, the private sector sets  $\pi^e$  rationally by taking expectations of (14) over the distribution of  $(\overline{y}, a, b, c)$ . The actual choice of the policy variables x and  $\pi_0$  depends on the realization of the shocks and it can be obtained substituting (1) and (2) into (13) and (14).

which is certainly satisfied for c < -(a/b), the condition for the FRF to be upward sloping.



Figure 3: Fiscal Leadership

Figure 3 depicts the fiscal leadership equilibrium, which is labelled FL; to ease the comparison, the Nash equilibrium both for the case with  $\theta_F(a/c+b)$  is positive and negative are also shown. When  $\theta_F(a/c+b)$  is negative, fiscal leadership results in higher output but lower inflation than under the Nash equilibrium. Since an increase in x reduces inflation when  $c \in (-a/b, 0)$ , fiscal leadership allows a more expansionary fiscal stance. When  $\theta_F(a/c+b)$  is positive, however, fiscal leadership results in lower output and higher inflation than under the Nash equilibrium. The fiscal authority prefers a less expansionary fiscal policy that delivers lower output but higher inflation.

## 7 Monetary Commitment

Here the monetary policy rule is fixed at step 1 and implemented at step 4 (a); fiscal policy is chosen after the monetary policy rule is fixed, say at step 4 (b).<sup>2</sup> Since the monetary policy rule  $\pi_0(\bar{y}, a, b, c)$  is chosen at an earlier stage, its decision must

<sup>&</sup>lt;sup>2</sup>Fiscal policy could be carried out at step 4 (a) and the monetary rule implemented at 4 (b) and the results would not change; all that matter is that the policy rule is already fixed and the central bank is committed to its implementation.

be made using the logic of subgame perfectness that takes into account the action of the fiscal authority later on in the game. One would think that commitment to a full state-contingent monetary rule would allow the central bank to get close to its ideal outcome or, at least, to do better than the case where monetary policy is discretionary. It turns out that, as long as fiscal policy is chosen in a discretionary manner (as it has been assumed so far), the monetary authority cannot improve upon the equilibrium that arises under monetary leadership with discretion. Hence, fiscal discretion destroys monetary commitment! As long as the central bank can move before the fiscal authority, commitment to a policy rule does not matter. Lately, much emphasis has been put on the importance of monetary rules to maintain inflation low and stable. This argument is thought to be particularly relevant for monetary unions, such as the EMU, where each government may engage in fiscal expansions to increase its own GDP expecting to pass the cost of its profligacy to other members in the form of higher inflation and interest rates. Our finding suggests that commitment to a monetary rule *not* accompanied by commitment to a fiscal rule is not enough.

To solve for the monetary rule chosen at step 1, we must use backward induction and start from the choice of fiscal policy at step 4 (b). The fiscal authority minimizes the loss function (3) with respect to x with  $\pi_0$  fixed. The first-order condition with respect to x is, once again, the FRF (5). One can therefore solve for fiscal policy as a function of the stochastic shocks, the monetary rule and private sector's expectations by substituting (1) and (2) into (5) and obtain

$$x(z) = \frac{1}{c \left[\theta_F \left(\frac{a}{c} + b\right)^2 + 1\right]} \left\{ -\left[\theta_F b \left(\frac{a}{c} + b\right) + 1\right] \pi_0(z) \right.$$
(15)  
$$\left. -\theta_F \left(\frac{a}{c} + b\right) \left[\overline{y} - y_F - b \pi^e\right] + \pi_F \right\}.$$

Output and inflation, as of step 1 and taking into account the choice of the fiscal authority at step 4 (b), are

$$y(z) = \frac{1}{\theta_F \left(\frac{a}{c} + b\right)^2 + 1} \left\{ -\frac{a}{c} \pi_0(z) + \overline{y} - b \pi^e + \theta_F \left(\frac{a}{c} + b\right)^2 \left[ y_F + \frac{\pi_F}{\theta_F \left(\frac{a}{c} + b\right)} \right] \right\},\tag{16}$$

and

$$\pi(z) = \frac{1}{\theta_F \left(\frac{a}{c} + b\right)^2 + 1} \left\{ \theta_F \frac{a}{c} \left(\frac{a}{c} + b\right) \pi_0(z) - \theta_F \left(\frac{a}{c} + b\right) \left[\overline{y} - y_F - b\pi^e\right] + \pi_F \right\}.$$
(17)

Proceeding by backward induction, we now consider the private sector that sets its expectations rationally at step 2. More precisely, expectations are

$$\pi^e = \int \pi(z),\tag{18}$$

with  $\pi(z)$  given by (17) and the integral is four-dimensional with respect to the joint distribution of z.

