Problem Set #3 (Revised)

Economics 240A, Second Half
Fall 2009

Due November 30

Questions from Goldberger's text:
Chapter 23: Exercise 23.2
Chapter 24 (optional): Exercise 24.1
Chapter 25 (optional): Exercise 25.1

Questions from Previous Midterm: This was a 25 point exam; students had 80 minutes to complete it. All subsections had equal weight (and each will have the same weight as an exercise from Goldberger's text for this problem set). This was a closed book exam, but one sheet of notes was permitted. All needed statistical tables were appended.

1.   **True/False/Explain** (10 points): For the three statements below, determine whether it is correct, and, if correct, explain why. If not, state precisely why it is incorrect and give a modification which is correct. (In the exam, only two questions were to be answered; if all three were answered, only the **lowest** two scores were counted toward the exam total.)

(a) For the partitioned model $y = X_1\beta_1 + X_2\beta_2 + \varepsilon$, if the expectation of ε is nonzero, but depends linearly on X_1 only (i.e., $E(\varepsilon) = X_1\gamma$ for some ε), and if the classical least squares coefficient estimators b_1 and b_1 of the regression of y on X_1 and X_2 are calculated (assuming X is nonstochastic with full column rank), then the estimator b_2 of β_2 is unbiased.

(b) Suppose the LS regression of y on X (which has the "summer vector" l as the first column of X) gives the residual vector $e = y - Xb$ and the coefficient of determination R^2; then if e is regressed on the "demeaned" vector $y - \bar{y}l$ of observations, the slope of this regression will be R^2.

(c) In the classical regression model, since $\varepsilon \equiv y - X\beta$ is based upon the true value of β while $e \equiv y - Xb$ uses the estimated b, it follows that $E(e'e) > E(\varepsilon'\varepsilon)$.

2.   (5 points) A researcher wants to fit the linear model

$$E(y_i) = \beta_1 + \beta_2 X_{i1} + \beta_3 X_{i3}$$

by least squares, but lacks the capability of inverting a 3×3 matrix, and doesn't know about taking deviations from means. Instead, he computes simple bivariate regression estimates for each pair of variables, obtaining the following least squares fits:

$$\hat{y}_i = \text{constant}_1 + 2X_{i2}, \qquad \hat{y}_i = \text{constant}_2 + 0.5X_{i3},$$
$$\hat{X}_{i2} = \text{constant}_3 + 0.3X_{i3}, \qquad \hat{X}_{i2} = \text{constant}_4 + 0.625y_i,$$
$$\hat{X}_{i3} = \text{constant}_5 + 3X_{i2}, \qquad \hat{X}_{i3} = \text{constant}_6 + 0.25y_i.$$

Use these simple regression results to obtain the least squares estimators b_2 and b_3 of the slope coefficients β_2 and β_3 in the "long regression" of y on a constant term (the "summer vector"), X_2, and X_3.

3.   (10 points) A microeconomist fits a Cobb-Douglas production function of the form

$$\ln(Q_i) = \beta_1 + \beta_2 \ln(L_i) + \beta_3 \ln(M_i) + \varepsilon_i$$

using $N = 25$ observations on Q_i (output), L_i (labor input), and M_i (materials input) for firms in a particular industry. The slope coefficients β_2 and β_3 are the objects of interest; their least-squares estimates and estimated variance-covariance matrix are

$$\begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0.3 \\ -0.3 \end{bmatrix}, \qquad \hat{V}\left(\begin{bmatrix} b_2 \\ b_3 \end{bmatrix}\right) = \begin{bmatrix} 0.40 & -0.10 \\ -0.10 & 0.05 \end{bmatrix}.$$

The maintained hypotheses of the classical normal regression model are assumed to be valid here, and the estimates were calculated under those assumptions.

(a) Construct a 95% confidence interval for $\theta = \beta_2 + \beta_3$. Does the point $\theta_0 = 1$ (corresponding to constant returns to scale in production) fall in this interval?

(b) Previous studies of this industry suggest that constant returns to scale does indeed hold, and that the labor coefficient is four times the value of the coefficient on materials: that is, $\beta_2 + \beta_3 = 1$ and $\beta_2 = 4\beta_3$. Taking these two linear restrictions as a joint null hypothesis, can they be rejected at a 5% significance level?