At step 1, the monetary authority minimizes the following loss function

$$\int L_M(z) = \frac{1}{2} \int \left[ \theta_M(y(z) - y_M)^2 + (\pi(z) - \pi_M)^2 \right], \tag{19}$$

where y(z),  $\pi(z)$  are given by (16) and (17), respectively. The monetary authority minimizes (19) with respect to the function  $\pi_0(\cdot)$  and with respect to  $\pi^e$ , subject to the constraint (18). The Lagrangean for this problem is as follows

$$\mathcal{L}_{\mathcal{M}} = \int \left\{ \frac{1}{2} \left[ \theta_M (y(z) - y_M)^2 + (\pi(z) - \pi_M)^2 \right] - \lambda \pi(z) \right\} + \lambda \pi^e, \tag{20}$$

where  $\lambda$  is the Lagrangean multiplier.

The first-order condition with respect to the function  $\pi_0(z)$  is given by

$$(y(z) - y_M) - \frac{\theta_F}{\theta_M} \left(\frac{a}{c} + b\right) (\pi(z) - \pi_M - \lambda) = 0.$$
(21)

The first-order condition with respect to  $\pi^e$  is given by

$$\lambda + \int \frac{b}{\theta_F \left(\frac{a}{c} + b\right)^2 + 1} \left[ -\theta_M (y(z) - y_M) + \theta_F \left(\frac{a}{c} + b\right) (\pi(z) - \pi_M) \right] = 0.$$
(22)

Using (21), the first-order condition (22) simplifies to

$$\lambda = 0.$$

When all the  $\pi_0$  are chosen ex-ante optimally, the rational expectations constraint is on the borderline of not binding. Using  $\lambda = 0$ , (21) becomes

$$(y(z) - y_M) - \frac{\theta_F}{\theta_M} \left(\frac{a}{c} + b\right) (\pi(z) - \pi_M) = 0, \qquad (23)$$

which is exactly (9), the first-order condition for  $\pi_0$  in the case where monetary policy is discretionary with monetary leadership! In fact, the state-by-state outcomes can be found by solving the discretionary fiscal reaction function (5) and the monetary rule (23), which is identical to (9). The outcome under monetary commitment is therefore exactly the same as the outcome under monetary discretion with monetary leadership.

Notice that the monetary rule (23) is a very general optimal state-contingent one. Much of the literature on monetary rules has imposed a condition of linearity so  $\pi_0$  is a linear function of  $(\bar{y}, a, b, c)$  and then calculated the expected loss and minimized it with respect to the coefficients of the linear function. We do not require linearity and solve a more general calculus of variations problem. Our monetary rule is not a linear function of  $(\bar{y}, a, b, c)$ , and the reason is that even though the model is linear-quadratic, our stochastic shocks are not in general additive.

The intuition for the result that fiscal discretion makes monetary commitment unnecessary can be grasped by looking at figure 2. As long as the fiscal authority can choose x discretionary and either be the last mover or face a committed monetary authority, output and inflation in equilibrium lie along the FRF. And the best the monetary authority can do, either by monetary discretion and leadership or by committing to a monetary rule, is to choose her preferred allocation along the FRF.

#### 8 Monetary Union

In this section, we extend our model to many countries in a monetary union.

The monetary union consists of i = 1, 2, ..., n countries. The union has a common monetary authority that chooses the common  $\pi_0$ . Each country has a fiscal authority, and the fiscal policy variables are denoted by  $x_i$ . These policies result in GDP levels  $y_i$  in the separate countries, and a common inflation rate  $\pi$ . As before,  $\pi^e$  denotes rational expectations of  $\pi$ . The stochastic shocks are

$$z = (\bar{y}, A, b, c),$$

which is a  $n \times (n+3)$  matrix.

The GDP levels of the countries are given by

$$y_i = \overline{y}_i + \sum_j a_{ij} x_j + b_i (\pi - \pi^e).$$
(24)

Each  $a_{ii}$  shows the effect on GPD of that country's own fiscal policy, and the  $a_{ij}$  for  $j \neq i$  are the spillovers of one country's fiscal policy on others. We require that the matrix  $A = (a_{ij})$  is nonsingular. The last term of the right-hand side of (24) is the usual supply effect of surprise inflation, and its magnitude can differ across countries.

The equations (24) for all countries can be collected into one, using the obvious vector and matrix notation, as

$$y = \overline{y} + A x + (\pi - \pi^e) b \tag{25}$$

The common inflation level is given by

$$\pi = \pi_0 + \sum_i c_i x_i = \pi_0 + c' x \tag{26}$$

For simplicity, in the remainder we assume that  $c_i > 0$ ,  $\forall i$ .

The fiscal authorities want to minimize their respective loss functions defined by

$$L_i^F i = \frac{1}{2} \theta_i^F (y_i^F - y_i)^2 + \frac{1}{2} (\pi - \pi^F)^2, \qquad (27)$$

The common central bank minimizes

$$L^{M} = \frac{1}{2} \sum_{i} \theta_{i}^{M} (y_{i} - y_{i}^{M})^{2} + \frac{1}{2} (\pi - \pi^{M})^{2}.$$
(28)

Let  $\Theta^M$  be the diagonal matrix with entries  $\theta_i^M$  and  $y^M$  be the vector of output goals for the monetary authority. The loss function for the central bank can be rewritten as

$$L^{M} = \frac{1}{2}(y(z) - y^{M})'\Theta^{M}(y(z) - y^{M}) + \frac{1}{2}(\pi - \pi^{M})^{2}.$$

The first-order condition with respect to  $\pi_0$  gives

$$\sum_{i} \theta_{i}^{M} (y_{i} - y_{i}^{M}) b_{i} + (\pi - \pi^{M}) = 0, \qquad (29)$$

and the first-order condition with respect to  $x_i$  is

$$\theta_i^F(y_i - y_i^F)(a_{ii} + b_i c_i) + (\pi - \pi^F) = 0.$$
(30)

Substituting for  $y_i$  from the fiscal first-order condition into the monetary first-order condition and solving

$$\pi = \frac{\pi^{M} - \sum_{i} k_{i} \pi_{i}^{F} - \sum_{i} \theta_{i}^{M} b_{i} (y_{i}^{F} - y_{i}^{M})}{1 - \sum_{i} k_{i}}$$
(31)

where

$$k_i = \frac{\theta_i^M}{\theta_i^F} \frac{b_i}{b_i + a_{ii}/c_i}.$$

We assume that all fiscal authorities are more expansionary than the monetary authority; hence

$$\pi^M < \pi^F_i, \quad y^M_i < y^F_i, \quad \frac{\theta^F_i}{n} > \theta^M_i, \text{ for all } i.$$

Then,  $k_i < 1/n$  and  $\sum_i k_i < 1$ . Then

$$\pi < \frac{\pi^M - \sum_i k_i \pi^M}{1 - \sum_i k_i} = \pi^M < \pi^F_i, \quad \forall i$$
(32)

and

$$y_{i} = y_{i}^{F} - c_{i}\theta_{i}^{F}(c_{i}b_{i} + a_{ii})(\pi - \pi_{i}^{F}) > y_{i}^{F} > y_{i}^{M}$$
(33)

Again, extreme outcome arise when  $c_i > 0$ ,  $\forall i$ .

Now we are going to show that monetary commitment is equivalent to monetary leadership with discretion. Let H be the diagonal matrix with entries  $h_i$ ,

$$h_i \equiv \theta_i^F (\frac{a_{ii}}{c_i} + b_i).$$

Hence, the first-order conditions with respect to x can be stacked as follows

$$H[y(z) - y^{F}] + \pi(z)e - \pi^{F} = 0, \qquad (34)$$

where e is the unit vector of dimension  $n \times 1$ . Substituting for output and inflation, fiscal policy is given by

$$x(z) = J^{-1}[-(Hb - e)\pi_0(z) - H(\bar{y} - b\pi^e - y^F) + \pi^F]$$
(35)

where  $J \equiv H(A + bc') - ec'$ . Inflation and output as of the beginning of step 1 and taking into account the action of the fiscal authority at step 4 (b) are

$$\pi(z) = [1 - c'J^{-1}(Hb - e)]\pi_0(z) - c'J^{-1}[H(\bar{y} - b\pi^e - y^F) - \pi^F]$$
(36)

and

$$y(z) = y^{F} - H^{-1} \left\{ [1 - c'J^{-1}(Hb - e)]\pi_{0}(z) - c'J^{-1}[H(\bar{y} - b\pi^{e} - y^{F}) - \pi^{F}] \right\} e + H^{-1}\pi^{F}.$$
(37)

Under monetary leadership, namely when the monetary authority moves before the fiscal authorities, the central bank minimizes her loss function (28) suject to (34). The first-order condition for the monetary authority is

$$-(y(z) - y^{M})'\Theta^{M}H^{-1}e + \pi(z) - \pi^{M} = 0.$$
(38)

Consider now the case where the central bank can commit to a monetary rule. The Lagrangean for the monetary authority's choice of the function  $\pi_0(\cdot)$  in the *n*-country case is

$$\mathcal{L}_{\mathcal{M}} = \int \left\{ \frac{1}{2} (y(z) - y^{M})' \Theta^{M} (y(z) - y^{M}) + \frac{1}{2} (\pi - \pi^{M})^{2} - \lambda \pi(z) \right\} + \lambda \pi^{e}, \quad (39)$$

where  $\lambda$  is the Lagrangean multiplier. The first-order condition with respect to  $\pi_0(z)$  gives

$$-(y(z) - y^{M})'\Theta^{M}H^{-1} + \pi(z) - \pi^{M} - \lambda = 0$$
(40)

and the first-order condition with respect to  $\pi^e$  gives

$$\lambda + \int \left[ -(y(z) - y^M)' \Theta^M H^{-1} + \pi(z) - \pi^M - \lambda \right] [c' J^{-1} H b] = 0$$
(41)

which, making use of (40), simplifies to

 $\lambda = 0.$ 

The constraint is on the border line of not binding, exactly as in the one-country case. Using  $\lambda = 0$ , (40) simplifies to

$$-(y(z) - y^M)'\Theta^M H^{-1}e + \pi(z) - \pi^M = 0,$$

which, together with (34), gives the solution to the monetary leadership case.

## 9 Conclusions

to be written

## A Appendix

We consider a one-country general equilibrium monetary model. There is a central bank that runs monetary policy and a fiscal authority that runs fiscal policy in the country. The economy is inhabited by N individual monopolistic producers, indexed by j, each of whom produces a single differentiated good. Each producer faces a downward sloping demand function and chooses the nominal price and the level of production of her good. Production makes only use of labor and, since labor supply is elastic, production is endogenously determined.

Each producer is also a consumer, who derives utility from the consumption of all goods, real money balances, and effort put in production. Producer-consumer (producer for short) j has the following utility function

$$U_j = \left(\frac{C_j}{\gamma}\right)^{\gamma} \left(\frac{M_j/P}{1-\gamma}\right)^{1-\gamma} - \left(\frac{d}{\beta}\right) Y_j^{\beta}, \quad \gamma \in (0,1), \quad d > 0, \quad \beta \ge 1,$$
(A.1)

where the variable  $C_j$  is a real consumption index

$$C_j = N^{\frac{1}{1-\theta}} \left[ \sum_{z=1}^N C_{zj}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1,$$
(A.2)

where  $C_{zj}$  is the *j*-th individual consumption of good *z*. The price deflator for nominal money is the consumption-based money price index corresponding to the consumption index (A.1)

$$P = \left[\frac{1}{N} \left(\sum_{z=1}^{N} P_z^{1-\theta}\right)\right]^{\frac{1}{1-\theta}},\tag{A.3}$$

where  $P_z$  is the price of good z.

Producer j's utility depends positively on consumption  $C_j$  and on real balances  $M_j/P$ , and negatively on the effort spent in producing the j-th good,  $Y_j$ . The utility function (A.1) is homogeneous of degree one in consumption and real balances, as well as separable in consumption and real money on one side, and leisure, on the other. This assumption is made for simplicity because it produces a constant marginal utility of wealth.

The consumption index  $C_j$  is a symmetric function of the consumption of each good  $C_{zj}$ ; we have assumed a constant elasticity of substitution  $\theta$  among all goods. To guarantee the existence of an equilibrium, we must assume that  $\theta > 1$  so that the goods are close substitutes and the elasticity of demand facing each producer is larger than one.

Real balances enter the utility function directly; more precisely, an increase in real balances increases the demand for goods. Of course, this is a shortcut for modelling the role of money in a more satisfactory way, which would require a dynamic model. Money is implicitly used to purchase goods in this model; hence, the appropriate price index P is the deflator for the consumption index  $C_i$ .

Production creates disutility because it reduces leisure. Notice that all individuals have identical preferences and disutility of effort d.

Producer j has the following budget constraint:

$$\sum_{z=1}^{N} P_z C_{zj} + M_j = P_j Y_j (1-\tau) - PT + \overline{M}_j \equiv I_j.$$
 (A.4)

The government has the budget constraint:

$$I_g \equiv \sum_{j=1}^N P_j Y_j \tau + NPT = NPG, \quad \alpha > 0.$$
 (A.5)

The individual budget constraint (A.4) says simply that nominal consumption expenditure plus the demand for money must equal nominal income. It is assumed that individuals pay (receives) taxes (transfers) proportional to their revenues,  $\tau > 0(<0)$ , and pay lump-sum taxes (transfers) PT, and have an initial holding of money,  $\overline{M}_j$ . Hence, nominal income is equal to nominal after-transfer revenues from selling the produced good, minus lump-sum taxes, plus the initial money holding. Both  $\tau$  and T can be either positive or negative.

(A.5) is the government budget constraint. The government levies per-capita lumpsum taxes T and spends the proceeds either by redistributing it back to the producers via distortionary transfers  $\tau < 0$ ; alternatively, it levies distortionary taxes  $\tau > 0$  to purchase from the producers the per-capita amount G of the same composite good consumed by private individuals,<sup>3</sup> or simply to spend the resources on services that are not explicitly modelled here (hence outside the economy). The budget constraint (A.5) has been written in a form general enough to accommodate different fiscal policies.

The solution of the model requires three steps. First, we characterize the optimal allocation of wealth between consumption and money taking individual wealth as given. Second, taking individual conditional demand functions, we obtain the aggregate demand facing each producer. Third, we solve the producer's optimization problem and derive the equilibrium price and quantities.

The first order condition with respect to  $C_{zj}$  and  $M_j$ , respectively, imply

$$C_{zj} = \left(\frac{P_z}{P}\right)^{-\theta} \frac{\gamma I_j}{NP},\tag{A.6}$$

$$M_j = (1 - \gamma)I_j. \tag{A.7}$$

As usual, the demand for each good is linear in wealth and depends on its relative price with elasticity  $-\theta$ . The demand for money is also linear in wealth.

The demand facing producer z can be obtained by aggregating individual demand over consumers

$$Y_z^d = \sum_{j=1}^N Czj = \left(\frac{P_z}{P}\right)^{-\theta} \left[\frac{\gamma I}{NP} + \frac{\chi I_g}{NP}\right]$$
(A.8)

where  $I \equiv \sum_{j=1}^{N} I_j = N I_j$  and  $\chi$  is an indicator function that is equal to 1 if government revenues are used to purchase the goods produced in the economy, and equal to 0 otherwise. Hence,  $\chi I_g/(NP)$  is government demand.

Substituting the demand for goods and money into the utility function (A.1), using the individual budget constraint (A.4) and the demand function facing producer j

$$\max G = N^{\frac{1}{1-\theta}} \left[ \sum_{z=1}^{N} G_z^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1,$$

subject to its budget constraint. Hence, the government's demand for good z will have the same form of the individual demand for good z.

<sup>&</sup>lt;sup>3</sup>This implies that the government chooses consumption  $G_j$  so as to

(A.8), we find an indirect utility function for the *j*-th individual:

$$U_j = (1 - \tau) \left(\frac{\gamma I}{NP} + \frac{\chi I_g}{NP}\right)^{\frac{1}{\theta}} Y_j^{\frac{\theta - 1}{\theta}} - T + \frac{\overline{M}_j}{P} - \left(\frac{d}{\beta}\right) Y_j^{\beta}.$$

Maximizing the indirect utility above with respect to the relative price, we obtain the price, and therefore output, chosen by producer j

$$\frac{P_j}{P} = \left[\frac{\theta d}{(\theta - 1)(1 - \tau)} \left(\frac{\gamma I}{NP} + \frac{\chi I_g}{NP}\right)^{\beta - 1}\right]^{\frac{1}{1 + \theta(\beta - 1)}}.$$
(A.9)

This equation shows that the relative price set by producer j increases with aggregate wealth I/P and with the disutility of effort, d.

Total production is the sum of output over the producers living in the country, namely

$$Y = \sum_{j=1}^{N} Y_j = N \left[ \frac{(\theta - 1)(1 - \tau)}{\theta d} \left( \frac{\gamma I}{NP} + \frac{\chi I_g}{NP} \right)^{\frac{1}{\theta}} \right]^{\frac{\eta}{1 + \theta(\beta - 1)}}.$$
 (A.10)

So far, the model has been solved by taking wealth as given. Consider the case where prices are flexible and there is no uncertainty. In equilibrim, all relative prices set by producers in a country must be equal. When prices are flexible, then  $P_z/P = 1$ , for all goods z produced in the country. Consider three alternative fiscal policies:

- 1. The government uses distortionary taxes  $\tau$  to finance its budget; the revenues are either thrown away or spent on services not modelled here.
- 2. The government uses lump-sum taxes T to finance its budget; the revenues are redistributed to the producers via a transfer  $\tau < 0$  proportional to sales.
- 3. The government uses distortionary taxes  $\tau$  to finance its budget; the revenues are spent to purchase the composite good G.

Consider fiscal policy 1. In this case,  $\chi = 0$ , G = 0. Aggregate wealth is given by

$$\frac{I}{P} = \frac{1}{(1-\gamma)(1+\frac{\gamma}{1-\gamma}\tau)} \overline{\frac{M}{P}},$$
(A.11)

where  $\overline{M} \equiv \sum_{j=1}^{N} \overline{M}_{j}$ . Using (A.11), we can now solve for aggregate output and relative prices as a function of the policy variables,  $\overline{M}$  and  $\tau$ , and the underlying parameters. Output is given by

$$Y = \frac{\gamma}{(1-\gamma)(1+\frac{\gamma}{1-\gamma}\tau}\overline{M}.$$
 (A.12)

Given the government budget constraint, one can back out the value of  $\tau$  as a function of government spending

$$\tau = \frac{1 - \gamma}{\gamma} \left[ \frac{NG}{\overline{MP} - NG} \right],\tag{A.13}$$

and the relative price for producer j (and for clarity, we indicate the nominal price of her good  $P_j$ ) is

$$\frac{P_j}{P} = \left\{ \frac{\theta d}{(\theta - 1)(1 - \tau)} \left[ \frac{\gamma}{N(1 - \gamma)(1 + \frac{\gamma}{1 - \gamma}\tau)} \frac{\overline{M}}{P} \right]^{\beta - 1} \right\}^{\frac{1}{1 + \theta(\beta - 1)}}.$$
 (A.14)

Consider now fiscal policy 2. In this case,  $\chi = 0, G = 0, \tau < 0, T > 0$ . We assume there are dead-weight losses inherent in the operation of the government; the government budget constraint is now

$$\sum_{j=1}^{N} P_j Y_j \tau(1+\alpha) + NPT = 0, \ \alpha > 0$$

To distribute transfers  $\sum_{j=1}^{N} P_j Y_j \tau$ , the government needs to raise  $1 + \alpha$  times that amount in the form of lump-sum taxes. Notice that the higher the nominal transfers, the higher the nominal dead-weight loss associated with government intervention. This is a shortcut for modelling the distortions and output losses induced by a system of taxes and transfers; however, it captures the idea that fiscal policy has costs and the larger the fiscal intervention, the larger the costs associated with it. The distortionary transfer  $\tau$  raises the incentive to produce, while the lump-sum tax T reduces income and thereby consumption;

Using the government budget constraint (5), we can solve for aggregate wealth as

$$\frac{I}{P} = Y(1 - \alpha\tau) + \frac{\overline{M}}{P},\tag{A.15}$$

We can now solve for aggregate output and relative prices as a function of the policy variables,  $\overline{M}$  and  $\tau$ , and the underlying parameters. Output is given by

$$Y = \frac{\gamma}{(1-\gamma)[1+\frac{\alpha\gamma}{1-\gamma}\tau]} \frac{\overline{M}}{P}.$$
 (A.16)

Hence, aggregate wealth is

$$\frac{I}{P} = \frac{1}{(1-\gamma)[1+\frac{\alpha\gamma}{1-\gamma}\tau]} \frac{\overline{M}}{P},$$
(A.17)

and the relative price for producer j is

$$\frac{P_j}{P} = \left[\frac{\theta d}{(\theta - 1)(1 - \tau)} \left(\frac{\gamma}{N(1 - \gamma)[1 + \frac{\alpha\gamma}{1 - \gamma}\tau]} \frac{\overline{M}}{P}\right)^{\beta - 1}\right]^{\frac{1}{1 + \theta(\beta - 1)}}.$$
 (A.18)

Consider now fiscal policy 3. In this case,  $\chi = 1$  and G = T. Using the government budget constraint (A.5), we can solve for aggregate wealth

$$\frac{I}{P} = \frac{1}{1 - \gamma} \frac{\overline{M}}{P}.$$
(A.19)

Since tax revenues are spent to purchase the goods produced in the economy, each producer receives her lump-sum tax back in the form of government demand for her product. Notice that, unlike the consumers, the government does not allocate any part of its budget to real money. Aggregate output and relative prices can be found as functions of the policy variables,  $\overline{M}$  and G, and the underlying parameters. Output is given by

$$Y = \frac{\gamma}{1 - \gamma} \, \overline{\frac{M}{P}} + NG. \tag{A.20}$$

The relative price for producer j is

$$\frac{P_j}{P} = \left\{ \frac{\theta d}{\theta - 1} \left[ \frac{\gamma}{N(1 - \gamma)} \left( \frac{\overline{M}}{P} \right) + G \right]^{\beta - 1} \right\}^{\frac{1}{1 + \theta(\beta - 1)}}.$$
(A.21)

Let d, the disutility of labor, be stochastic. Suppose the fraction  $0 < \mu < 1$  of prices must be set at the beginning of the period, before the shock and the policy variables are chosen; we denote by  $\bar{P}_j$  the price of the *j*-th good when set in advance. The fraction  $1 - \mu$  of prices, on the other hand, can adjust instantaneously; these prices are simply denoted by  $P_j$ . This assumption is a short cut for modelling nominal rigidities in the economy, but it captures the fact that some prices are not flexible. Under this assumption, the fraction  $\mu$  of producers must set their prices rationally in advance without knowing the realization of  $d, \bar{M}$  and the fiscal variable  $\tau$ .

Let d be lognormally distributed with mean  $E\delta$  and variance  $s_d^2$ ; hence,  $\pi = \log P, m = \log \overline{M}, y = \log Y, \tau$  are conditionally normally distributed (and this follows from the assumption that d is lognormally distributed), with means  $E\pi, Em, Ey, E\tau$ . Then, producer j's expected indirect utility maximization when  $P, \overline{M}$  and  $\tau$  are not known becomes

$$\bar{P}_{j}^{1+\theta(\beta-1)} = \frac{\theta}{\theta-1} \exp\left\{E\left[\log d + \theta\beta\log P + \beta\log W\right] + 1/2Var\left[\log(dP^{\theta\beta}W^{\beta})\right]\right\}$$
(A.22)

\* exp 
$$\left\{ E \left[ \log(1-\tau) + (\theta-1)\log P + \log W \right] + 1/2Var \left[ \log(P^{\theta-1}(1-\tau)W) \right] \right\}^{-1}$$
,

where  $W \equiv \gamma I/(NP)$ . We make use of (12) and of the approximation that  $\log(1+x) = x$ , for x small; let  $\bar{\pi}_j = \log \bar{P}_j$  and  $\pi_j = \log P_j$ . Taking logarithms of the above expression one obtains

$$\bar{\pi}_j = hE\pi + (1-h)Em - lE\tau + qE\delta + k, \qquad (A.23)$$

where

$$h = \frac{1 + (\theta - 1)(\beta - 1)}{1 + \theta(\beta - 1)} \in (0, 1), \quad c = \frac{1 + (\beta - 1)\gamma/(1 - \gamma)}{1 + \theta(\beta - 1)} > 0,$$
$$l = \frac{1}{1 + \theta(\beta - 1)} > 0,$$
$$k = \frac{1}{1 + \theta(\beta - 1)} \left[ \log \frac{\theta}{\theta - 1} + (\beta - 1)(\log \frac{\gamma}{1 - \gamma} - \log N) + \chi_0 - \chi_1 \right],$$

where  $\chi_0, \chi_1$  are the variances of the term at the numerator and denominator of (A.22). Taking logarithms of (A.12), one obtains

$$y = \log \frac{\gamma}{1 - \gamma} - \frac{\gamma}{1 - \gamma} \tau + m - p.$$
 (A.24)

Linearization of (A.3) implies

$$\pi = \mu \bar{\pi}_j + (1 - \mu) \pi_j \tag{A.25}$$

to be completed

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